SHORTER CONTRIBUTIONS

Note on the Numerical Integration of the Equations of Meteorological Dynamics

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The work of CHARNEY (1949 a, 1949 b and 1950) and his collaborators on the numerical integration of the equations of meteorological dynamics is of considerable theoretical interest and of potential practical importance. In view of the promising nature of Charney's earlier results an independent examination has been carried out in the Forecasting Research Division of the Meteorological Office at Dunstable. The purpose of the present note is to report the results of some calculations performed along the lines of those of CHARNEY (1949 a, 1949 b) and also to indicate the results of experience which we have gained from some small-scale computational experiments with a baroclinic model used by SUTCLIFFE (1947) and now being studied by CHARNEY (1950).

The one-dimensional method of computing the change in the 500 mb height profile given by CHARNEY (1949 a) has been applied to various latitudes on a total of about 60 charts. Significant success was achieved; the correlation coefficient between predicted and observed height changes was + 0.54 along latitude 50° N and + 0.35 along latitude 40° N in a summer series; it was + 0.34 along latitude 45° N on a winter series. However the success was definitely less than that achieved by conventional forecasting methods on the same charts. On the other hand our results showed that better results were obtained from the computations than would have been achieved if it had merely been assumed that features of the pressure profile moved eastward with the zonal current. This fact gives one confidence that the one-dimensional barotropic model, despite its manifest limitations, does contain some essential features which are important in large-scale atmospheric motions, although it does not yet provide a basis for practical use in forecasting.

It appeared also from the cases studied that the failures of the predicted changes were most serious when the basic assumptions underlying the one dimensional model were not satisfied; for example, when the 500 mb contours included a closed centre, a flat area with practically no zonal flow, or a trough or ridge of large amplitude and distorted shape, and also when isotherms and contours crossed at a considerable angle. As CHARNEY himself has pointed out (1950) the assumption that isotherms and contours are parallel is probably a serious shortcoming of his earlier models. We therefore considered a model used by SUTCLIFFE (1947) and also more recently by CHARNEY (1950), in which the thermal wind has the same direction in all layers but is not necessarily parallel to the wind direction. In this latter model the equation for the motion at the equivalent barotropic level contains an additional term (see CHARNEY (1950) equation (32) which is proportional to

$$V_T \cdot \nabla \zeta_T$$

where V_T and ζ_T are the thermal wind and the vorticity of the thermal wind through a layer of the troposphere. Evaluation of this term at points along a parallel of latitude showed that its inclusion in the equation for the equivalent barotropic motion would have involved corrections to the 24-hr height changes predicted by Charney's one-dimensional method which were substantially greater than the changes actually occurring. It would probably be more appropriate to evaluate the corrections from values of $V_T \cdot \nabla\zeta_T$ averaged over a latitude band of some 20 degrees, but even so the corrections remain substantial.

We were also led to consider that the barotropic model had serious short-comings from the results of two calculations of the theoretical height tendency $\frac{\partial z}{\partial t}$ along the lines suggested by CHARNEY (1949 b). The numerical solution of his equation (45) was carried out on the assumption that the 500 mb level corresponded to the equivalent barotropic level. Although some correspondence between the predicted and actual areas of 500 mb height change was apparent the correlation between the two was very low, about + 0.4 in one example and less than + 0.1 in the other. Also root mean square errors of the forecast at a selection of points were greater than those obtained from a forecast of 'no change'. An examination of the charts showed that the absolute vorticity was not conserved in the motion at 500 mb, and although some of the discrepancies might be accounted for by errors of analysis and errors associated with computations on a finite difference grid, we were also led to regard the effect of baroclinic development as important.

Numerical calculations have also been made by us on Sutcliffe's baroclinic model of the atmosphere from a slightly different point of view, our purpose being primarily to evaluate the vertical velocities in baroclinic development. SUTCLIFFE (1947) uses the relative divergence between two levels in the troposphere as a measure of the cyclonic development. In cases where the changes of thickness of atmospheric layers are due solely to horizontal advection (i. e. adiabatic motion and neutral vertical stability) he obtains an expression for the relative divergence between two levels as follows: —

$$f\,(\mathrm{div}\ V - \mathrm{div}\ V_0) = -\,V_T \cdot \nabla\,(f + \zeta + \zeta_0) \quad (1)$$

where V and V_0 are the velocities at the upper and lower levels, ζ and ζ_0 are the corresponding vorticities and V_T is the thermal wind through the layer, f is the coriolis parameter, and divergences are evaluated in isobaric surfaces.

Fields of vertical velocity computed from equation (1) show very plausible agreement with areas which the synoptician generally regards as areas of upward motion and subsidence. The areas of substantial upward velocity as computed agree well with observed rain areas, but comparison with the amounts of rainfall suggests that the computed vertical velocity is too great.

SUMNER (1950) had modified Sutcliffe's equation slightly to take account of the effect on the thickness pattern of vertical motion in a stably strafied atmosphere. Summer gives an equation corresponding to (1) of the form

$$f(\mathrm{div}\ V - \mathrm{div}\ V_0) = -\,V_T \cdot \nabla\,(f + \zeta + \zeta_0) + \\ + \frac{R}{f}\int_{p_1}^{p_0}\left\{\Delta\left(\Gamma_p\frac{dp}{dt}\right)\right\} d\,(\log p) \quad (2)$$

where R is the gas constant and Γ_p is the rate of change of temperature with regard to pressure, $\partial T/\partial p$, in the air column minus its value in an atmosphere with a dry (or saturated) adiabatic lapse rate. Evaluations of mean vertical velocity based on the numerical solution of equation (2) give values of the vertical motion from ¼ to ½ those based on equation (1) and values of the corresponding rate of change of thickness of the 1,000—500 mb-layer accord fairly closely with the changes observed.

The success of these experiments in deriving values of the thickness tendency suggests that the tbaroclinic model of the atmosphere with the hermal wind direction the same at all levels may prove a useful basis for numerical integration. Equation (2) can be used in conjunction with an equation for the thickness change given by SUTCLIFFE and FORSDYKE (1950) to give a method of computing $\partial z'/\partial t$, the rate of change of thickness of a selected isobaric layer (conveniently chosen to include most of the troposphere), in terms of the existing distribution of z' and $\bar{z}$, the height of a selected isobaric level (conveniently chosen as Charney's equivalent barotropic level). Charney's equation for the motion at the equivalent barotropic level modified by the inclusion of the term $V_T \cdot \nabla\zeta_T$ gives $\frac{\partial \bar{z}}{\partial t}$ in terms of z' and $\bar{z}$, so that we have a closed system of equations which might possibly be adapted to numerical integration, provided suitable assumptions can be made about the lapse rate and the effect of condensation on it.

Quite clearly, from the experience of CHARNEY (1950), the work of numerical integration will be very heavy and may tax the capacity of even modern electronic methods. However it does appear to us that the baroclinic model chosen, including the effect of vertical stability is the simplest which we can hope will provide a basis for useful numerical forecasting, and it is most desirable that its potentialities should be explored.

REFERENCES

CHARNEY, J. G. and A. ELIASSEN, 1949 a: A Numerical method for predicting the perturbations of the middle latitude westerlies. *Tellus*, 1. 2. pp. 38—54.

CHARNEY, J. G., 1949 b: On a physical basis for numerical prediction of large-scale motions in the atmosphere. *Journal of Met.* 6. pp. 371—385.

CHARNEY, J. G., R. FJÖRTOFT and J. VON NEUMANN, 1950: Numerical Integration of the barotropic vorticity equation. *Tellus*, 2. 4. pp. 237—254.

SUMNER, E. J., 1950: The significance of vertical stability in synoptic development. *Quart. J. R. Met. Soc.* 76, pp. 384—392.

SUTCLIFFE, R. C. 1947: A contribution to the problem of development. *Quart. J. R. Met. Soc.* 73, pp. 370—383.

SUTCLIFFE, R. C. and A. G. FORSDYKE, 1950: The theory and use of upper air thickness patterns in forecasting. *Quart. J. R. Met. Soc.* 76, pp. 189—217.

Photographs of the "Complete Precipitation" of a Cumulus Cloud

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It is well known that in higher latitudes the lower parts of cumulonimbus may precipitate completely leaving only the anvil. It is also well known (KOTSCH, 1947) that in tropical latitudes precipitation occurs from clouds which have not reached the freezing level. Finally, it is known, at least to most tropical meteorologists, that quite commonly in the tropics a cumulus cloud will precipitate completely.

PALMER and ELLSAESSER (1949) describe this last phenomenon as follows: "In low latitudes the entire cloud may precipitate and disappear, and in the last stages you observe precipitation falling from a clear sky. ... the cloud begins to show evidence of shear, and precipitation usually begins. The top part then breaks away and the bottom dissolves. In the final stage the falling precipitation from the top portion of the cloud is white and fibrous and is sometimes mistaken for cirrus (Figure 3)." I have reproduced this figure 3 as my figure 1. In a recent article CRADDOCK (1949) has presented several photographs of this phenomenon. In addition the reader may find other published examples in DEPPERMANN's (1937) collection of cloud photographs, although Depperman does not specifically call attention to them. Both these sources, however, have portrayed clouds dissipating inland over a coastal area. In this note we present photographs of this occurring over the sea, with its horizontally uniform properties as a source of heating and of moisture.

During the summer of 1950 there occurred Operation MIDPAC—a geophysical research cruise jointly sponsored by the U.S. Navy, through its Office of Naval Research, and the University of California, through the Scripps Institute of Oceanography and the Institute of Geophysics of the University of California. The author was a member of the meteorological group aboard the Scripps' vessel *Horizon*. This group, headed by D.S. Johnson of the Department of Meteorology, UCLA, was the meteorological component of MIDPAC. It was during this cruise that the photographs, which are the reason for this note, were taken.

On 20 August 1950 at approximately 0800 local time the color photographs of figure 3, reproduced

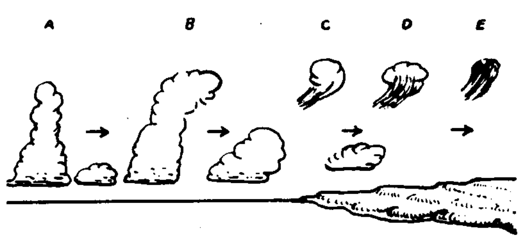

Fig. 1. The complete precipitation of cumulus (after Palmer and Ellsaesser).

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