A Note on the Kinetic Energy Balance of the Zonal Wind Systems

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Broadly speaking, there are two processes that may produce the needed transport of angular momentum from the zones of surface easterlies to those of surface westerlies. One is a large-scale mean convective process, with ascending motions in lower latitudes and descending motions in higher latitudes. The other is the large-scale eddy motions. Although some types of mean meridional circulations might be able to account for the necessary transport, the existence of such circulations is hard to establish observationally. On the other hand, the investigations conducted at the Massachusetts Institute of Technology and at the University of California have demonstrated that the eddy process not only produces enough angular momentum transfer but also produces sufficient transfer of energy to maintain the atmosphere in radiational equilibrium. This suggests that the general circulation is maintained mainly by the large-scale eddy process.

However, before any definite conclusion is drawn, one must also investigate the relative efficiency of the two processes as regards the kinetic energy balance of the mean zonal flow. For this purpose each velocity component may be decomposed into a mean value, defined as the average along a latitude circle, and a deviation from this mean so that $u = \bar{u} + u'$, $v = \bar{v} + v'$, $w = \bar{w} + w'$, where u, v, w are respectively the eastward, northward and upward components, and u', v', w' are the eddy components. In spherical coordinates the zonal momentum equation may be written

$$\frac{\partial \varrho u}{\partial t} = -\frac{\partial(\varrho u v \cos^2 \Phi)}{a \cos^2 \Phi\, \partial \Phi} - \frac{\partial(\varrho u w)}{\partial z} + f \varrho v - \frac{\partial(p + \varrho u^2)}{a \cos \Phi\, \partial \lambda}$$

where Φ is latitude, λ longitude, z elevation, f the coriolis parameter, a earth's radius, ϱ density, p pressure and t time. The effect of the vertical component of the deflecting force is small and is neglected. This equation may be multiplied by the mean zonal velocity $\bar{u}$ and integrated over the volume V bounded by two complete latitude circles and over the entire column of air in the vertical. For this particular problem the effect of the variation of density along latitude circles and with time is small so that $\int \bar{u} \frac{\partial \varrho u}{\partial t}\, d\lambda = \pi \partial \overline{\varrho u^2} / \partial t$. It follows that

$$\frac{\partial E_x}{\partial t} = -2 \pi a^2 \iint \left[\bar{\dot{\lambda}} \frac{\partial(\overline{\varrho u v} \cos^2 \Phi)}{\partial \Phi} + \bar{u} \cos \Phi \frac{\partial(\overline{\varrho u w})}{\partial z} - f \bar{u}\, \overline{\varrho v} \cos \Phi \right] d\Phi\, dz =$$

$$= \int \left[\overline{\varrho u' v'} \cos \Phi \frac{\partial \bar{\dot{\lambda}}}{\partial \Phi} + \overline{\varrho u' w'} \frac{\partial \bar{u}}{\partial z} \right] dV +$$

$$+ \int \left[\overline{\varrho w}\, \bar{u} \frac{\partial \bar{u}}{\partial z} + \left(f + \cos \Phi \frac{\partial \bar{\dot{\lambda}}}{\partial \Phi} \right) \overline{\varrho v}\, \bar{u} \right] dV +$$

$$+ \int \overline{\varrho u v_n} \cdot \bar{u}\, ds_1 + \int (\overline{\varrho u w})_0\, \bar{u}_0\, ds_0$$

where $E_x = \frac{1}{2} \int \overline{\varrho} \overline{u}^2 dV$ is the total kinetic energy of the mean zonal motion, $\overline{\dot{\lambda}}$ is the mean zonal current in angular measure ds_1 and ds_0 are the area elements of the lateral walls and bottom respectively, v_n is inward meridional velocity and the limits of integration are as stated. This equation is of basic importance for the study of the general circulation. It is the writer's desire to call attention to the integral requirement which it expresses. It may be noted that this equation is similar to the Reynold's energy equation for the mean motion, except for the term involving f due to earth's rotation. However, it is derived without the assumptions of incompressible motion and the vanishing of the velocities along the boundaries. The pressure force does not appear in this equation. According to this equation, the kinetic energy of the mean zonal flow can change not only by the presence of mean meridional circulations, as represented by the terms containing $\overline{\varrho v}$, $\overline{\varrho w}$, but also by a *conversion of the eddy kinetic energy into that of mean zonal flow*. This rate of conversion is given by the products of the eddy stresses $\overline{\varrho u'v'}$ and $\overline{\varrho u'w'}$ into the horizontal and vertical shears of the mean zonal wind. Because the vertical velocity w generally differs from zero at the level where surface wind is measured, we also retained the last term, where the subscript zero denotes surface values. This term may be interpreted as representing the dissipation of the kinetic energy of mean zonal flow through ground friction. The dissipation of mean zonal kinetic energy directly through molecular viscosity in the free atmosphere is too small and is neglected. The equation obtained may also be looked upon as a continuity equation expressing the balance of mean zonal kinetic energy for the atmosphere. Thus the volume integrals can be looked upon as sources (or sinks) and the surface integrals as net inward transports of this energy across the boundaries.

To the extent that data are available it is possible to examine whether the eddy process can provide sufficient kinetic energy to account for the frictional loss. This is given by the first two terms containing $\overline{\varrho u'v'}$ and $\overline{\varrho u'w'}$ in the last member of the equation. These terms as they stand include the effects of small scale eddies as well as of large scale. It is probable that the small scale eddies lead to dissipation so that only large eddies need be considered.

To estimate the order of magnitude of the first term arising from the large eddies, we may make use of the values of $\overline{\varrho u'v'}$ obtained by STARR (1951) over the American continent. Although these values may not be very representative for the hemisphere, the order of magnitude and the trend of the meridional distribution are probably correct. In Starr's data, the meridional shear of the zonal wind, $\partial\overline{\dot{\lambda}}/\partial\Phi$ varies very little with height above 700 mb, therefore one may compute this rate of conversion by using the integrated values of the angular momentum transfer $\tau = 2\pi a^2 \cos^2\Phi \int \overline{\varrho u'v'} dz$ (actually up to 100 mb level), and take the meridional wind shear at the 500 mb level as the mean wind shear. Such values are given in table 1.

Integrating from 25° N to 70° N, the result is that $\int \tau\, \partial\overline{\dot{\lambda}}/\partial\Phi\, d\Phi \sim 125 \times 10^{19}$ gm cm² sec⁻³. There is also a net influx of about 100×10^{19} units through the latitude walls. A zero value of τ at around 10° N is suggested by an as yet unfinished study. Integration from 10° N to 70° N gives a total of about 200×10^{19} gm cm² sec^{-3}. Each of these computations gives an average rate of conversion of about 10^{-4} watts per cm².

As for the second term, we still lack any reliable determinations. However, since almost all the large disturbances in middle latitudes have their axes inclined toward west with increasing elevation (one reason for this inclination is the thermal asymmetry), the correlation $\overline{u'w'}$ is mostly negative in middle latitudes. This conclusion has been confirmed by a few indirect computations by WHITE (1950). In low latitudes the correlation $\overline{u'w'}$ is probably positive. Since $\partial\overline{u}/\partial z$ is either negative or small there, the total contribution of the second term from the large eddies is probably negative. Thus this

Table 1

Values of τ in units of 10^{25} gm cm² sec^{-2} and of $\partial\overline{\dot{\lambda}}/\partial\Phi$ in 10^{-5} sec^{-1}

Lat. (deg. N)	10°	25°	35°	45°	55°	63°	76°
τ	(0)	+ 56.0	+ 41.5	+ 8.40	— 6.20	— 2.00	+ 0.60
$\partial\overline{\dot{\lambda}}/\partial\Phi$	+ 0.63	+ 1.2	+ 0.57	+ 0.17	— 0.23	— 2.10	— 2.80

term may be combined with the last term and the parts of the first term arising from small eddies and looked upon as constituting the total dissipation through skin and turbulent friction.

According to BRUNT (1941) and others, the rate of dissipation of the total kinetic energy in the surface layer and in the free atmosphere is about 5×10^{-4} watts per cm². Since less than one-third of this total dissipation can be attributed to the loss of the kinetic energy of the mean zonal flow, and since the use of long period averages of $\overline{u'v'}$ and $\partial\bar{\bar{\lambda}}/\partial\Phi$ gives an underestimation, it seems that the rate of conversion obtained is of the right order of magnitude and is sufficient to account for the frictional loss.

This seems to suggest that the eddy process in the atmosphere is a self consistent and self sufficient mechanism in maintaining the zonal wind systems. One or two consequences of the energy equation may be noted. Although this equation itself does not depend upon the scale of the motion, as already mentioned, the properties of the eddy stresses may be quite different for the small scale turbulent motions and for the large-scale disturbances. In the ordinary turbulence regime, the momentum flux is generally in the direction of the gradient of the mean velocity so that a virtual coefficient of viscosity K can be defined such that $\overline{\varrho u'v'} = - K\varrho\,\partial\bar{u}/\partial y$, where y is in the direction of increasing $\bar{u}$. However, this is not necessarily true for large-scale eddies. When the flow is stable and the disturbances are sustained by some other processes, the reverse may happen and the disturbances may add their momentum and energy to the main flow. The values of $\overline{\varrho u'v'}$ and $\partial\bar{\bar{\lambda}}/\partial\Phi$ cited above show that this is really the case for the large-scale disturbances in the atmosphere. This is not only true for momentum, but also true for other properties. Thus even when the absolute vorticity of an element of air is nearly conserved during its motion, it does not necessarily follow that vorticity will be transported in the direction of the gradient; rather the reverse is what should be expected when the flow is relatively stable for horizontal disturbances. Only when the flow is quite unstable so that disturbances develop at the expense of the mean flow and a thorough mixing is produced, the momentum and vorticity will be transported along the gradient. One must not therefore use the concept of virtual viscosity in discussing the effect of the large-scale eddy motions in the atmosphere (unless the possibility of a negative viscosity coefficient is admitted).

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