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On the comparison between two sets of 500 mb geopotential height eigenfunctions

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ABSTRACT

When the largest eigenvalues of an Empirical Orthogonal Function (EOF) analysis are of similar magnitude, the corresponding EOFs do not hold great interest when they are considered one at a time; only the vector subspace generated by the first EOFs is significant. In this case, the comparison between two EOF analyses may be summarized by a distance index between the two subspaces. Such a method is applied to EOF analyses over the northern hemisphere of the 500 mb geopotential height from 10 winters of simulation (using a version of the ECMWF spectral model) and 13 observed winters. For more than the first three low-order EOFs, the agreement between simulation and observation is statistically significant.

1. Introduction

Empirical Orthogonal Function (EOF) analysis is a useful way to study the space and time variability of a meteorological field (Lorenz, 1956; Holmström, 1963) especially for 500 mb geopotential height (Rinne et al. 1981). When comparing the results of a General Circulation Model (GCM) with a series of observations of 500 mb height fields, the normal way, after looking at time-averages, is to consider the first EOFs. They indeed represent the modes with the most variability. This has been done with results from the European Center for Medium Range Weather Forecasts (ECMWF) GCM by Volmer et al. (1984). The first four EOFs on the Europe-Atlantic area are very similar to the observed ones and this shows that this model varies around its mean state in the same manner as the real atmosphere, but with a smaller amplitude. If we carry out such an analysis over the whole northern hemisphere, a problem appears. Because the eigenvalues decrease very slowly, we get a kind of degeneracy, since some of the eigenvalues are almost multiple eigenvalues. As a result, a rotation of a small number of consecutive EOFs may lead to almost the same series of eigenvalues and therefore the same solution to the EOF analysis.

Because of this degeneracy, due to the too large number of degrees of freedom having similar variability, a direct comparison of the first model EOF to the first observed EOF, the second one to the second one, etc... has little meaning. The first EOF given by the diagonalisation of the covariance matrix is strongly dependent on the input data. In order to get patterns which have more physical significance, some techniques have been introduced in factor analysis and applied to meteorological fields. They consist of rotating a subset of low-order EOFs in order to maximize a criterion which allows one to make a distinction between the new vectors and the corresponding components of the analyzed field. This distinction is more accurate than the usual criterion based on the % of the total variance that a given EOF is responsible for. In the Varimax method, applied to a normalized analysis, (Kaiser, 1958, Horel, 1981), the criterion which is used in the spatial variance of the square loadings (squared coordinates of the eigenvector weighted by the eigenvalue); the new vectors have simpler structures. In the Persistent Component Analysis (Déqué, 1983), the criterion is the time autocorrelation of the scores (spatial projections of the field on the eigenvectors); the new scores have a smoother time-evolution and are less dependent on each other. Both methods may be called "orthogonally rotated analyses", since the new scores remain uncorrelated. There exist other methods, called "obliquely rotated" (Richman, 1981), which do not have this property.

As we can see, when the first EOFs are not clearly separated, we should not consider the EOFs on an individual basis but rather analyze the whole vector subspace they generate. In order to compare the large-scale variability of data generated by a model with observations, we must directly compare the 2 vector subspaces. At this stage, two questions may be asked:

(i) how many primary EOFs is it necessary to retain?

(ii) how is it possible to measure the proximity between the simulation and the observation subspaces?

To answer the first question, some rules have been introduced in the literature often based on comparison with EOFs generated by a random process (Overland and Preisendorfer, 1982; von Storch and Hannoschöck, 1985).

To answer the second question, we introduce in Section 2 an index which measures the distance between two vector subspaces. In section 3, this index is used to study two meteorological EOF analyses. Finally in Sections 4 and 5, we give some estimates of the statistical properties of this index when the two sets of vectors are statistically independent.

2. An index describing the distance between two sets of vectors

Let $(A_1, \ldots A_p)$ and $(B_1, \ldots B_p)$ by two sets of n-dimensional orthogonal normalized vectors: $A_i'A_j = B_i'B_j = 1$ if $i = j$, 0 if $i \neq j$, where the prime indicates transposition. The distance d_i between the vector A_i and the vector subspace generated by $(B_1 \ldots B_p)$ is defined by: $d_i^2 = 1 - \Sigma_{j=1}^{p}(A_i'B_j)^2$.

This distance is maximum (i.e. $=1$) when A_i is orthogonal to each B_j and minimum (i.e. $=0$) when A_i is a linear combination of the B_j. The distance index between $(A_1, \ldots, A_p)$ and $(B_1, \ldots B_p)$ is defined as the mean squared distance:

$$\delta = (1/p)\sum_{i=1}^{p} d_i^2 = 1 - 1/p\sum_{i=1}^{p}\sum_{j=1}^{p}(A_i'B_j)^2.$$

This index is symmetrical with respect to $(A_i)_{i=1,p}$ and $(B_j)_{j=1,p}$. Its maximum value is 1, when the two vector subspaces are disjoint. Its minimum value is 0 when the two vector subspaces are identical.

By simple continuity arguments, it can be said that two sets of vectors are close to each other when this index is small. When the vectors are randomly distributed, and assuming that $\langle(A_i'B_j)^2\rangle$ is independent of i and j ($\langle\,\rangle$ indicates the expected value), $\langle\delta\rangle$ is equal to $1 - p/n$.

This can be proven by setting $\langle(A_i'B_j)^2\rangle = a$. If we take n vectors B_j, $\Sigma_{j=1}^{n}(A_i'B_j)^2 = 1$ since (B_j) is an orthonormal base and A_i a normalized vector. Then $a = 1/n$ and $\langle\delta\rangle = 1 - pa = 1 - p/n$. The assumption, as the conclusion, may be verified with Monte Carlo simulation (see Section 4). When $p = n$, the index goes to zero because the A_i's and B_j's generate the whole vector space. When p is small compared to the vector dimension n, the index will be close to 1 in the case of statistical independence of the two sets.

This index involves only the scalar products of the A_i's and the B_j's and thus remains unchanged when all the A_i's and the B_j's are rotated by the same n-dimensional operator. It also remains unchanged when (A_i) is substituted for a new set of orthogonal normalized vectors of the same subspace.

3. Comparison between GCM-generated and observed EOFs

The 15 level, T21 truncated spectral ECMWF model has been integrated for a 10-year climate simulation (Volmer et al., 1983, 1984). We consider here the winter (December to February) values of 500 mb height in the northern hemisphere. This field is described on a 215-point polar stereographic grid. The grid points are located north of 20° N and are equidistant. We then have to consider 10 × 91 values of this field.

The model values are to be compared with observed values of daily analyses made by the French weather service from 1965 to 1977. EOF analyses are performed on both data sets. Since we consider only winter height values, the seasonal cycles do not appear in the first EOF. Fig. 1 shows the first two EOFs of simulation, (a) and (b); and observation, (c) and (d). The first EOF

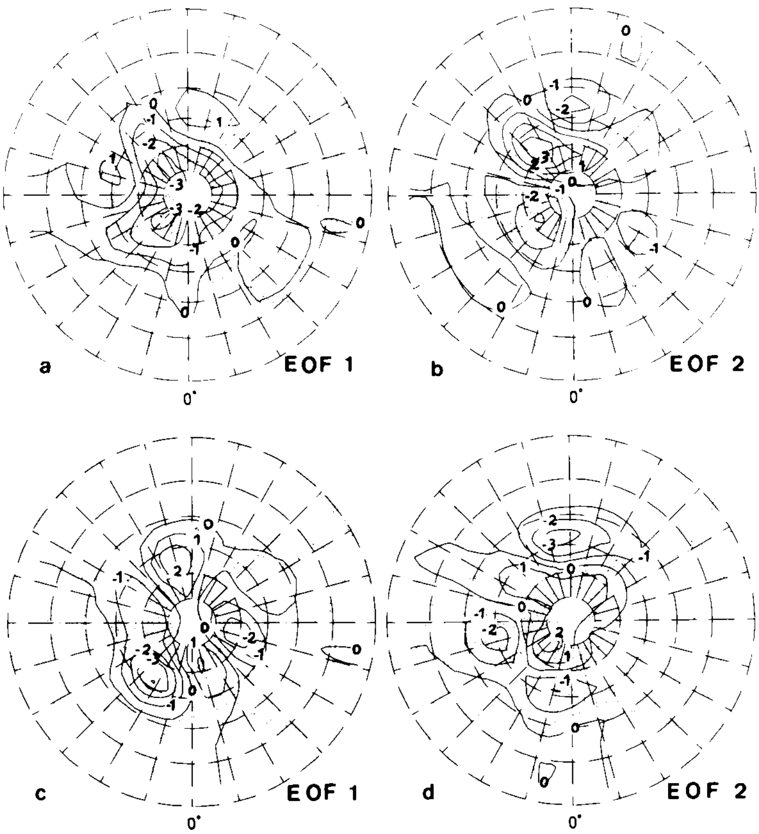

Fig. 1. Contours of the first two EOFs of the simulation (a,b) and of 13 observed winters (c,d) from 1965 to 1977 for 500 mb height. (Dimensionless units; spatial quadratic average is 1.)

Table. 1. *%s of variance explained by the nth EOF for observation (P_o) and simulation (P_s), and the distance indices between the first n EOFs of observation and simulation (δ)*

n	1	2	3	4	5	10	15	20	25	30
P_o	8.3	8.0	6.5	5.6	5.2	2.9	1.9	1.4	1.0	0.8
P_s	8.3	6.5	5.4	4.8	3.8	2.6	2.0	1.5	1.1	0.9
δ	0.95	0.72	0.50	0.50	0.49	0.43	0.32	0.28	0.24	0.22

consists of a negative structure at high latitudes in the simulation, whereas a negative teleconnection between Atlantic + Asia and Pacific + Europe is found in the observation. The second EOFs show complex patterns, but a slight similarity may be found between observation and simulation: a positive structure over Alaska is associated with two negative structures over the Atlantic and Central Pacific.

Comparison of the *n*th simulated EOF to the *n*th observed EOF does not yield good results even for $n = 1$. As can be seen in Table 1, the eigenvalues decrease very slowly. The index decreases quickly down to 0.50 with $n = 3$. For larger values of n, the decrease becomes slower and the index reaches 0.22 for $n = 30$.

4. Statistical significance of the distance index

The question we may ask at this point is whether, for example, a 0.28 index value could be obtained with two independent sets of 20 EOFs. When the sets are independent, they are not orthogonal (orthogonality is a dependence relation) and the expected distance index is $\langle\delta\rangle = 1 - 20/215 = 0.91$, as seen in Section 2. We have performed a Monte Carlo estimation based on 1000 simulations of 2 subspaces generated by 20 random vectors, each consisting of 215 independent values, randomly distributed. The mean value is then about 0.91, and is relatively independent of the random distribution. The fifth percentile (the probability to get a value of δ less than $\delta_{5\%}$ is 5%) varies between 0.84 for a Cauchy distribution (frequency $f(x) = 1/\pi(1 + x^2)$) and 0.90 for a Gaussian distribution. This indicates that the value of 0.28 found in Section 3 for δ is definitely significant.

But this method is not very satisfactory, since the first EOFs have a spatial structure which is physically significant, and consequently their coordinates are not independent. We have developed another method which leads to lower values of $\langle\delta\rangle$ and $\delta_{5\%}$ and thus makes the test more severe. In order to produce vectors which look like geopotential height EOFs, we have performed 10 EOF analyses on the 10 simulated winters and 13 EOF analyses on the 13 observed winters. We thus have 130 pairs of sets of vectors and we can calculate 130 indices, a mean index and a fifth percentile. If there were no correspondence between simulation and observation, these values would be the same as for independent samples, and an index found in the previous section would be greater than the fifth percentile (with a probability of 95%). If there is a correspondence between simulation and observation, the average and the fifth percentile will be lower than those corresponding to independent samples. But the index calculated in the previous section will be even lower, because the larger size of the samples provides a larger statistical significance. The first EOFs of a single winter better represent the characteristic features of this winter than the general features of winter circulations. Comparing the value of the index with the fifth percentile of the one-winter EOF analyses allows us to accept or reject the hypothesis of agreement between simulation and observation. Fig. 2 shows that with more than three EOFs, the agreement is significant. The mean index (dashed line) is lower than in the random case (where $\langle\delta\rangle = 1 - p/215$) because an EOF is not completely randomly distributed and because there is some correspondence between simulation and reality

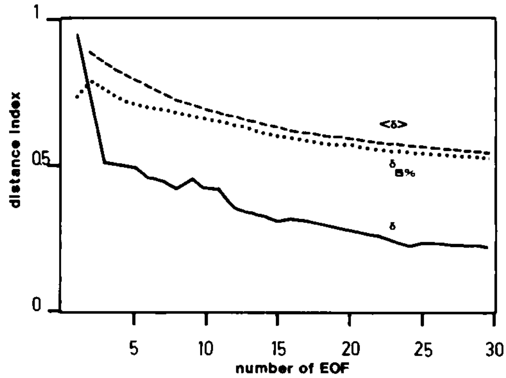

Fig. 2. Distribution of the distance index δ as a function of the number of EOFs retained. The solid line corresponds to a comparison between EOFs of the simulation (10 winters) and observations (13 winters). Dashed line (dotted line) indicates the average (fifth percentile) of 130 indices between observed and simulated single winters.

even for one winter. The fifth percentile (dotted line) is not much smaller than the mean index, as in the random case, which shows that the index has a weak dispersion.

5. Calibration

At this stage, we know that the distance index between the first 30 EOFs of the simulation and of the observation is 0.22. We get the same value when the observed set consists of 6 winters of ECMWF analyses (from 1980 to 1985) instead of the 13 winters considered above. If the two climates were identical, the index would be 0. If they were as far as two random climates, the index would lie between 0.60 and 1. In order to better locate this value of 0.22 between 0 and 0.60, some other values need to be given. If we consider the first 6 versus the last 6 winters of the observation data, the index is also 0.22. This shows that a large part of the distance between observation and simulation may be explained by the year-to-year variability. If we consider the 13 observed winters versus the 13 observed summers, the index is 0.30. This index has also been computed to compare the 13 observed winters with a 10-year simulation with a lower resolution GCM. This GCM (Royer et al., 1983) has 10 vertical levels, and T10-13 spectral truncations (the associated Gaussian grid has 20 latitude circles and 32 longitudes). Though this model reproduces the winter mean state fairly well, the distance index to observation is 0.37. As we can see, the value of 0.22 shows a satisfactory agreement. A smaller value may be found if we consider the 10-day forecasts by the ECMWF operational prediction model versus the corresponding analyzed situations during 6 winters (1979/80 to 1984/85). In this case the index is 0.17.

6. Conclusion

The ability of the ECMWF model to produce the large-scale variability of 500 mb height is easily shown for a limited domain, such as Europe-Atlantic, by comparing the first simulated EOFs to the observed ones. On the northern hemisphere, such a comparison becomes impossible. The number of independent phenomena having similar amplitudes is too large to allow a clear separation of the individual patterns by an EOF analysis based upon a 10-year sample. The direct comparison of the maps of the low order EOFs show a discrepancy between model and reality, even for the first EOF. If the model has the same modes of large-scale variability as reality on the Europe-Atlantic area, there is little reason why the same should not be true on the whole northern hemisphere. The comparison between model and reality must be done by looking at a set of low-order eigenvectors. With more than 3 eigenvectors, there is a fairly good agreement between model and reality. The distance index introduced here is thus very useful to compare two EOF analyses when the ratio: number of degrees of freedom of a field over the number of independent observations available, is not small enough to avoid numerical degeneracy, which is often the case in meteorology. This index may also be used when the EOFs are well separated, since it provides a quantitative similarity criterion between the maps.

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