Tellus (1987), *39A*, 390–396

Predicted hazard area for a polar low

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(Manuscript received 13 June 1986; in final form 12 January 1987)

ABSTRACT

Autoregressive models are fitted to 57 polar low trajectories sampled by satellite and weather maps from The Norwegian Sea neighbourhood at approximately $\Delta t = 3$ h intervals. The data tentatively suggest that no velocity model is significantly better than a Brownian. When a low has been observed, the predicted probability of a low encounter is estimated to be significant over an area with 800 km characteristic length and width.

1. Introduction

A polar low is a weather event with characteristic horizontal diameter of the order of 100 km with risk of simultaneous hurricane wind velocity and heavy icing. It may cause damage to ocean systems such as boats and oil rigs and/or their operation. Optimal design and control of these systems do therefore require risk estimates for polar low encounters.

There seems to be a common conviction among meteorologists that a trajectory can be accurately predicted by conventional dynamical models. These models predict expected fields given the initial conditions. Comparisons between predicted and actual trajectories often indicate insignificant differences even over lead times of the order of 24 h. Small prediction errors imply that such a prediction would be most informative for estimating the hazard area of a polar low. However, these models are too rich and complicated relative to the scattered data to be properly validated (cf. Goodwin and Payne, 1977 and references therein). Also the initial error for an intense small scale polar low flow field with as scarce observation as in the Norwegian Sea must be large. A normal atmospheric velocity field error is estimated as about 4 m s^{-1} (Hollingworth and Lönnberg, 1986). The prediction accuracy of tropical cyclones has been investigated by Neumann and Pelissier (1980). They estimated small differences between different prediction methods. Since meteorological prediction errors are likely to have large integral scales, and they increase with the lead time (Lorenz, 1985), it appears that the prediction error for polar low position over a lead time of 12 h, can hardly be less than about 200 km. This suggests that the prediction error should be objectively estimated and also taken into account for hazard area prediction.

The probabilistic properties of a polar low trajectory is here studied by means of autoregressive models. They may extrapolate optimally a trajectory based upon easily available information on the trajectory history. The lead time for a prediction is imagined to be in the range of 12 h. This objective of optimal extrapolation is the same as identification of the best timeseries model (compare for instance Box and Jenkins, 1970). A significant estimate of characteristic simple features is considered to be more useful than uncertain estimates of more sophisticated features. When the simple results obtained appear to disagree with common subjective conviction, this is particularly important. Since the total number of trajectories is only 57 with about 12 observations each, the data are so few that maximum averaging is sought. A classification with respect to for instance the larger scale flow, is therefore judged to be inappropriate in the present analysis. Even then

the data are so few that the results should only be considered as tentative.

2. Estimation

2.1. Data

The data, consisting of polar low centre longitude λ and latitude ϕ at different times, are obtained from satellite photographs and weather maps from the Norwegian Sea and its neighbourhood. Since the time of observation almost always is assigned to synoptic hours (0, 3, 6, 9, ...), linear interpolation and extrapolation are done so that the data consist of length coordinates, east and north, at synoptic times $x_1(t) = (R\cos\phi(t))\lambda(t)$, $x_2(t) = R(\phi(t) - \phi_0)$. Here R is earth radius and t is the time index. The timestep is $\Delta t = 3$ h. The origin of the coordinate system is chosen at ($\lambda = 0°$ E, $\phi_0 = 65°$ N). A few lows with lifetimes less than 15 h are not included. An example of a trajectory is shown in Fig. 1. The starting date and position of each trajectory analysed is given in Table 1 and important mean characteristics in Table 2.

The observation error of the polar low centre, when not knowing the future trajectory development, cannot be much smaller than the typical size of a polar low, say $\Delta x \approx 20$ km. Assuming that the position errors at different times are stochastically independent, the mean velocity estimate based upon neighbouring observations has an error of about 2 m s^{-1}. The present data have been obtained with information on the whole trajectory. This means that smoothing has probably occurred and small scale randomness is underestimated in the present study.

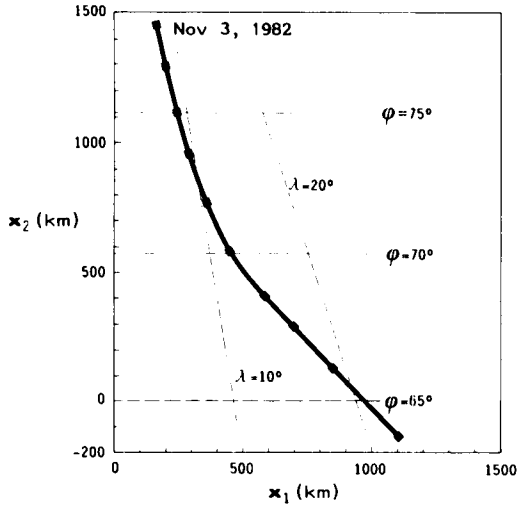

Fig. 1. A polar low trajectory in $x_1 = (R\cos\phi)\lambda$, $x_2 = R(\phi - \phi_0)$ coordinates. R: earth radius. λ: longitude. ϕ: latitude. $\phi_0 = 65°$ N. Data at synoptic time intervals. Units: km.

2.2. Model identification

Table 1 indicates that once a polar low has been observed, there is a large probability that it will persist for at least 24 h. The lifetime is therefore supposed to be sufficiently long to be noninteresting for the present purpose.

Since the polar low two-dimensional position vector $\boldsymbol{x}(t) = (x_1(t), x_2(t))^T$ is highly nonstationary, difference models are sought (Lagrangian velocity). The autoregressive models tested are then of the form (cf. Box and Jenkins, 1970; Priestly, 1981):

$$\sum_{i=0}^{p} \boldsymbol{A}(i;p)\,\boldsymbol{u}(t-i) = \sum_{i=0}^{p} \boldsymbol{A}(i;p)\,\boldsymbol{\mu}(t-i) + \boldsymbol{a}(t;p), \tag{1}$$

with $\boldsymbol{A}(0;p) = \mathbf{1}$. The velocity vector is $\boldsymbol{u}(t) = (\Delta t)^{-1}[\boldsymbol{x}(t) - \boldsymbol{x}(t-1)] = (\Delta t)^{-1}\nabla\boldsymbol{x}(t)$, with expected value $\boldsymbol{\mu}(t) = E\boldsymbol{u}(t)$. $\boldsymbol{A}(i;p)$ are the autoregressive matrices. The one step ahead (innovation) error vector $\boldsymbol{a}(t;p)$ is assumed to be nearly Gaussian stationary and white with variance matrix $\boldsymbol{V}(1;p)$. All models considered belong to the class (1).

The real unknown model order p could be very large or infinite. The first goal is to identify to each timeseries the simplest possible model that is sufficiently rich as compared to the available data. Objective identification techniques as summarized by Andel (1982) require a larger sample size than available in this study, $n \approx 12$. Both theory and meteorological data (Katz, 1981; Eidsvik, 1985) suggest that Akaike's (1974) information criterium tends to propose large-order models. Therefore, if this measure identifies small and simple models, these are probably sufficiently rich as compared to the data also for small samples sizes. Akaike's Information criterium is: $\mathrm{AIC}(p) = n\ln(\mathrm{Det}\,\hat{\boldsymbol{V}}(1,p)) + 2k^2 p$. Here n is the number of data, $\hat{\boldsymbol{V}}(1;p)$ the esti-

Table 1. *Polar low trajectories analyzed; initial time and position relative to (0°E, 65°N) in km; lifetime in h*

Date	h	$x_1(0)$	$x_2(0)$	$n\Delta t$
Nov03, 1982	3	167	1472	27
Dec 05, 1982	0	−193	778	36
Dec 11, 1982	12	370	626	27
Dec 12, 1982	3	321	1235	60
Dec 14, 1982	3	808	1122	27
Dec 15, 1982	3	707	854	45
Dec 16, 1982	15	250	635	48
Dec 21, 1982	12	231	423	42
Jan 07, 1983	3	−158	790	27
Jan 13, 1983	6	504	868	36
Jan 17, 1983	18	−623	−78	30
Jan 30, 1983	9	802	892	69
Jan 30, 1983	9	266	719	96
Feb 10, 1983	3	155	1076	27
Mar09, 1983	6	−753	−78	30
Mar09, 1983	12	−957	−211	39
Sep 25, 1983	18	−50	1535	69
Oct 19, 1983	15	90	454	24
Nov01, 1983	9	807	1225	39
Nov10, 1983	3	1427	1178	27
Nov13, 1983	12	−455	745	27
Nov17, 1983	9	355	1205	36
Nov21, 1983	0	438	1068	75
Nov24, 1983	9	699	1202	57
Nov24, 1983	18	709	648	48
Dec 05, 1983	9	−445	−141	18
Dec 09, 1983	3	607	1227	42
Dec 20, 1983	12	−162	1224	87
Jan 07, 1984	6	−230	−624	27
Jan 17, 1984	6	−737	−758	18
Jan 17, 1984	12	−557	−556	51
Jan 20, 1984	6	−183	659	24
Jan 20, 1984	6	361	1012	24
Feb 14, 1984	15	−303	756	24
Feb 26, 1984	18	−569	567	36
Mar01, 1984	12	227	1063	21
Mar05, 1984	18	308	901	30
Mar06, 1984	3	522	1083	36
Mar15, 1984	6	266	−534	63
Mar29, 1984	6	257	1288	51
Nov20, 1984	9	646	857	18
Nov28, 1984	12	−449	−367	30
Dec 02, 1984	18	192	1012	27
Jan 23, 1985	3	1237	802	27
Jan 27, 1985	0	87	535	30
Jan 29, 1985	15	578	887	24
Feb 05, 1985	0	1093	866	27
Mar12, 1985	18	334	897	69
Mar14, 1985	3	285	−110	36
Mar14, 1985	21	−452	−389	45
Mar15, 1985	9	−204	−4	36
Mar26, 1985	3	−123	755	39
Mar26, 1985	0	368	1079	66
Mar28, 1985	12	−315	−667	18
Apr 07, 1985	12	858	856	18
Apr 25, 1985	0	303	−222	42
Apr 26, 1985	15	302	673	24

mate of $V(1,p)$, and $k=2$ the dimension of $\boldsymbol{u}$. For each p, the best model is estimated by means of Yule Walker's equation to give $\hat{A}(i;p)$ and $\hat{V}(1;p)$. These are then compared with Akaike's AIC-measure. The model giving the best compromise between data fit and uncertainty, i.e., the minimum AIC is chosen. Assuming that $\boldsymbol{\mu}$ is constant for each experiment, it turns out that the best model is identified as $p=1$ for all experiments of sufficient length. Trajectories with lifetimes below ~24 h are identified as $p=0$ processes. Schwarz's (1978) BIC-measure identifies most trajectories as $p=0$ processes. It is therefore appropriate to conclude that $p=1$ is sufficiently rich for a polar low velocity model as compared to the data. The order is chosen so, and comparable parameter matrices $A(1,1)=A$, $V(1,1)=V_a$ are estimated for all experiments.

Table 2. *One-step prediction error for different models, $V=E[\boldsymbol{u}(t+1)-\hat{\boldsymbol{u}}_t(1)][\boldsymbol{u}(t+1)-\hat{\boldsymbol{u}}_t(1)]^T$; weighted averages over all experiments; average lifetime is 36 h; average autoregressive matrix for the best $p=1$ model is: $A_{11}=-0.44$, $A_{12}=0.08$, $A_{21}=-0.10$, $A_{22}=-0.47$*

		Matrix elements ($m^2\ s^{-2}$)			Norm ($m\ s^{-1}$)
$\hat{\boldsymbol{u}}_t(1)$	V	1 1	1 2	2 2	$\|\cdot\|^{1/2}$
$\boldsymbol{\mu}$	V_μ	22.70	−0.12	19.13	5.84
$\boldsymbol{u}(t)$	V_d	15.32	0.17	12.23	4.60
$2\boldsymbol{u}(t)-\boldsymbol{u}(t-1)$	V_{dd}	34.37	0.05	25.60	7.00
$\boldsymbol{\mu}-A(\boldsymbol{u}(t)-\boldsymbol{\mu})$	V_a	12.22	0.91	10.67	4.24

Since $\boldsymbol{x}$ can be expressed as a sum of $\boldsymbol{a}(t)$, $\boldsymbol{a}(t-1)$, ..., it is supposed to be nearly Gaussian. The optimal time prediction of the statistical structure for a polar low trajectory is then given by the conditional expectation $\hat{\boldsymbol{x}}_t(l)=$

$E\{\boldsymbol{x}(t+l)/\boldsymbol{x}(t), \boldsymbol{x}(t-1)\ldots\}$ and its prediction error variance matrix $\boldsymbol{G}(l)$. For the purpose of illustration, this prediction is shown in eqs. (2) and (3):

$$\hat{\boldsymbol{x}}_t(l) = \left[\sum_{i=0}^{l} (-1)^i \boldsymbol{A}^i\right] \boldsymbol{x}(t) + \left[\boldsymbol{A} \sum_{i=0}^{l-1} (-1)^i \boldsymbol{A}^i\right] \boldsymbol{x}(t-1) + \sum_{i=0}^{l-1} (l-i)(-1)^i \boldsymbol{A}^i [\boldsymbol{1} + \boldsymbol{A}]\, \hat{\boldsymbol{\mu}}\, \Delta t, \tag{2}$$

$$\boldsymbol{G}(l) = E\{(\boldsymbol{x}(t+l) - \hat{\boldsymbol{x}}_t(l))(\boldsymbol{x}(t+l) - \hat{\boldsymbol{x}}_t(l))^{\mathrm{T}}\} = \sum_{i=0}^{l-1} \left[\sum_{j=0}^{i} (-1)^j \boldsymbol{A}^j\right] \times \boldsymbol{G}(1) \left[\sum_{q=0}^{i} (-1)^q \boldsymbol{A}^q\right]^{\mathrm{T}}. \tag{3}$$

The one step ahead position error covariance matrix is,

$$\boldsymbol{G}(1) = (\Delta t)^2 \{\boldsymbol{V}_a + [\boldsymbol{1} + \boldsymbol{A}] \times E[\hat{\boldsymbol{\mu}} - \boldsymbol{\mu}][\hat{\boldsymbol{\mu}} - \boldsymbol{\mu}]^{\mathrm{T}} [\boldsymbol{1} + \boldsymbol{A}]^{\mathrm{T}}\}.$$

It is a sum of $\boldsymbol{V}$ and the estimation error for mean velocity. The best $p = 1$ model is identified under the assumption that the estimation error for the mean velocity is zero. If (2, 3) were to be actually applied, both $\boldsymbol{\mu}$ and its error would have to be predicted.

Table 2 shows estimated weighted mean values over the experiments of the one step ahead error for different prediction methods,

$$\boldsymbol{V} = E[\boldsymbol{u}(t+1) - \hat{\boldsymbol{u}}_t(1)][\boldsymbol{u}(t+1) - \hat{\boldsymbol{u}}_t(1)]^{\mathrm{T}}$$

The first method is velocity prediction with known constant mean and variance matrix $\boldsymbol{V}_\mu = E(\boldsymbol{u} - \boldsymbol{\mu})(\boldsymbol{u} - \boldsymbol{\mu})^{\mathrm{T}}$. The second is a Brownian velocity model $\nabla \boldsymbol{u}(t) = \boldsymbol{a}(t)$ with variance matrix $\boldsymbol{V}_d = E[\boldsymbol{u}(t) - \boldsymbol{u}(t-1)][\boldsymbol{u}(t) - \boldsymbol{u}(t-1)]^{\mathrm{T}}$. The third is a Brownian acceleration model $\nabla^2 \boldsymbol{u}(t) = \boldsymbol{a}(t)$ with variance matrix $\boldsymbol{V}_{dd} = E[\boldsymbol{u}(t) - 2\boldsymbol{u}(t-1) + \boldsymbol{u}(t-2)][\boldsymbol{u}(t) - 2\boldsymbol{u}(t-1) + \boldsymbol{u}(t-2)]^{\mathrm{T}}$. The fourth is the best $p = 1$ model estimated for each experiment. Its velocity variance matrix is $\boldsymbol{V}_a = E\boldsymbol{a}(t)\boldsymbol{a}^{\mathrm{T}}(t)$. Euclidian norms (largest eigenvalues) are also shown. Since Fig. 2 indicates linear relations between $|\boldsymbol{\mu}|$, $\|\boldsymbol{V}_\mu\|^{1/2}$, $\|\boldsymbol{V}_d\|^{1/2}\|\boldsymbol{V}_{dd}\|^{1/2}$, and $\|\boldsymbol{V}_a\|^{1/2}$, most information is contained in mean values over the experiments, given in Table 2. The minimal average velocity prediction error over 3 h lead time is observed to be ~ 4 m s^{-1}, significantly larger than the measurement error, of ~ 2 m s^{-1}. The subjectively attractive Brownian acceleration model is objectively identified as very poor. Since the ratio of $\|\boldsymbol{V}_{dd}\|$ and $\|\boldsymbol{V}_d\|$ is approximately equal to 2, the mean acceleration for two successive intervals $[\boldsymbol{u}(t) - \boldsymbol{u}(t-1)]$ and $[\boldsymbol{u}(t-1) - \boldsymbol{u}(t-2)]$ is estimated to be stochastically independent. The acceleration cannot be predicted based upon the trajectory history. The "best" $p = 1$ model is not much better than predicting the velocity to be a known constant mean for each trajectory. Fig. 2 shows that the difference between $\|\boldsymbol{V}_a\|^{1/2}$ and $\|\boldsymbol{V}_d\|^{1/2}$ is also small for each individual experiment. It is recalled that except for the Brownian models, $\boldsymbol{\mu}$ must be estimated with an additional prediction error (eq. (3)). If, $\hat{\boldsymbol{\mu}}$, in eq. (2) is predicted as $(\Delta t)^{-1}[\boldsymbol{x}(t) - \boldsymbol{x}(t-1)]$, the equation turns out to be a simple Brownian velocity prediction. This $\boldsymbol{\mu}$-prediction is subjectively expected to be accurate as compared to estimates based upon steering winds from weather maps. In conclusion the Brownian velocity model is therefore estimated almost as accurately as possible in the autoregressive class, and a nearly optimal prediction is very simple:

$$\hat{\boldsymbol{x}}_t(l) = \boldsymbol{x}(t) + (\boldsymbol{x}(t) - \boldsymbol{x}(t-1))\, l \tag{4}$$

$$\boldsymbol{G}(l) = (\Delta t)^2\, \boldsymbol{V}_d \sum_{j=1}^{l} j^2. \tag{5}$$

From eq. 5 and Table 2, a characteristic prediction error over 12 h lead time is then about $\|\boldsymbol{G}(4)\|^{1/2} \approx 270$ km, similar to the rough estimate given in the introduction. This implies that there is about a 50% probability of predicting the trajectory within ± 150 km, which is comparable to the size of the low. However, there is also about 5% probability of predicting the low trajectory more than ± 500 km offcourse. With a mean transport velocity of $|\boldsymbol{\mu}| \approx 8$ m s^{-1} this implies that there is an approximately 5% probability of predicting the trajectory direction more than $\pm 25°$ in error.

If it turns out that significantly more accurate prediction models exist, the present estimates

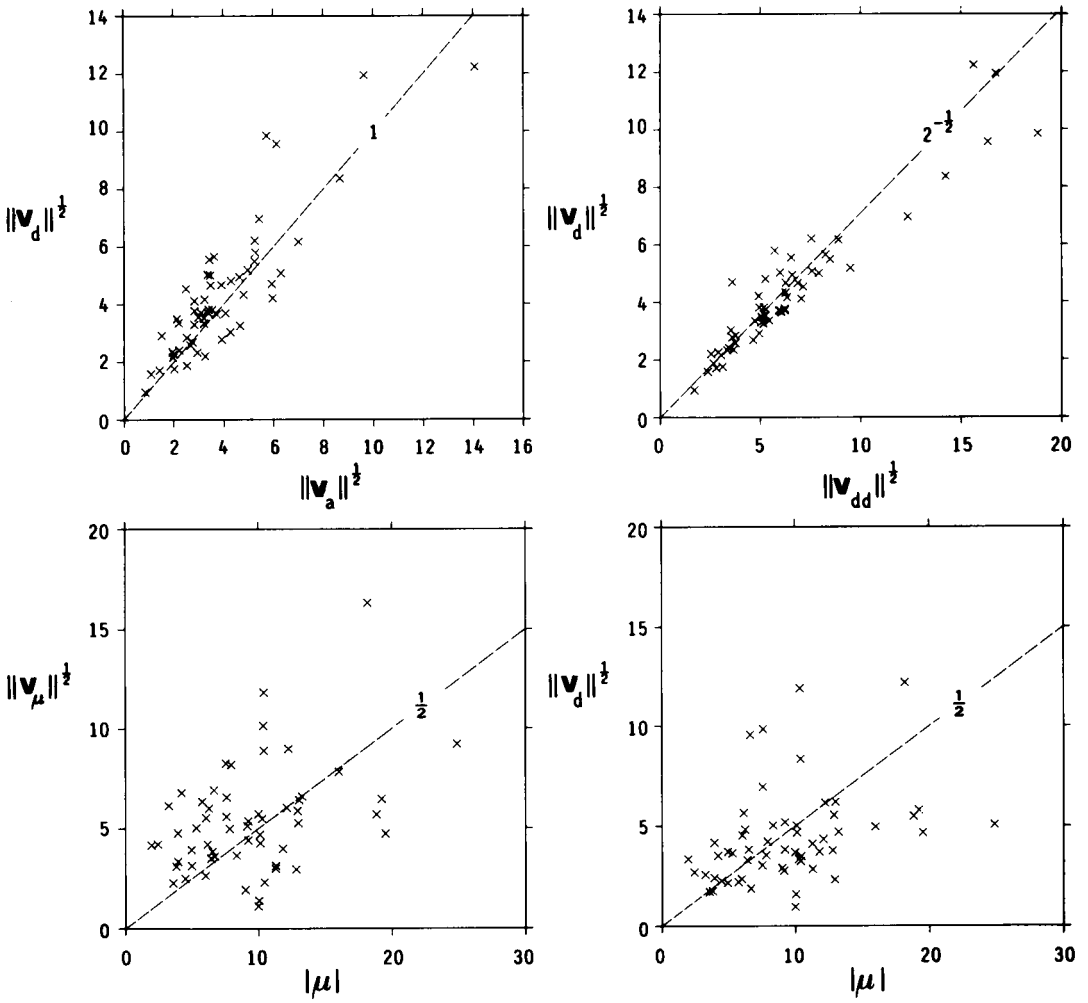

Fig. 2. Experimental values for Euclidean norms of one step ahead prediction variance matrixes and mean velocity vector. Unit: m s^{-1}.

give a simple reference for measuring improvements. However, until the opposite is quantitatively verified, it seems that the prediction error estimated by the present objective data analysis should also be considered as representative for the most sophisticated methods. Even if the predicted trajectory $\hat{x}_i(l)$ may then be subjectively more attractive than for the Brownian velocity model, it should be associated with a prediction error of the order of $\|G(l)\|^{1/2}$.

3. Predicted hazard probability

The user is supposed to be most interested in the predicted hazard probability at fixed Eulerian locations. This depends upon the conditional distribution of $x(t+1)$ given $x(t)$, $x(t-1)$ and upon how a system or operation can be affected by a polar low encounter. Assumptions about the conditional distribution cannot be tested with any reasonable sample size. For the present purpose

of tentative estimation it is supposed to be Gaussian, $N(\hat{x}_t(l), G(l))$. For the same purpose the conditional probability of damage or effect given a low encounter is characterized by a "damage diameter" $d(l)$. It is supposed to mean that the system or operation is almost certainly affected if the polar low centre comes closer than $d/2$.

It is convenient to define an orthogonal (possibly slightly curvilinear) coordinate system with origin at $x(t)$ and y_1 axis parallel and y_2 axis normal to $\hat{x}_t(l)$. l is extended from being a time index to a continuous, nondimensional trajectory parameter. When a realization $x(t+l)$; $l>0$ is given, there is a one to one correspondence between l and $y_1 = \bar{x}_1(l) \approx |x(t) - x(t-1)|l$, so that the trajectory may be expressed in Eulerian coordinates as $\bar{x}_2(y_1)$; $y_1 > 0$. Components of x in the y system are denoted $\bar{x}_1$ and $\bar{x}_2$. Damage occurs inside the Eulerian boundaries $\bar{x}_2(y_1) \pm d(y_1)/2$. At a given y_1 and y_2 damage then occurs when $\bar{x}_2(y_1) \in (y_2 - d(y_1)/2, y_2 + d(y_1)/2)$. Principally the distribution of $\bar{x}_2(y_1)$ is given by $N(\hat{x}_t(l), G(l))$ and the transformation $l = l(y_1)$. In the chosen coordinate system $\bar{x}_2(y_1)$ has expected value zero and variance $G_{22}(y_1)$. The hazard probability at y is then

$$p(y) \approx \int_{y_2 - d(y_1)/2}^{y_2 + d(y_1)/2} (2\pi G_{22}(y_1))^{-1/2} \times \exp(-\tfrac{1}{2}\bar{x}_2^2 G_{22}(y_1)^{-1})\, d\bar{x}_2 = \int_{(y_2 - d(y_1)/2)/G_{22}(y_1)^{1/2}}^{(y_2 + d(y_1)/2)/G_{22}(y_1)^{1/2}} N(0,1)\, dn. \quad (6)$$

In a given situation, with information on $x(t)$, $x(t-1)$, ..., the error $G_{22}(y_1)$ and the damage diameter $d(y_1)$ must be predicted. Analogous to the prediction of turbulent diffusion it seems reasonable to estimate Eulerian and Lagrangian variances as similar. For the present purpose of tentative estimation $G_{22}(y_1)$ is set equal to $\|G(l(y_1))\|$. For the same purpose $d(y_1)/2$ is set equal to the size of the low, assumed equal to the one step ahead prediction error $\|G(1)\|^{1/2}$. $\|G(l)\|^{1/2}$ is estimated from eq. (5) and Fig. 2 setting $\|\hat{V}_d\|^{1/2} \approx 0.5|\hat{\mu}| \approx 0.5(\Delta t)^{-1}|x(t) - x(t-1)|$. The integration limits in (6) are then given by $[2y_2|x(t) - x(t-1)|^{-1} \pm 1](\sum j^2)^{-1/2}$. Integration for $l = 1, 2, \ldots$ and interpolations to noninteger $l = y_1|x(t) - x(t-1)|^{-1}$ give isocurves for hazard probability as shown in Fig. 3.

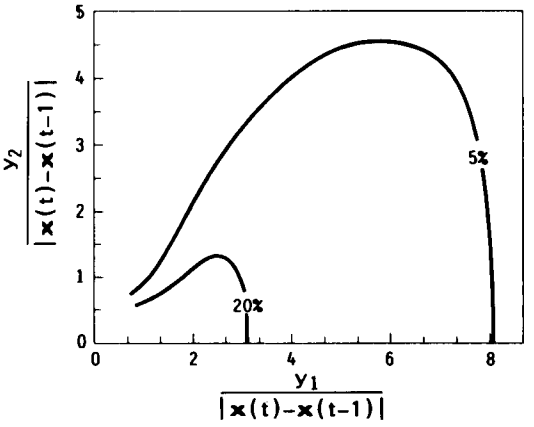

Fig. 3. Schematic isocurves for predicted hazard probability. y_1 axis along $\hat{x}_t(l) \approx (x(t) - x(t-1))l$.

If a hazard probability of about 20% can be accepted, the area is not much wider than the size of the low. However, when the consequences of a polar low encounter are supposed to be disastrous, a 5% hazard probability must be considered as high. In this case the maximum alongwind and transverse dimensions of the predicted hazard area are approximately given by $y_1(p = 5\%) \approx 8|x(t) - x(t-1)| \approx 800$ km and $y_2(p = 5\%) \approx 4|x(t) - x(t-1)| \approx 400$ km. At a lead time of 12 h, the width of the predicted hazard area is about 8 times the polar low "damage diameter". Although the present hazard estimates must necessarily be highly tentative, they suggest that a large and wide area may be exposed to significant risk once a polar low is under way and a lead time of the order of a day is needed.

4. Concluding remarks

The data on polar low position sampled at synoptic hours allow the following tentative suggestions: All trajectories of sufficient lifetimes are identified as a simple Markov velocity process. No systematic variation of the autoregressive matrix is detected. The innovation variance for the best model is not significantly smaller than for a simple Brownian process. The estimated Euclidian norms of the two covariance matrices are approximately equal and are highly

correlated. They both increase about quadratically with the mean speed of the low, $\|\boldsymbol{V}_d\|^{1/2} \approx 0.5|\boldsymbol{\mu}|$.

Until the opposite is quantitatively verified, it seems that more sophisticated models should be considered to be as inaccurate as the Brownian velocity model, and prediction errors should be accounted for when estimating the hazard area for a polar low. Estimates of predicted hazard probability at fixed locations indicate that an area with longitudinal and transverse distance comparable to 800 km should be predicted as hazardous once a polar low is under way and high risk cannot be afforded.

5. Acknowledgements

The present work is part of a larger study on polar low predictions, funded by oil companies with licences north of 62°N on the Continental Shelf of Norway.

REFERENCES

Akaike, H. 1974. A new look at the statistical model identification. *IEEE Trans., Autom. Control, AC-19*, 716–723.

Andel, J. 1982. Fitting models in time series. *Analysis. Math. Operationsforsch. Statist. Ser. Statistics. 13*, 121–143.

Box, G. E. P. and Jenkins, G. M. 1970. *Time series analysis, forecasting and control.* Holden Day, 553 pp.

Eidsvik, K. J. 1985. Large-sample estimates of wind fluctuations over the ocean. *Boundary-Layer Meteorol. 32*, 103–132.

Goodwin, G. C. and Payne, R. L. 1977. *Dynamic system identification. Experimental design and data analysis.* Academic Press, New York, 291 pp.

Hollingsworth, A. and Lönnberg, P. 1986. The statistical structure of short-range forecast errors as determined from radiosonde data. Part I: The wind field. *Tellus 38A*, 111–137.

Katz, R. W. 1981. On some criteria for estimating the order of a Markov chain. *Technometrics 23*, 243–249.

Lorenz, E. N. 1985. The growth of errors in prediction. In *Turbulence and predictability in geophysical fluid dynamics and climate dynamics*, pp. 243–265 (ed. by M. Ghil et al.). Elsevier.

Neumann, C. J. and Pelissier, J. M. 1980. Models for the prediction of tropical cyclone motion over the north Atlantic. An operational evaluation. *Mon. Wea. Rev. 109*, 522–538.

Schwarz, G. 1978. Estimating the dimenion of a model. *Ann. Stat. 6*, 461–464.

Priestley, M. B. 1981. Spectral analysis and time series, vol. 2: *multivariate series, prediction and control.* Academic Press, 890 pp.