The Meridional Flux of Sensible Heat over the Northern Hemisphere

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Abstract

The meridional eddy flux of sensible heat is examined by means of finite difference integration methods from northern hemisphere charts for the winter 1945—46. The eddy sensible heat flux is shown to occur principally in the lowest layers of the atmosphere and to decrease with elevation. The magnitude of the eddy sensible heat transports are compared with the total poleward energy flux required by the radiation balance, and are shown to comprise a very large fraction of this flux.

Introduction

The difference in the net heating of equatorial and polar latitudes necessitates a meridional transport of total energy directed from equatorial to polar regions. This transport may be expressed to a high degree of accuracy as

$$T^{\star} = h + \int_s (c_p \tau + ql + \Phi)\, \varrho v ds \qquad (1)$$

where h is the contribution to the poleward energy flux by the hydrosphere, c_p is the specific heat of air at constant pressure, τ is the temperature, q is the specific humidity, l is the latent heat of water vapor, Φ is the geopotential energy per unit mass, ϱ is the density, v is the northward component of the wind velocity and ds is an element of the conical surface above a given latitude. The integration is to be performed over the entire conical surface. Equation (1) may be written down directly from considerations of the conservation of energy. It may also be derived from the general physical energy equation and the equations of motion as shown by STARR and collaborators (1949).

Equation (1) simply states that the poleward flux of energy may be accomplished essentially as (a) an energy flux within the oceans, (b) a flux of sensible heat, (c) a flux of latent heat, (d) and a flux of potential energy. The term $\int_s c_p \tau \varrho v ds$ results from the combination of the flux of internal energy of dry air and the flux of kinetic energy transferred as work done by the pressure forces. These two energy forms are always transferred in the constant ratio c_v/R where R is the gas constant for dry air and c_v is the specific heat of air at constant volume. The transport of kinetic energy accomplished by the flow of existing kinetic energy has been neglected because of its smaller order of magnitude (see STARR 1949).

The partition of the poleward energy flux among the various terms in equation (1) is a problem of great importance for a better understanding of the mode of operation of the general circulation. The magnitude and variation of the flux of sensible heat across various latitudes upon a hemispherical basis will be examined below.

Computation procedure

Because of the lack of adequate observed wind data upon a hemispherical basis it

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becomes necessary to compute the meridional flux of sensible heat using geostrophic winds. Use of such a measure of the wind velocity to calculate this term introduces modifications into its meaning which are not without advantages.

The integrated flux of sensible heat across a latitude circle at a given geometrical level in the atmosphere is given by the expression

$$T_E = \oint c_p \tau (\varrho v)\, dx \tag{2}$$

where dx is an element of distance longitudinally and the integration is performed around a complete circle of latitude. If we place

$$\tau = \overline{\tau} + \tau' \tag{3}$$

$$\varrho v = \overline{\varrho v} + (\varrho v)' \tag{4}$$

where the primes denote deviations of the quantities from the mean around the latitude circle at the given level, and take the product of equations (3) and (4), then equation (2) may be written as

$$T_E = c_p \oint \overline{\tau}\, \overline{\varrho v}\, dx + c_p \oint \tau' (\varrho v)'\, dx \tag{5}$$

in which each of the terms has a distinct physical significance. The first term in the right hand member of equation (5) is the portion of the flux of sensible heat due to the non-zero value of $\overline{\varrho v}$ at the given level, i.e., the portion transported by virtue of a mass flux at the level and hence accomplished by mean meridional cells of the Hadley type.

The second term in equation (5) is that portion of the flux due to the correlation between τ and (ϱv) along the latitude circle. As such it represents the transport of sensible heat by large scale horizontal eddy processes associated with eddies of the size of migratory cyclones and anticyclones of middle latitudes.

If the terms in equation (5) are computed geostrophically, the first term must vanish and the only portion of the flux which does not vanish is that part due to the correlation between the temperature and the meridional component of the geostrophic wind. The question arises immediately as to the nature of the difference between the geostrophic eddy transport and the actual eddy transport. This point will be discussed in greater detail in a later part of this paper.

Estimates were made of the geostrophic eddy transport of sensible heat based upon complete northern hemisphere data by finite difference integration methods. The data which were used extended only up to 500 mb and were available in a mixed coordinate system. The surface data were available at the geometric height $z = 0$, whereas the upper air data were available along constant pressure surfaces.

The computation procedure is greatly simplified if the eddy flux term which depends solely on the correlation between the temperature and (ϱv) is calculated along constant pressure surfaces. The assumption which is introduced by this procedure is that the correlation between the temperature and the geostrophic mass flux along a constant level can be approximated by the correlation between the temperature and the geostrophic wind along some corresponding pressure surface. It is probable that the eddy flux computed from constant pressure charts resembles closely a similar flux calculated from constant level charts. For purposes of this investigation this condition is assumed to hold. In any case we may state that the eddy flux of sensible heat which has been calculated is that given by computations based upon constant pressure surfaces.

The data which are available for computational purposes are in the form of the sea level pressure distribution, and the height distributions of the 700 and 500 mb surfaces. In view of this it becomes more convenient to use the approximation that

$$\int_{z=0}^{z=z_1} \oint c_p \tau \varrho v_g\, dx\, dz \approx c_p/g \int_{500\,\text{mb}}^{p_0} \oint \tau v_g\, dx\, dp \tag{6}$$

where $z = 0$ is sea level, z_1 is equal to some constant height at approximately the 500 mb level, p_0 is taken as the sea level pressure and the element ds has been approximated by $dx\, dz$ where dx is an element of distance longitudially and dz is an element of distance in the vertical. Substituting the geostrophic relation for v_g we find

$$c_p/g \int_{500}^{p_0} \oint \tau v_g\, dx\, dp = c_p/f \int_{500\,\text{mb}}^{p_0} \oint \tau \frac{\partial z}{\partial x}\, dx\, dp \tag{7}$$

where f is the coriolis parameter.

Since temperature observations are not available within these layers over a wide network of stations it is necessary to substitute the mean virtual temperature of the layer in question for the actual temperature. Because the magnitude of the eddy transport across any given latitude circle depends not on the absolute value of the temperature but upon the correlation between the wind direction and the temperature and the variance of these quantities when integrated around a latitude circle, this substitution probably introduces only a slight error, the mean virtual temperature and the actual mean temperature of the layer being very highly correlated, and the variance of the mean virtual temperature being about equal to the variance of the actual mean temperature.

The sea level pressure was reduced to the height above sea level of the 1013 mb surface, assuming the NACA standard atmosphere. This reduction introduces a slight error into the calculations. We may write that the eddy transport of sensible heat evaluated up to 500 mb is composed of three terms as follows:

$$T_E = \frac{c_p}{f}\left[\int_{500\,\text{mb}}^{700\,\text{mb}} \oint \bar{\tau}_v \frac{\partial z}{\partial x} dx\, dp + \int_{700\,\text{mb}}^{1013\,\text{mb}} \oint \bar{\tau}_v \frac{\partial z}{\partial x} dx\, dp + \int_{1013\,\text{mb}}^{p_0} \oint \bar{\tau}_v \frac{\partial z}{\partial x} dx\, dp\right] \tag{8}$$

where $\bar{\tau}_v$ is the mean virtual temperature appropriate to the given layer. The integration of the first two terms with respect to pressure in the vertical is between two constant pressure surfaces and hence presents no problem in evaluation. The third term however has a variable upper limit of integration because the sea level pressure is not constant along a latitude circle. It seems highly probable that the transport given by the third term is no larger than the transport between two constant pressure surfaces separated by an amount equal to the average pressure difference between sea level and 1013 mb., i.e., by perhaps 30 mb, so that the third term is perhaps a tenth as large as the second. We may state then that in all probability the errors introduced by the conversion of the surface pressure to the height of the 1013 mb surface are minor in nature. The eddy transport of sensible heat was computed for the layers, 1013 to 700 mb, and 700 mb to 500 mb. The mean geostrophic wind for each layer defined as the mean of the geostrophic wind at the top and at the bottom of the layer in question was used. The product of the mean virtual temperature and the geostrophic wind was formed at 72 evenly spaced points at each circle of latitude and then the integration of equation (7) was accomplished by summing the 72 products so derived for each latitude circle and each layer for each day. The computations of the eddy transport of sensible heat were carried out by the General Circulation Project, M.I.T. partly by hand and partly by punch card methods.

The question may legitimately be raised as to the nature of the difference between the eddy transport of sensible heat by actual winds versus that by geostrophic winds. These eddy transports were compared over regions of North America, where sufficient observed upper wind data are available. Data for the months of January 1949 and December 1948 were selected over regions of the United States between latitudes 43° N and 50° N. Wherever possible, rawinsonde stations were selected to prevent bias toward fair weather synoptic patterns. The study was carried out only at 700 mb. The sources of the data were as follows:

(a) 700 mb actual winds and temperatures were gathered from the United States Weather Bureau Daily Upper Air Bulletin.[1]

(b) 700 mb geostrophic winds were computed from Daily United States Weather Bureau 700 mb charts. The ratio

$$\frac{\overline{\tau v_g} - \bar{\tau}\bar{v}_g}{\overline{\tau v_a} - \bar{\tau}\bar{v}_a} = \frac{\overline{\tau' v_g'}}{\overline{\tau' v_a'}} \tag{9}$$

was examined. Here τ is the temperature in degrees centigrade, v_g is the geostrophic wind velocity, v_a is the observed wind velocity. The bars denote means with respect to time and space, and the primes denote deviations

[1] The stations selected for this study were: Bismark N.D. 764; Buffalo, N.Y. 528; Caribou, Me. 712: Glasgow, Mont. 768; International Falls, Minn. 747; Lander, Wyo. 576; Rapid City, S.D. 662; St. Cloud, Minn. 644; Sault Ste. Marie, Mich. 735; Spokane, Wash. 785; Tatoosh, Wash. 798; Great Falls, Mont. 775.

Table I

The mean geostrophic eddy transport of sensible heat across various latitudes for the winter season 1945—46 for layers and periods specified. Units in cals $sec^{-1} \times 10^{+12}$

Month	Layer	Lat. 75° N	65° N	55° N	45° N	35° N
Nov. 1945	1013—700 mb	— 38.0	+ 147.0	+ 447.1	+ 371.0	+ 193.9
	700—500 mb	+ 17.9	+ 91.2	+ 189.9	+ 168,3	+ 74.3
	1013—500 mb	— 20.2	+ 238.2	+ 636.9	+ 539.3	+ 268.1
Dec. 1945	1013—700 mb	— 2.4	+ 172.9	+ 670.7	+ 781.9	+ 361.2
	700—500 mb	+ 23.3	+ 84.1	+ 221.5	+ 168.6	+ 107.6
	1013—500 mb	+ 20.9	+ 257.0	+ 892.3	+ 950.5	+ 468.8
Jan. 1946	1013—700 mb		+ 335.4	+ 745.8	+ 580.1	+ 177.0
	700—500 mb		+ 120.8	+ 198.7	+ 232.7	+ 122.5
	1013—500 mb		+ 487.1	+ 941.7	+ 812.7	+ 279.2
Feb. 1946	1013—700 mb	+ 41.1	+ 269.4	+ 683.7	+ 713.0	+ 320.0
	700—500 mb	+ 1.1	+ 60.2	+ 178.3	+ 153.7	+ 841.7
	1013—500 mb	+ 42.1	+ 329.6	+ 862.1	+ 866.8	+ 404.1
Mean	1013—700 mb	— 0.7	+ 226.4	+ 632.5	+ 612.3	+ 261.2
	700—500 mb	+ 14.5	+ 88.4	+ 197.6	+ 179,4	+ 96.6
	1013—500 mb	+ 13.7	+ 328.7	+ 834.2	+ 792.6	+ 354.6

from the means of these quantities in both time and space. The quantities $\overline{\tau' v_g'}$ and $\overline{\tau' v_a'}$ are proportional to the mean eddy flux of sensible heat over the limited length of the latitude circle. $\overline{\tau v_g}$ and $\overline{\tau v_a}$ are proportional to the total flux of sensible heat including the flux due to the correlations between τ and v and the flux due to a non-zero $\bar{v} \cdot \bar{\tau} \bar{v}_g$ and $\bar{\tau} \bar{v}_a$ represent the component of the flux due to the non-zero $\bar{v}$ over the limited length of the latitude circle and that component which we wish to subtract out.

The data were examined separately by months and also as a single sample. The total number of cases was 472. The value of this ratio for the entire sample was 0.82. Thus in this sample, the eddy transport by actual winds is about 20 per cent greater toward the north than the transport by geostrophic winds. For the individual months, this ratio was essentially constant, being 0.83 during December 1948, and 0.79 during January 1949. These results indicate that the geostrophic eddy transport of sensible heat is a good measure of the observed eddy transport of sensible heat.

Results

The mean monthly values of the geostrophic transport of sensible heat across various latitude circles are presented in Table I. One feature which is apparent is the concentration of the eddy transport of sensible heat in the layer between 1013 and 700 mb. Approximately 70 per cent of the eddy transport of sensible heat which is accomplished below 500 mb takes place in the lowest layer. This is in contrast to the horizontal meridional eddy transport of angular momentum which appears to be at a maximum in the high tropospheric layers (see STARR and WHITE 1951).

It should be pointed out that because of the irregularities of the earth's surface, and because artificially reduced sea level pressures were used, the eddy flux of sensible heat between 1013 and 700 mb includes a fictitious component. This component is probably largest in the latitudes of the Himalaya mountains where the mountain barriers are oriented in an east-west direction and extend to great heights. It is not possible with the data at hand to determine the magnitude of this fictitious component but it is felt to be of minor importance in view of the small portion of the latitude circle which is covered by such mountain ranges.

In all months the eddy transport of sensible heat reveals a maximum somewhere in middle latitudes, decreasing both poleward and equatorward. The location and intensity of the maximum eddy transport of sensible heat between 1013 and 500 mb varies from month to month. It is of interest to compare the mean monthly eddy transport with the character of the mean monthly northern hemisphere pressure distribution, Table II. However in view of the small number of months of data which have been examined, the results can only be considered as suggestive. Several factors must be taken into account in any attempt at such a comparison. The seasonal trend to stronger poleward transports from early to late winter must be considered. Thus the transport for November is somewhat smaller than those of other months.

As regards the three mid-winter months it appears that December 1945 which possesses the lowest sea-level zonal westerly index also possesses the most intense maximum (950.5×10^{12} cals sec^{-1}) which occurs at the relatively low latitude of 45° N.

Table II

The mean monthly meridional sea-level pressure profiles for the months as indicated. Units in mb

Month	Nov. 1945	Dec. 1945	Jan. 1946	Feb. 1946
Lat.				
80° N	1019.2	1022.3	1018.9	1026.9
75° N	1014.8	1019.8	1014.1	1021.2
70° N	1013.8	1019.7	1012.0	1016.9
65° N	1013.7	1020.1	1010.8	1013.5
60° N	1013.0	1018.7	1010.1	1011.3
55° N	1013.9	1016.6	1011.7	1011.0
50° N	1016.8	1014.1	1015.2	1012.9
45° N	1018.7	1012.7	1018.8	1015.7
40° N	1018.9	1014.4	1020.6	1017.8
35° N	1018.5	1016.8	1021.1	1018.7
30° N	1017.5	1018.1	1020.6	1019.1
25° N	1016.2	1018.0	1019.2	1018.4
20° N	1014.4	1016.1	1016.8	1016.7
15° N	1012.0	1013.4	1014.0	1013.7
10° N	1010.0	1011.2	1011.5	1011.2

On the other hand January 1946 has an abnormally high sea-level zonal westerly index with a comparable maximum (941.7×10^{12} cals sec^{-1}) which is displaced to a much higher latitude at 55° N. February 1946 which is a more normal month as regards the sealevel zonal westerlies has a less intense maximum (866.8×10^{12} cals sec^{-1}) located at 45° N. In February however the value of the transport across 55° N is almost equal to it in magnitude (862.1×10^{12} cals sec^{-1}) indicating that the maximum may lie somewhere between 45° N and 55° N.

It is of interest also to compare the actual magnitudes of the mean eddy transport of sensible heat between 1013 and 500 mb for the four months which have been investigated with estimates of the total energy transport which is required by the radiation balance of the earth and atmosphere. Such estimates are crude and vary within wide limits. The required yearly normal poleward total energy transport based upon the radiation data of ALBRECHT (1931) and another such preliminary estimate based upon data given by GABITES (1950) differ quite substantially but may be used as indicating possible limits of the required total energy transport. These data are given in Table III. Comparison with Table I where the mean geostrophic eddy transport of sensible heat for the four months November 1945 to February 1946 is given may be made.

Table III

The yearly normal total poleward energy flux across various latitudes required by the radiation balance to maintain the temperature difference between polar and equatorial regions based upon (a) radiation data according to Albrecht and (b) according to Gabites. Units in cals $sec^{-1} \times 10^{12}$

Lat.	(a)	(b)
30° N	+ 620	+ 903
40° N	+ 660	+ 1,044
50° N	+ 564	+ 917
60° N	+ 399	+ 670
70° N	+ 199	+ 364
80° N	+ 53	+ 105
90° N	0	0

Although rigid comparison of these two sets of figures is ruled out, because the eddy transports given in Table I have been computed for only one winter season and only up to 500 mb, and the transports in Table III are

Table IV

The mean monthly divergence of the eddy transport of sensible heat for the four months of the winter 1945—46 as indicated, in the layer from the surface to 500 mb within selected latitude belts. Unit in cals $sec^{-1} \times 10^{+12}$

Month	Lat. Belt				
	35—45° N	45—55° N	55—65° N	65—75° N	75—90° N
Nov.	+ 271.2	+ 97.6	— 398.7	— 258.4	+ 20.2
Dec.	+ 481.7	— 58.5	— 635.3	— 236.1	— 20.9
Jan.	+ 533.5	+ 129.0	— 454.6	— 487.1	← (65°—90° N)
Feb.	+ 462.7	— 4.7	— 532.5	— 287.5	— 42.1
Mean	+ 438.0	+ 41.6	— 505.5	— 315.0	— 13.7

yearly normals, some information about the efficiency of the eddy transport processes relative to other modes of meridional energy flux may be obtained because of the following considerations. The mean geostrophic eddy flux of sensible heat for the winter of 1945—46 has been computed up to 500 mb and if used as a typical yearly value will be considerably in underestimate because of the neglect of the transport above 500 mb, but on the other hand will be considerably in overestimate because only the winter season has been sampled. The extent to which these two factors tend to cancel will determine the appropriateness of the use of these mean geostrophic eddy transports of sensible heat in conjunction with the total energy transport which has been estimated on a yearly normal basis.[1]

It appears then from comparisons of Tables I and III that eddy sensible heat transport processes account for a major portion of the required total energy flux.

The relation between the mean monthly pressure profiles and the distribution of the divergence of the transport of sensible heat may be examined. The magnitude of the divergence of the eddy transport of sensble heat is proportional to the generation of kinetic energy, to the extent that horizontal eddy transport processes comprise the principal mode of transport in the atmosphere. The mean monthly divergence of the transport of sensible heat by latitude belts is given in Table IV. The divergence of the transport of sensible heat is represented by the difference between the rate of inflow of sensible heat into a given region and the rate of outflow from the same region. When a divergence of this transport exists, i.e., when there is more going out of a given latitude belt than is coming in, then there must exist a generation of kinetic energy within this belt. On the other hand a convergence of the transport of sensible heat signifies a disappearance of kinetic energy.

According to the present supposition it may be inferred from an examination of Table IV that in all months there exists a region of dissipation in more northerly latitudes. This systematic arrangement of the regions of kinetic energy generation and dissipation agrees with recent suggestions made by STARR and colloborators (1949) that a primary source of kinetic energy for the atmosphere lies in the subtropics.

[1] The extent to which the factors mentioned above actually do cancel was studied for regions of North America from rawin and rason observations which reach to great heights. It was found that in middle latitudes approximately 70 per cent of the total eddy transport of sensible heat takes place below 500 mb. On the other hand a measure of the wintertime rate of transport in middle latitudes was approximately 45 per cent greater than a measure of the yearly mean rate of transport. This study suggested that the geostrophic eddy transport of sensible heat computed during the winter months and only up to 500 mb may be a good approximation to the yearly mean. For further details of this study see WHITE (1950).

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