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SHORT CONTRIBUTION

On the numerical evaluation of the energy conversion integral

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ABSTRACT

Numerical results of the "$\alpha\omega$-" and the "$v \cdot \nabla\phi$-formulation" of the atmospheric energy conversion integral, as derived from ECMWF-data, are presented and discussed. It is shown that even global integrals of either formulation are considerably different. If the integration area is restricted to hemispheres, the differences — though growing somewhat larger — are of similar magnitude. The results suggest that numerical evaluations of the baroclinic energy conversion process are still associated with a considerable degree of uncertainty.

1. Introduction

Since the concept of available potential energy was introduced by Lorenz (1955), numerous large-scale energetics studies based on the so-called "Lorenz energy cycle" have been published. However, due to the lack of data sets with global extension, it has been impossible to verify a fundamental statement within Lorenz's concept, namely the identity of the global release of available potential energy and the global production of kinetic energy. As a consequence, the simultaneous presence of two different versions of the energy conversion integral, the "$\alpha\omega$-formulation" and the "$v \cdot \nabla\phi$-formulation" (Peixoto and Oort, 1974) caused some problems concerning the interpretation of spatial distributions or limited area integrals of the terms.

Recently, a new objective analysis set covering the whole globe has been made available by the European Centre for Medium Range Weather Forecasts (ECMWF) and hence we shall take a new approach to the problems mentioned above. It will be demonstrated that numerical results of both versions of the global energy conversion integral show rather large differences from one another. The effect of several kinds of restriction of the global integration will also be discussed. Special emphasis will be given to the possibility of interpreting the amount of the conversion integral as a measure of the intensity of baroclinic processes in the atmosphere. Only a short time-period (February 1979) will be considered, and no attempt will be made to give long-term representative values of the quantities under discussion.

2. The baroclinic energy conversion rate

It is common to speak in terms of (baroclinic) conversion of available potential energy to kinetic energy, when numerical results of $-\alpha\omega$ or $-v \cdot \nabla\phi$ are considered. The reason for this identification of adiabatic release of total potential energy and adiabatic production of kinetic energy is the identity of the global integrals of both

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terms:

$$\int \mathrm{d}p\, g^{-1} \widehat{\nabla_3 \cdot (v_3 \phi)} = 0$$

$$= \int \mathrm{d}p\, g^{-1} \widehat{v \cdot \nabla\phi} - \int \mathrm{d}p\, g^{-1} \widehat{\alpha\omega}. \tag{1}$$

From the simple relation

$$C = -\int \mathrm{d}p\, g^{-1} \widehat{\alpha\omega} = -\int \mathrm{d}p\, g^{-1} \hat{\alpha}\hat{\omega}$$

$$- \int \mathrm{d}p\, g^{-1} \widehat{[\alpha]''[\omega]''} - \int \mathrm{d}p\, g^{-1} \widehat{\alpha^* \omega^*}, \tag{2a}$$

it is apparent that the interpretation of C as baroclinic energy conversion is certainly justified, as it is related to processes involving the rise of relatively warm and the subsidence of relatively cold air. The first integral of the right-hand side vanishes as it contains the pressure-level mean of ω.

Moreover, formulation (2a) has the advantage that it offers a natural decomposition of C into contributions of the mean meridional circulation and the eddies. Each contribution has its identical counterpart in the equivalent relation within the $v \cdot \nabla\phi$-formulation:

$$C = -\int \mathrm{d}p\, g^{-1} \widehat{v \cdot \nabla\phi}$$

$$= -\int \mathrm{d}p\, g^{-1} \widehat{[v] \cdot [\nabla\phi]} - \int \mathrm{d}p\, g^{-1} \widehat{v^* \nabla\phi^*}. \tag{2b}$$

In view of the following discussion, we note that the $v \cdot \nabla\phi$-term can be further expanded, treating the contribution of the rotational wind component v_R and the divergent one v_D separately:

$$C = -\int \mathrm{d}p\, g^{-1} \widehat{v \cdot \nabla\phi}$$

$$= -\int \mathrm{d}p\, g^{-1} \widehat{v_R \cdot \nabla\phi} - \int \mathrm{d}p\, g^{-1} \widehat{v_D \cdot \nabla\phi}. \tag{3}$$

The first integral on the right-hand side vanishes and therefore does not contribute anything to the global baroclinic conversion, which is determined by the divergent wind alone.

Unfortunately, the designation "baroclinic energy conversion" of the terms in eq. (1) is sometimes retained, if the integration is restricted to parts of the atmosphere or even if spatial distributions of the integrands are considered. Thereby the following (quantitative and interpretative) inconsistencies are introduced.

(1) Any restriction of the global integration domain results in non-vanishing values of the left-hand integral in eq. (1). These may be interpreted as boundary fluxes of potential energy ($v \cdot \nabla\phi$-formulation) or pressure work terms ($\alpha\omega$-formulation), according to Oort and Peixoto (1974).

(2) The mean vertical velocity over a non-closed pressure level does not vanish either. Resulting contributions of $-\hat{\alpha}\hat{\omega}$ have no significance with respect to baroclinic conversion and must be eliminated. Other problems concerning the reference state for open regions are discussed by Johnson (1970) and Robertson (1985).

(3) Considering eq. (3), $v_R \cdot \nabla\phi$ does not integrate to zero over any limited atmospheric volume. This term represents local sources of kinetic energy due to purely barotropic processes, which have no counterpart in the budget of available potential energy. However, even $-v_D \cdot \nabla\phi$ does not describe a purely baroclinic (local) conversion either, because of the non-vanishing boundary fluxes of geopotential.

A refined method of gaining more detailed information about the baroclinic conversion process is the expansion of the eddy conversion integral into components of discrete zonal wave numbers. The corresponding analytical framework was provided by Saltzman (1957). Theory requires the identity of each spectral component of the integrals over $v^* \cdot \nabla\phi^*$ and $\alpha^* \omega^*$ as long as there is no restriction of the integration domain in the φ- and p-directions.

3. Discussion of the numerical results

The aim of this paper is to demonstrate which extent of disagreement is to be expected, if the results of energy conversion computations based on $\alpha\omega$- or $v \cdot \nabla\phi$-formulations are compared. For the terms which should agree theoretically, an impression of the amount of the differences related to numerical or data problems can be gained. Knowing these "implicit" differences is fundamental for the interpretation of those terms which disagree on principle.

3.1. Basic data

Our calculations are based on initialized FGGE-level IIIb-analyses of ECMWF for an arbitrarily chosen period of time (February 1979). The data are described by Bengtsson et al. (1982). Zonal means of $\alpha^* \omega^*$ and $v^* \cdot \nabla\phi^*$ are computed as spectral sums over the wavenumber range $1 \leqslant n \leqslant 35$. A conventional trapezoidal integration scheme over 15 pressure levels between 1000 mb and 10 mb was used for the vertical integration. Hence, no values had to be assumed for $p = p_0$ and $p = 0$.

3.2. *Global means*

Global integrals of $[v]\cdot[\nabla\phi]$ and $[\alpha]''[\omega]''$ as well as those of $v^*\cdot\nabla\phi^*$ and $\alpha^*\omega^*$ are presented in Table 1 and Table 2, respectively.

The contribution of the mean meridional circulation to the total conversion shows, as usual, negative values throughout. Sometimes, considerable differences can be noticed, but due to compensation effects, the monthly mean yields similar values for either formulation. The disagreement grows more serious for the eddy energy conversion rate. The development in time is similar but the absolute values differ by large amounts. For the monthly means, a difference of 0.3 W/m^2 is yielded, which is larger than the standard deviation of either time series.

Table 1. *Vertically integrated global averages of* $-[v]\cdot[\nabla\phi]$ *and* $-[\alpha]''[\omega]''$ *for February 1979 (00 GMT for each day). Monthly mean and corresponding standard deviation are included. Units: Wm*$^{-2}$

February 1979	$-[v]\cdot[\nabla\phi]$	$-[\alpha]''[\omega]''$
1	0.02	−0.07
2	−0.46	−0.47
3	−0.53	−0.48
4	−0.35	−0.37
5	−0.45	−0.43
6	−0.24	−0.26
7	−0.28	−0.35
8	−0.10	−0.23
9	−0.17	−0.26
10	−0.22	−0.27
11	−0.26	−0.31
12	−0.43	−0.49
13	−0.43	−0.48
14	−0.40	−0.45
15	−0.23	−0.36
16	−0.52	−0.57
17	−0.52	−0.56
18	−0.43	−0.48
19	−0.20	−0.32
20	−0.25	−0.36
21	−0.27	−0.29
22	−0.23	−0.16
23	−0.46	−0.38
24	−0.43	−0.34
25	−0.40	−0.39
26	−0.27	−0.27
27	−0.22	−0.24
28	−0.34	−0.36
∅	−0.32	−0.36
σ	0.13	0.12

Further it can be seen from Table 2 that the differences are not confined to a certain wavenumber range. A remarkably large part of the total difference is already inherent in the group of planetary waves. Thus we conclude that problems of data resolution, e.g., the influence of the ECMWF data assimilation scheme on the vertical velocity field, should not be the main reason for the smaller values arising from use of the $\alpha\omega$-formulation. Vertical eddy geopotential fluxes over 1000 mb or 10 mb cannot be blamed either, as the amount and the variability of ECMWF's initialized ω-fields on these levels are known to be very weak. More likely, the mass balance in the lower troposphere is spoiled by the interpolation of the data fields from model levels to pressure levels. Sometimes this "interpolation" includes extrapolation below ground (Arpe et al., 1986). Such a procedure should cause a stronger influence on the results arising from the $v\cdot\nabla\phi$-formulation, which receive larger contributions from the lower levels (see Kung and Tanaka, 1983). Hence, without more careful treatment of the lower boundary conditions, the $\alpha\omega$-formulation may be the more reliable one. We recall that Kung and Tanaka (1983), who approached the problem otherwise, draw a different conclusion and seem to have little confidence in the ECMWF's vertical velocity fields. They were, however, not aware of the differences between $v^*\cdot\nabla\phi^*$- and $\alpha^*\omega^*$-integrals. A far more detailed investigation of the spatial structure of the fields is required to explain this point conclusively.

3.3. *Hemispheric means*

A comparison of hemispheric means is reasonable only for the eddy contribution of the energy conversion. The total conversion integral would now contain large contributions due to the hemispheric mean of ω, while consideration of the mean meridional circulation makes little sense as the Hadley cell is cut off at a random position.

Hemispheric means of $-\alpha^*\omega^*$ and $-v\cdot\nabla\phi^*$ are listed in Table 3. For the monthly mean

Table 2. *Vertically integrated global averages of* $-v^*\cdot\nabla\phi^*$ *and* $-\alpha^*\omega^*$ *for February 1979 (00 GMT for each day). Contribution of planetary and intermediate scale waves to the total conversion are listed in separate columns. Monthly mean and corresponding standard deviation are included. Units:* Wm^{-2}

February 1979	$-v^*\cdot\nabla\phi^*$ $1\leqslant n\leqslant 3$	$-v^*\cdot\nabla\phi^*$ $4\leqslant n\leqslant 9$	$-v^*\cdot\nabla\phi^*$ $1\leqslant n\leqslant 35$	$-\alpha^*\omega^*$ $1\leqslant n\leqslant 3$	$-\alpha^*\omega^*$ $4\leqslant n\leqslant 9$	$-\alpha^*\omega^*$ $1\leqslant n\leqslant 35$
1	0.65	1.13	2.16	0.40	1.27	1.97
2	0.78	1.35	2.36	0.47	1.39	2.05
3	0.76	0.97	2.08	0.48	0.97	1.76
4	0.65	0.91	1.80	0.57	0.74	1.47
5	0.47	1.07	1.81	0.45	1.00	1.65
6	0.68	0.89	1.90	0.60	0.83	1.72
7	0.68	1.30	2.28	0.51	1.13	1.88
8	0.57	1.01	1.92	0.49	0.82	1.59
9	0.76	1.09	2.07	0.65	0.85	1.67
10	0.67	1.02	1.99	0.49	0.94	1.59
11	0.67	1.26	2.43	0.58	1.06	1.99
12	0.77	1.20	2.37	0.70	1.11	2.13
13	1.01	1.13	2.48	0.89	1.02	2.19
14	0.74	1.38	2.52	0.63	1.29	2.26
15	0.81	1.28	2.51	0.74	1.23	2.28
16	0.74	1.28	2.50	0.89	1.17	2.40
17	0.93	1.01	2.25	0.96	0.85	2.03
18	0.92	0.97	2.18	0.94	0.96	2.10
19	0.60	1.08	2.11	0.62	1.01	1.90
20	0.77	1.12	2.40	0.49	0.99	1.83
21	0.64	1.41	2.49	0.31	1.18	1.85
22	0.62	1.52	2.73	0.22	1.43	2.06
23	0.75	1.24	2.46	0.41	1.27	1.99
24	0.76	1.21	2.24	0.54	1.14	1.88
25	0.56	1.13	1.96	0.54	1.04	1.86
26	0.63	1.09	1.98	0.58	1.06	1.83
27	0.78	0.90	1.90	0.73	0.85	1.73
28	0.99	0.85	2.25	0.96	0.65	1.96
∅	0.73	1.14	2.22	0.60	1.04	1.92
σ	0.13	0.17	0.25	0.19	0.19	0.23

values, the difference turns out to be of similar order as the difference between the global means (Table 2), suggesting that the amount of the neglected equatorial fluxes is comparable to the uncertainties due to problems connected with data quality and data processing. However, this does not hold for daily values, for which both formulations can differ by up to 60%.

It is of essential practical interest, under which circumstances the computation of limited area conversion integrals without special attention to horizontal boundary fluxes, gives reasonable results, which may be interpreted as contributions to the global baroclinic energy conversion. The results presented above do not allow a definite answer to this question with respect to hemispheric integrals. We would suggest that a more detailed knowledge of the spatial structure of the term $-v_D\cdot\nabla\phi$ could be useful to come closer to a solution of this problem. Pearce (1974) rightly pointed out that the spatial structures of $-v_D'\cdot\nabla\phi'$ and $-\alpha'\omega'$ can be quite different from each other. However, the degree of similarity of related limited area integrals should be checked for data of different quality.

4. Conclusion

The conclusions of the results above are summarized as follows.

Table 3. *Vertically integrated hemispheric averages of* $-v^*\cdot\nabla\phi^*$ *and* $-\alpha^*\omega^*$ *for February 1979 (00 GMT for each day). Monthly mean and corresponding standard deviation are included. Units:* Wm^{-2}

February 1979	$-v^*\cdot\nabla\phi^*$ N.H.	$-\alpha^*\omega^*$ N.H.	$-v^*\cdot\nabla\phi^*$ S.H.	$-\alpha^*\omega^*$ S.H.
1	2.86	2.65	1.36	1.17
2	3.21	2.69	1.43	1.30
3	3.00	2.58	1.11	0.84
4	2.49	2.08	1.05	0.78
5	2.25	1.91	1.31	1.29
6	2.14	1.79	1.58	1.55
7	2.51	2.12	1.94	1.54
8	2.07	1.82	1.72	1.27
9	2.25	1.94	1.79	1.31
10	2.76	2.31	1.14	0.78
11	3.42	2.86	1.38	1.02
12	3.30	3.13	1.34	1.03
13	3.63	3.39	1.22	0.88
14	2.94	2.91	1.95	1.52
15	3.39	3.34	1.53	1.10
16	3.59	3.53	1.31	1.17
17	3.13	3.00	1.26	0.95
18	2.70	2.77	1.55	1.32
19	2.47	2.30	1.65	1.42
20	2.90	2.20	1.79	1.36
21	2.83	1.81	2.06	1.81
22	2.97	1.80	2.39	2.23
23	2.68	1.58	2.14	2.32
24	2.74	1.77	1.68	1.89
25	2.46	2.18	1.40	1.45
26	1.95	1.78	1.92	1.77
27	2.09	1.87	1.64	1.51
28	2.59	2.36	1.82	1.43
∅	2.76	2.37	1.59	1.36
σ	0.47	0.57	0.34	0.39

(1) The theoretical identity of the global integrals of either $[\alpha]''[\omega]''$ and $[v]\cdot[\nabla\phi]$ or $\alpha^*\omega^*$ and $v^*\cdot\nabla\phi^*$ should *never* be assumed for any data-set without special verification. This holds all the more so for the spectral components of $\alpha^*\omega^*$ and $v^*\cdot\nabla\phi^*$.

(2) The way of data processing used in this study is most common, and the used data are — at the present stage — widely regarded as being the best available. Nevertheless, the results do not fulfill the required identity constraints. As a matter of fact, the differences between the results arising from both formulations of the energy conversion rate are so large that little information can be gained with respect to the possibility of interpreting limited area integrals of the terms as a quantitative measure of the intensity of baroclinic energy conversion processes.

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6. Appendix

∇	2-dimensional del-operator on isobaric surfaces
∇_3	3-dimensional del-operator
λ	geographic longitude
φ	geographic latitude
ϕ	geopotential
α	specific volume
$\omega = dp/dt$	generalized vertical velocity
g	acceleration of gravity
n	zonal wavenumber
p	pressure
p_0	sea level pressure
v	2-dimensional wind-vector
v_3	3-dimensional wind-vector
v_R, v_D	rotational and divergent wind component ($v = v_R + v_D$)
$\hat{X}$	global area average over an isobaric surface
$[X]$	zonal average
$X^* = X - [X]$	deviation from zonal average
$X'' = X - \hat{X}$	deviation from global area average
X'	deviation from limited area average
$\int dp\, g^{-1}\ldots$	vertical integration between 1000 mb and 10 mb

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