Possible Heavy Turbulent Exchange between the Extratropical Tropospheric Air and the Polar Stratospheric Air

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Abstract

In 1941 the present author pointed out that there exists a criterion for the development of heavy (horizontal) turbulence in zonal motion which deviates widely from th ewellknown criterion of dynamic stability. If the anticyclonic wind shear is greater than the critical anticyclonic shear given by the condition of zero absolute vorticity, the zonal motion is dynamically unstable; and if the wind shear is greater than a proper critical cyclonic shear, heavy dynamic turbulence results. The wind shear reaches this critical cyclonic value in a narrow zone just north of the latitude of the jet stream. Then, there must be a zone of heavy turbulent exchange of air masses between the extratropical tropospheric air and the stratospheric air through the slit between the extratropical and polar tropopauses.

Through investigations by SOLBERG (1939,) KLEINSCHMIDT (1941), VAN MIEGHEM (1946 a, b, 1948) and others, the question of the stability of zonal motion has been introduced in meteorology. In the case of stability the absolute angular momentum of the zonal flow must decrease with increasing latitude, and in the case of instability the absolute angular momentum must increase with increasing latitude. In the case of transition to instability the absolute vorticity vanishes. According this criterion the critical value of shear is given by

$$\frac{\partial u}{\partial y} = f + \frac{u}{R} \tan \Phi, \qquad (1)$$

where y is the meridional coordinate (positive to the north), u is the zonal (gradient) wind, Φ the latitude, $f = 2\,\omega \sin \Phi$ the Coriolis parameter, ω the angular velocity of the earth's rotation and R the radius of the earth. If the anticyclonic shear is greater than the value given by the formula (1), dynamic instability results; if the shear is smaller, the motion is dynamically stable (fig. 1).

As early as in 1941, the present author published three papers (1941 a, b, c) containing the same criterion as the one given by SOLBERG, KLEINSCHMIDT, VAN MIEGHEM and others. Moreover he also pointed out the existence of another criterion for turbulent motion in zonal flow which deviates widely from the dynamic instability criterion well known in recent meteorological literature.

According to this theory, the critical cyclonic shear is given by

$$-\frac{\partial u}{\partial y} = \omega \sin \Phi + \frac{2\,u}{R} \tan \Phi, \qquad (2)$$

where the symbols have the same meaning as in Eq. (1). If the shear is cyclonic $\left(-\frac{\partial u}{\partial y} > 0\right)$ and greater than the value given by (2), heavy dynamic turbulence results. Formula (1) gives the critical anticyclonic shear, and if the wind shear is greater than the value given by (1), the motion is dynamically unstable. Formula (2) gives the critical cyclonic shear, and if the wind shear is less than the value

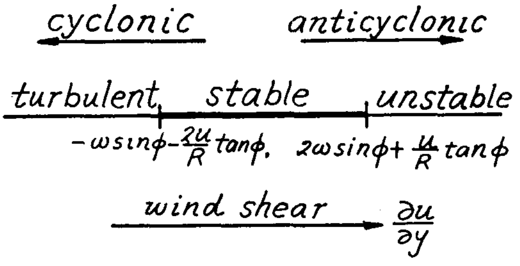

Fig. 1. Graphical representation of the turbulent, stable and unstable zones as function of wind shear.

given by (2), dynamical heavy turbulent motion results (fig. 1). Assuming a west wind of 80 m sec^{-1} at latitude 45° N, the critical value given by (1) is $(10+1) \times 10^{-5}$ sec^{-1} corresponding to a shear of 11 m sec^{-1} (100 km)$^{-1}$; while the critical value given by (2) is $(5 + 2.5) \times 10^{-5}$ sec^{-1} corresponding to a shear of 7.5 m sec^{-1} (100 km)$^{-1}$. According to PALMÉN (1948) and PALMÉN, NAGLER (1948) the anticyclonic wind shear reaches the critical value determined by formula (1) in a rather narrow zone (of a width not in excess of 300 km) just south of the latitude of the "jet stream". Furthermore, the cyclonic shear reaches the critical value determined by formula (2) in a rather narrow zone just north of the latitude of the "jet stream".

Assuming the flow to be zonal, the absolute angular momentum of the westerlies, Ω, is composed of the angular momentum of the earth at a given latitude, Ω_e, and the relative angular momentum Ω_u of the westerly flow. Thus

$$\Omega = \Omega_e + \Omega_u, \tag{3}$$

where

$$\Omega_e = \omega R^2 \cos^2 \Phi, \quad \Omega_u = R\, u \cos \Phi. \tag{4}$$

The present author has discussed the turbulent motion of zonal flow under the assumption that the absolute angular momentum of an individual parcel is conserved while it travels cross-stream. This absolute angular momentum Ω may be a function of the Φ-direction normal to the mean zonal flow. Assuming as a working hypothesis that the parcel, as a result of the turbulent motion, is displaced in the Φ-direction a certain distance $l = R\, d\Phi$ before it loses its individuality through mixing with its new environment. The distance l is called the mixing length after PRANDTL. When a parcel of air is displaced from a position $\Phi - d\Phi$ to Φ, the wind speed $u^\star$ of the moving element in its new position Φ is then given by

$$\begin{aligned} \omega R^2 \cos^2 \Phi + R u^\star(\Phi) \cos \Phi &= \\ = \omega R^2 \cos^2 (\Phi - d\Phi) &+ \\ + R u(\Phi - d\Phi) \cos (\Phi - d\Phi), \end{aligned}$$

or approximately,

$$\begin{aligned} u^\star(\Phi) - u(\Phi) &= \\ &= \left(2\,\omega R \sin\Phi + u \tan \Phi - \frac{\partial u}{\partial \Phi}\right) d\Phi. \end{aligned}$$

The wind speed of the surroundings is $u(\Phi)$. The difference between the wind speed of the displaced air parcel and the wind speed of the surroundings is given by

$$u' = l\left(-\frac{\partial u}{R\,\partial \Phi} + \frac{u}{R} \tan\Phi + 2\omega \sin\Phi\right), \tag{5}$$

or

$$u' = -l\, \frac{\partial \Omega}{R^2 \cos \Phi\, \partial \Phi}. \tag{6}$$

This value is at least an approximation to the turbulent velocity component u'. Thus, whenever the angular momentum decreases with increasing latitude the relative west wind increases in a poleward moving air parcel but decreases in a parcel moving equatorward.

The centrifugal force acting on the displaced parcel per unit volume is

$$\varrho\, (u + R\omega \cos\Phi + u')^2 / R \cos\Phi,$$

approximately, while the centrifugal force on the surrounding air per unit volume is given by

$$\varrho\, (u + R\omega \cos\Phi)^2 / R \cos \Phi.$$

Stability prevails when the centrifugal force on the surrounding air is smaller than that on the displaced parcel, i.e., stability demands that for the displacement $l > 0$. If $u' > 0$ for $l > 0$, then the displaced parcel will be pushed back to its original position since the centrifugal force acting on it is larger than the centrifugal force on the surrounding air. While being forced back, the parcel of air acquires kinetic energy and therefore tends to overshoot its equilibrium position. In this manner a stable oscillation around the equilibrium position is brought about. This type of stability is called dynamic stability.

If the centrifugal force acting on the parcel were smaller than the centrifugal force acting on the environment, the particle would move further from its original position. This distribution of the zonal motion would tend to create instability. If finally the centrifugal force on the parcel were equal to that on the surrounding air, no force would be acting on a displaced air parcel; it would everywhere be in indifferent equilibrium.

The stability condition $u' > 0$ for the displacement $l > 0$ can also be written in the form

$$\frac{\partial \Omega}{\partial \Phi} < 0, \text{ or } \frac{\partial u}{R \partial \Phi} < 2\omega \sin \Phi + \frac{u}{R} \tan \Phi, \quad (7)$$

which states that stability with respect to small perturbations exist if the absolute angular momentum decreases with increasing latitude in the atmosphere. If the absolute angular momentum is constant, we have indifferent equilibrium; if the angular momentum increases with increasing latitude, instability prevails. Those results have been obtained by SOLBERG and others, as given by Eq. (1).

RICHARDSON (1920) has given a criterion to determine whether turbulence will increase or decrease. The principle on which his criterion is based is that the flow will remain turbulent if the rate of supply of energy by the eddy stresses is at least as great as the work which has to be done to maintain the turbulence against gravity or other stabilizing forces.

The mean excess of centrifugal force per unit volume which drives a displaced parcel back to its original position is half of its extreme value, or

$$\frac{\varrho}{2} \left\{ (u + R\omega \cos \Phi + u')^2 - \right.$$
$$\left. - (u + R\omega \cos \Phi)^2 \right\} / R \cos \Phi \approx$$
$$\approx \varrho \left(\omega + \frac{u}{R \cos \Phi} \right) u'.$$

When a parcel of air is displaced from a position $\Phi - d\Phi$ to Φ ($R d\Phi = l$), the work done against this excess centrifugal force is $\varrho\, l \left(\omega + \frac{u}{R \cos \Phi} \right) u' \sin \Phi$. If v' is the instantaneous normal velocity to an element of the vertical area F standing on a latitude circle, then the volume of air flowing across an element of this area per unit time is $v' dF$. The time rate of the work done on the air flowing across the vertical area F is, therefore, $\int \varrho\, l \left(\omega + \frac{u}{R \cos \Phi} \right) \times \sin \Phi\; u' v' dF$. From the stability condition (7), the product $u' v'$ is always positive. The amount of

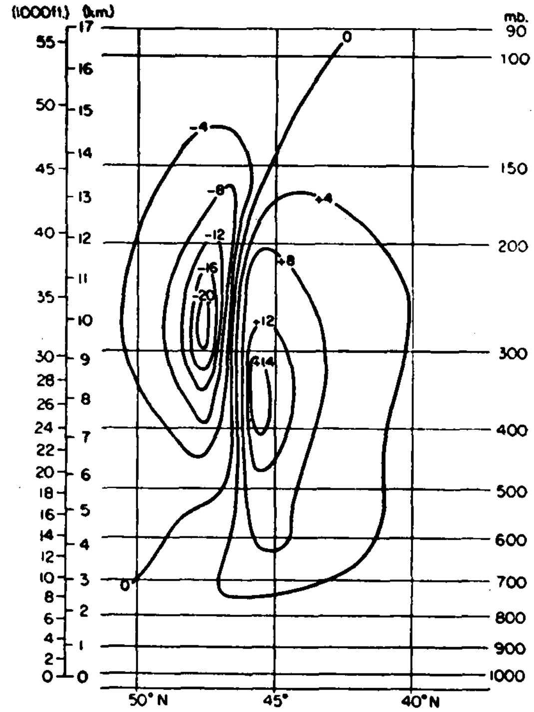

Fig. 2. Increase of the geostrophic wind computed along isentrophic surfaces. (Acc. PALMÉN and NAGLER 1948).

the work required to maintain the turbulence under stable conditions is always positive. Determining the sign in appropriate ways, the time rate of the work to be done on the air flowing across the vertical area F in order to maintain the turbulence against the centrifugal force is, therefore,

$$F \varrho\, |l| \left(\omega + \frac{u}{R \cos \Phi} \right) \sin \Phi \cdot \overline{u' v'}, \qquad (8)$$

where the bar indicates that the mean value of the product is to be taken.

The time rate of work done by the eddy stresses per unit volume is $-\varrho \overline{u' v'} \left(\frac{\partial u}{R \partial \Phi} + \frac{\tan \Phi}{R} u \right)$, where $-\varrho \overline{u' v'}$ stands for the Reynolds or eddy stress. Now consider a parallelepiped with horizontal edges $|l|$ and bounded by one pair of parallel vertical planes F. Then the time rate of work done on the air whose volume is $F \cdot |l|$ by the eddy stresses becomes

$$-\varrho F \cdot |l| \cdot \overline{u' v'} \left(\frac{\partial u}{R \partial \Phi} + \frac{\tan \Phi}{R} u \right). \qquad (9)$$

From Eqs. (8) and (9), the condition that turbulence should increase may therefore be written,

$$-\left(\frac{\partial u}{R\partial \Phi}+\frac{\tan \Phi}{R}u\right) > \left(\omega+\frac{u}{R\cos\Phi}\right)\sin\Phi.$$

This inequality may be written in another form,

$$-\frac{\partial u}{R\partial \Phi} > \omega \sin \Phi + \frac{2u}{R}\tan \Phi, \qquad (10)$$

or

$$\frac{\partial}{\partial \Phi}\left(\frac{u+R\omega\cos\Phi}{\cos^2\Phi}\right) < 0,$$

which is the relation expressed as Eq. (2). The criterion seems to be valid in this form, with reservations for a numerical factor which appears on the right-hand side of the inequality (10). It is readily seen that the quantity $\overline{u'v'}$ in Eq. (8) need not of necessity be equal to the quantity $\overline{u'v'}$ in Eq. (9), and this circumstance may give rise to a different factor on the right hand side of the above inequality.

PALMÉN (1948) and PALMÉN, NAGLER (1948) showed that the strongest wind or jet stream can regularly be observed just south the latitude where the tropopause is split in two, i.e., the extratropical tropopause and the polar tropopause (fig. 2). The wind shear reaches the critical value determined by formula (2) in a rather narrow zone just north of the latitude of the jet stream. Then there must be a zone of heavy turbulent exchange of air masses between the extratropical tropospheric air and the polar stratospheric air. It might be emphasized that the turbulent exchange between the extratropical tropospheric air and the polar stratospheric air through the slit of two tropopauses will be an important problem in the general circulation of the atmosphere.

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