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A study of the eddy kinetic energy of the Southern Hemisphere during January and July with emphasis on the FGGE year

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ABSTRACT

The eddy kinetic energy (EKE) of the Southern Hemisphere was studied for January 1979 and July 1979 in terms of zonal wavenumber using the FGGE analyses of the European Centre for Medium Range Weather Forecasts. The EKE, the production of EKE, the transfer of EKE to the zonal mean, and the transfer of energy among waves by non-linear interactions were computed for wavenumbers from 1 to 10. In January, the EKE was a maximum at wavenumber 5 and in July it was a maximum at wavenumber 1 with a secondary maximum at wavenumber 5. In these 2 months and 10 wavenumbers, the major source of energy in the hemispheric EKE budget was the conversion of EAPE to EKE which had maxima at wavenumber 5. In general, each of the wavenumbers transfers energy to the zonal mean with the maximum at or near wavenumber 5. In January, the non-linear interaction term is a source of energy in the medium scales and a sink of energy in shorter and in longer scales while the opposite was the case in July. Some of the above calculations were done for an average of 5 Januaries and 5 Julys using the global analyses of the US National Meteorological Center and the character of the energy cycle revealed by the high quality FGGE data set was also evident in the operational analyses.

1. Introduction

There are very few data sets of sufficient quality available which permit the computation of atmospheric energetics statistics in the Southern Hemisphere. Price (1973) used the data collected during the International Geophysical Year (IGY) to compute estimates of the monthly means of zonal available potential energy (ZAPE), eddy available potential energy (EAPE), zonal kinetic energy (ZKE), and eddy kinetic energy (EKE) in the lower troposphere for the period from July 1, 1957 to December 31, 1958. Computation of similar statistics for the Northern Hemisphere revealed differences between the two hemispheres. Annual cycles in the energetics have less amplitude in the Southern Hemisphere than in the Northern Hemisphere. The partitioning of the total energy between zonal and eddy components results in a larger proportion of the total energy in the zonal component in the Southern Hemisphere and that over a period of a year the Southern Hemisphere contains more energy than the Northern Hemisphere.

The Global Weather Experiment (FGGE) produced an even higher quality more extensive data set which permits the calculation of more detailed energetics statistics of the Southern Hemisphere. The present study will give statistics for the first ten zonal wavenumbers of not only the EKE but also statistics of the transfer of kinetic energy among various wavenumbers and the production of EKE.

For comparison purposes, some of the above calculations were done for an average of 5 Januaries and 5 Julys using the global analyses of the American National Meteorological Center (NMC).

2. Data and analysis

The EKE can be expressed in terms of zonal wavenumbers. In a manner similar to Saltzman (1957), the vertically-integrated, time-averaged, and area-weighted budget for each of the spectral components at a given latitude can be written,

$$\frac{\partial}{\partial t} K(m,\phi) = L(m,\phi) - M(m,\phi) + C(m,\phi) - D(m,\phi), \qquad (1)$$

where K is the kinetic energy, L is the exchange of energy among waves, M is the exchange between waves and the zonal mean, C is the production of energy by conversion of EAPE to EKE and the convergence of the meridional flux of potential energy, and D is the dissipation of energy. The expressions used to compute K, L, M, and C are contained in the Appendix. Eq. (1) can be averaged in latitude over the Southern Hemisphere and upon separation of the boundary terms following Kanamitsu et al. (1972), one obtains

$$\frac{\partial}{\partial t} \langle K(m) \rangle = \langle L(m) \rangle - \langle M(m) \rangle + \langle C(m) \rangle - \langle D(m) \rangle + \mathrm{CB}(m) + \mathrm{LB}(m), \qquad (2)$$

where CB is the work done by the pressure forces at the boundaries and LB is the boundary flux of EKE.

During the FGGE year, 2 primary level III-b data sets were produced; one by the European Centre for Medium Range Weather Forecasts (ECMWF), and the other by the Geophysical Fluid Dynamics Laboratory (GFDL). Kung and Tanaka (1983) made a comparison of global spectral energetics calculations from the two datasets during the two Special Observing Periods (SOP) of FGGE. Their results showed that there was reasonable agreement between the two sets of values computed for the kinetic energy storage term $K(m)$, the wave-zonal interaction term $M(m)$ and the wave-wave interaction term $L(m)$ and that the agreement was better during the second SOP. The agreement between the two estimates for the APE to KE conversion term was poor with the ECMWF values being lower than those of GFDL. Kung and Tanaka computed this term using the vertical motion and the specific volume and attributed the difference between the two estimates as arising from differences in the initialization schemes used in the data production. The ECMWF used an adiabatic non-linear normal mode initialization procedure which underestimated the vertical motion especially in the tropics. The GFDL scheme was less restrictive and produced stronger vertical motions. This result suggests that estimates of the APE to KE conversion term computed from the initialized ECMWF analyses will be less reliable than those obtained from the GFDL analyses.

One avoids the initialized ECMWF analyses when computing the EKE production term which is expressed in terms of the horizontal wind and the geopotential, neither of which has been initialized. In addition to the initialization procedures, there are other differences in the two data assimilation schemes which are, as Kung and Tanaka point out, reflected in the analyses. The ECMWF assimilation scheme is subject to a geostrophic constraint which is absent in the GFDL system and this results in higher levels of divergence in the GFDL analyses which in turn influences the values of the EKE production term. This was also discussed in Kung and Tanaka who displayed the vertical profiles of the global averages of $-\mathbf{V}\cdot\nabla\Phi$ computed from the two datasets. Although this result shows that there are differences in the two values, it is not obvious which analyses are preferable for a given application.

The primary data used for the present study were the January and July ECMWF level III-b FGGE analyses which are available as point values on a 1.875° by 1.875° latitude-longitude grid at 15 pressure levels from 10 mb to 1000 mb. The grid point fields were converted to an array of spherical harmonic coefficients with a triangular truncation at 40 waves. The terms K, L, M, and C of (1) were evaluated every 12 h at 31 Gaussian latitudes covering the entire Southern Hemisphere. All horizontal derivatives were evaluated exactly and finite-differences were used to compute derivatives with respect to pressure.

The secondary data used were the global analyses of the NMC for 5 Januaries and 5 Julys from 1979 to 1983. The most significant difference between these analyses and those of ECMWF is that they are operational analyses and as a result do not have an augmented observing system available to them such as the one that was implemented during FGGE. Even the operational analyses produced during the FGGE year were subject to a data cut-off time of 10 h which prevented most of the data from the special observing systems from being incorporated into the NMC analyses. Consequently, these analyses will be less reliable than the level III-b analyses produced during the FGGE year which do contain the data from the augmented observing network.

The NMC analyses are available as point values on a 2.5° by 2.5° latitude-longitude grid at 12 pressure levels from 50 mb to 1000 mb. The fields were spectrally analyzed in terms of spherical harmonic coefficients with a triangular truncation at 32 waves and calculations with the divergence removed from the wind field and with the omega field set to zero were made every 12 h at 25 Gaussian latitudes covering the Southern Hemisphere for the 5 Januaries and the 5 Julys.

3. Results

The results of the computation of the $\langle K(m)\rangle$, $\langle L(m)\rangle$, $\langle M(m)\rangle$, $\langle C(m)\rangle$, and LB(m) from eq. (2) for the Southern Hemisphere for January 1979 from the ECMWF analyses are given in Table 1. Although the calculations were done with a truncation at 40 waves, only the first 10 wavenumbers are displayed. The boundary term CB(m) is not displayed since its absolute value was always less than 5.0×10^{-4} W m^{-2} and the $\langle D(m)\rangle$ term is not given because it was not possible to compute this term explicitly. The largest source of energy is provided by the conversion of EAPE to EKE which has a maximum at $m = 5$ and a secondary maximum at $m = 1$. Some of this energy is converted to ZKE as described by the $\langle M(m)\rangle$ term. The maximum transfer of EKE to ZKE also takes place at $m = 5$, and at $m = 1$ there is a weak conversion in the opposite sense from ZKE to EKE. The wave-wave interaction term, $\langle L(m)\rangle$, is a source of energy for $3 \geqslant m \geqslant 6$ and is a sink for the remaining wavenumbers given in Table 1. This is a rather unusual behaviour since in the Northern Hemisphere wave-wave interactions normally extract energy from the medium scales and contribute energy to both longer and shorter scales, Saltzman (1970) and Tomatsu (1979).

Table 1. *Terms of the eddy kinetic energy budget for the Southern Hemisphere for January 1979 from the ECMWF level III-b analyses. The units of* $\langle K\rangle$ *are* 10^4 *J m*$^{-2}$ *and the units of* $\langle C\rangle$, $\langle L\rangle$, $\langle M\rangle$, *and LB are* 10^{-2} *W m*$^{-2}$

m	$\langle K\rangle$	$\langle C\rangle$	$\langle L\rangle$	$\langle M\rangle$	LB
1	6.2	33.1	−1.1	−0.1	−2.3
2	4.8	26.1	−8.4	2.8	−0.5
3	6.3	29.4	0.3	2.1	−0.1
4	5.1	26.3	4.0	3.9	−0.5
5	10.9	33.6	1.0	15.9	−0.2
6	6.6	31.0	3.1	11.4	−0.4
7	5.3	24.6	−3.1	6.5	−0.1
8	3.1	22.7	−6.2	6.5	−0.1
9	2.4	17.3	−1.1	3.8	−0.1
10	1.6	9.9	−0.2	3.3	−0.0

The prominence of wavenumber 5 in the Southern Hemisphere summer has been reported in the literature. Salby (1982) observed a large amplitude wavenumber five in the Southern Hemisphere mid-latitude height and temperature fields during FGGE and Hamilton (1983) used the height field analyses of the Australian Bureau of Meteorology (ABM) for the period 1972 to 1979 to show a similar spectral peak which occurred most often at $m = 5$ and there were a few cases of $m = 4$ or $m = 6$. These investigations also showed that wave propagated steadily eastward with a period averaging about eleven days and that it had a slight westward tilt with height.

Fig. 1 is a wavenumber-latitude plot of the vertically-integrated, time-averaged, and area-weighted EKE, $K(m, \phi)$, from eq. (1). This clearly shows the $m = 5$ maximum at 47°S. Also evident is the fact that the EKE decreases from the $m = 5$ maximum more rapidly with decreasing wavenumber than with increasing wavenumber. A similar result was obtained by Kung and Tanaka (1983) who computed the $K(m, \phi)$ for the first six wavenumbers during the first SOP (Jan. 5, 1979 to March 5, 1979) from the ECMWF, GFDL and NMC analyses. Fig. 2, which is a wavenumber-latitude plot of the EKE production term, $C(m, \phi)$, shows that at 47°S this term is largest at $m = 5$ and $m = 6$ and decreases gradually with increasing wavenumber. Between $m = 4$ and $m = 5$, there is a change in the sign of this term. It is probable that the weak EKE in the low

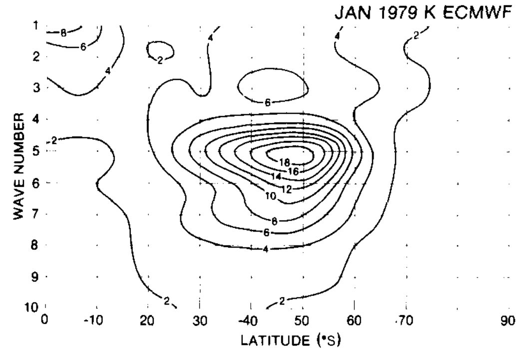

Fig. 1. A latitude-wavenumber plot of the time-averaged, vertically-integrated, and area-weighted kinetic energy storage term K for the Southern Hemisphere for January 1979 from the FGGE analyses of the ECMWF. The contours are labelled in units of 10^4 J m^{-2}.

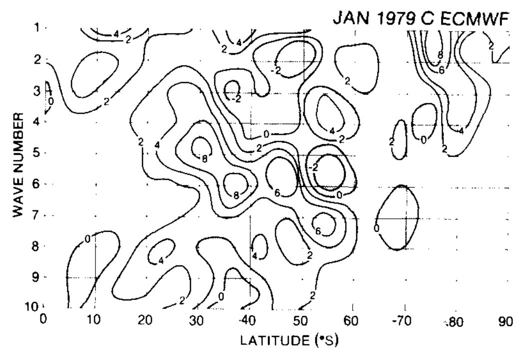

Fig. 2. Same as Fig. 1 except for the EKE production term C with the contours labelled in units of 10^{-1} W m^{-2}.

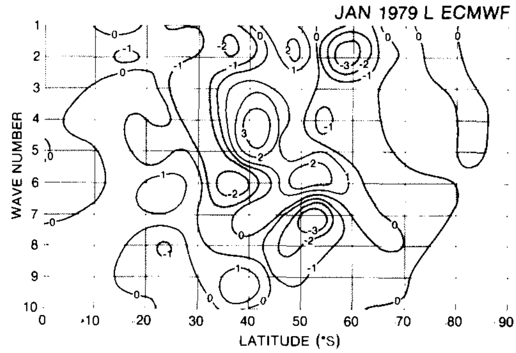

Fig. 3. Same as Fig. 2 except for the wave-wave interaction term L.

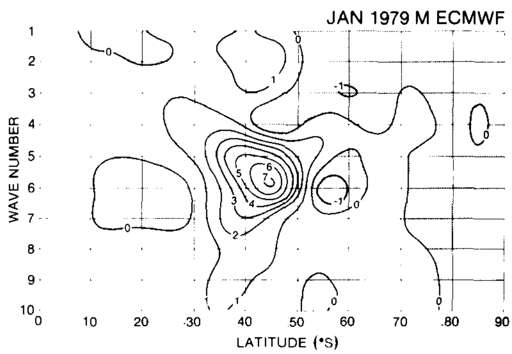

Fig. 4. Same as Fig. 2 except for the wave-zonal conversion term M.

wavenumbers results from the lack of production of EKE in the large scales and suggests that the spectrum of the $C(m, \phi)$ term is important in determining the form of the EKE spectrum. The $m = 5$ EKE maximum is accentuated by the wave-wave interaction term, $L(m, \phi)$, which is given in Fig. 3. In interpreting these results, it must be remembered that exchanges in energy are possible in both wavenumber and latitude and no attempt has been made to separate the two processes. At 47°S the $L(m, \phi)$ term changes sign between $m = 6$ and $m = 7$ with the result that it is contributing to the growth of the EKE maximum and reducing the amplitudes of the shorter waves. Fig. 4 displays the wave-zonal conversion term $M(m, \phi)$. This term is a sink of energy at 47°S for all wavenumbers except $m = 4$ and the main result of this term is the shifting of the latitude of the EKE maximum southward.

Kung and Tanaka (1984) computed the latitudinal distribution of the APE to KE conversion term computed from the vertical motion and the specific volume fields for wavenumbers from 0 to 5 for the first SOP but their results are not directly comparable to the January $C(m, \phi)$ of the present study which were computed in terms of the horizontal wind and geopotential. However, it can be seen that both results show strong conversions at wavenumber five and relatively weak values for $m < 5$ in the Southern Hemisphere.

The hemispheric results for July 1979 using the ECMWF analyses are given in Table 2 and they

Table 2. *Same as Table 1 except for July 1979*

m	$\langle K \rangle$	$\langle C \rangle$	$\langle L \rangle$	$\langle M \rangle$	LB
1	16.4	34.7	17.7	6.1	0.4
2	7.9	21.5	7.6	10.0	0.3
3	9.1	33.9	−1.9	6.0	0.3
4	8.7	48.9	−3.1	12.7	0.1
5	10.1	56.8	−15.5	12.4	0.1
6	7.5	45.4	−10.9	7.5	0.1
7	4.5	31.0	−6.6	5.6	0.1
8	3.8	22.8	−2.7	3.9	0.1
9	2.3	14.6	−2.6	2.4	0.0
10	1.7	7.8	3.9	2.2	0.0

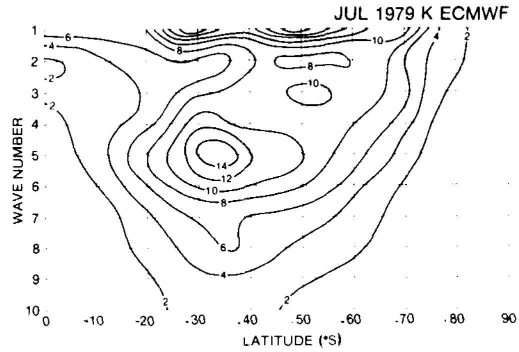

Fig. 5. Same as Fig. 1 except for July 1979.

show that the EKE maximum is at $m = 1$ and that there is also a peak at $m = 5$. The main source term for all wavenumbers is the EAPE to EKE conversion which is a maximum at $m = 5$. A portion of the energy resulting from this conversion is transferred to ZKE as described by the $\langle M(m) \rangle$ term with, the largest transfers at $m = 4$ and $m = 5$. The wave-wave interaction term, $\langle L(m) \rangle$, is a sink of energy in the range $3 \leqslant m \leqslant 9$ and is a source for $m = 1$, $m = 2$, and $m = 10$. Many studies of the Northern Hemisphere winter show an anomalous behaviour of $\langle L(2) \rangle$ in that it is a large sink in the wavenumber two energy budget. This has led Saltzman and Teweles (1964) and Tsay and Kao (1978) to attribute the anomalous behaviour to the effects of the ocean-continent distribution in the Northern Hemisphere. The results of this study lend support to this idea since $\langle L(2) \rangle$ does not exhibit an anomalous character in the Southern Hemisphere winter.

Fig. 5 displays the EKE storage term, $K(m, \phi)$, for July 1979. There is a maximum at $m = 5$ and 33°S which is broader and less intense than the one for January 1979. Fig. 6 shows the wavenumber-latitude plot for the EKE production term, $C(m, \phi)$. At the latitude of the $m = 5$ peak, this term is a source of energy at all wavenumbers and has a broad maximum at $m = 5$. It does not exhibit an abrupt change of sign at low wavenumbers as it did in January. Fig. 7 gives the wave-wave interaction term, $L(m, \phi)$. At the latitude of the EKE $m = 5$ maximum, this term is a relatively large sink of energy near the spectral peak and is a weaker sink or a source at other wavenumbers and as a result is acting to broaden and reduce the $m = 5$ maximum in the EKE. The wave-zonal transfer term, $M(m, \phi)$, is displayed in Fig. 8. It is a weak source of energy at 33°S and results in southward movement of the EKE maximum.

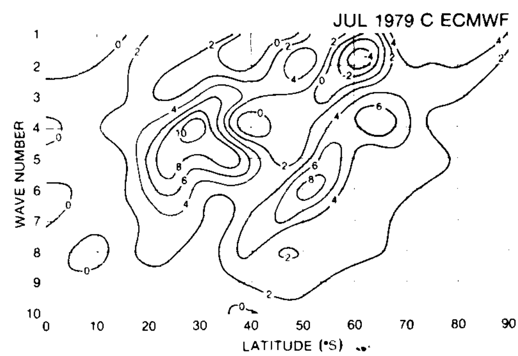

Fig. 6. Same as Fig. 2 except for July 1979.

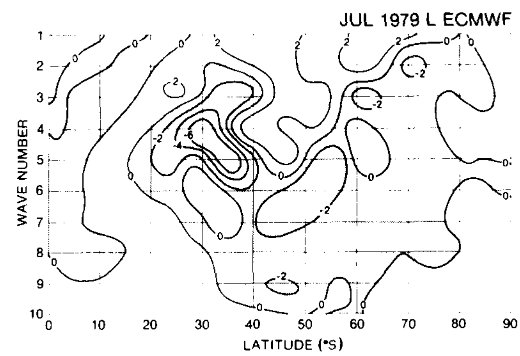

Fig. 7. Same as Fig. 3 except for July 1979.

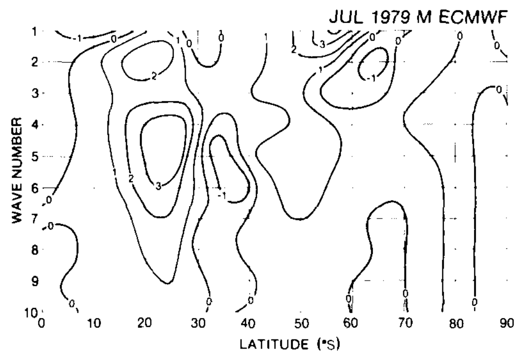

Fig. 8. Same as Fig. 4 except for July 1979.

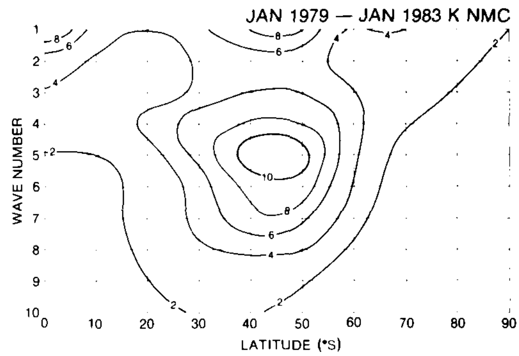

Fig. 9. A latitude-wavenumber plot of the time-averaged, vertically-integrated, and area-weighted kinetic energy storage term K for the Southern Hemisphere for an average of the 5 Januaries between 1979 and 1983 from the NMC analyses. The contours are labelled in units of 10^4 J m^{-2}.

Table 3. *Same as Table 1 except for an average of the 5 Januaries between 1979 and 1983 from the NMC analyses*

m	$\langle K\rangle$	$\langle L\rangle$	$\langle M\rangle$	LB
1	8.4	−1.3	0.7	0.1
2	5.4	−2.9	0.6	−0.2
3	5.6	−3.3	−0.9	0.2
4	6.8	1.5	7.5	−0.1
5	6.8	2.0	6.9	−0.1
6	5.4	−2.0	5.9	−0.0
7	4.6	−2.1	6.7	−0.1
8	3.1	−0.9	4.7	−0.1
9	2.2	0.8	3.1	0.0
10	1.7	0.1	2.6	0.0

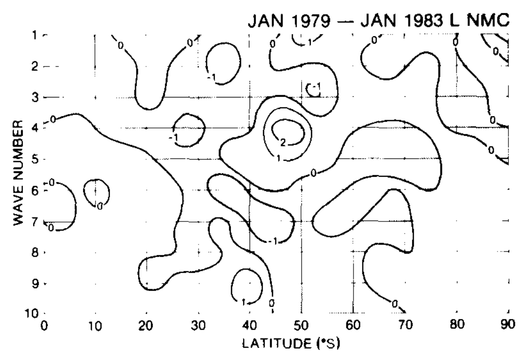

Fig. 10. Same as Fig. 9 except for the wave-wave interaction term L with the contours labelled in units of 10^{-1} W m^{-2}.

Table 3 displays the first ten wavenumbers of $\langle K(m)\rangle$, $\langle L(m)\rangle$, and $\langle M(m)\rangle$ which were calculated with a truncation at 32 waves for an average of 5 Januaries using the NMC global analyses. It was felt that the quality of these analyses would not support the calculation of the EKE production term and as a result, the $\langle C(m)\rangle$ and the $C(m,\phi)$ terms were not computed. The budget quantities behave in a similar fashion to those of the preceding ECMWF results for January 1979; i.e., the EKE has a local maximum at $m=4$ and $m=5$, the wave-wave interaction term, $\langle L(m)\rangle$, is a source of energy in this wavenumber band, and in general there is a transfer of EKE to ZKE with the maximum value at $m=4$.

Fig. 9 shows the wavenumber-latitude plot for $K(m,\phi)$. This shows an EKE maximum at $m=5$ and 45°S. The EKE maximum in 1979 was at $m=5$, in 1980 at $m=5$, in 1981 at $m=4$, in 1982 at $m=5$, and in 1983 at $m=6$. This variability in the position of the maximum results in the spectral peak for the 5-year average being less pronounced than that of Fig. 1. Fig. 10 gives the wave-wave interaction term, $L(m,\phi)$. This term has a similar behaviour to that of January 1979 from the ECMWF analyses in that it is sharpening the EKE maximum at $m=5$. The wave-zonal conversion term, $M(m,\phi)$, displayed in Fig. 11, transfers energy to ZKE at all wavenumbers and the maximum occurs at $m=6$.

The NMC analyses were also used to compute

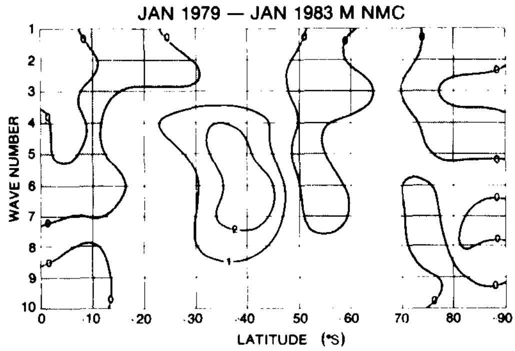

Fig. 11. Same as Fig. 10 except for the wave-zonal conversion term M.

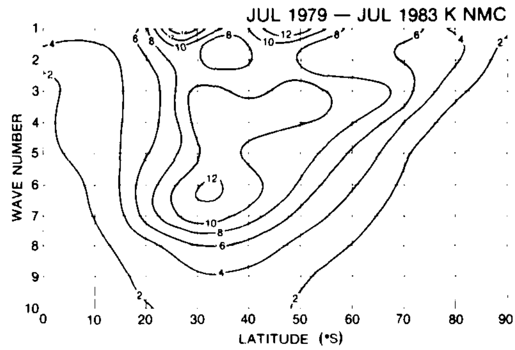

Fig. 12. Same as Fig. 9 except for an average of 5 Julys between 1979 and 1983.

Table 4. *Same as Table 1 except for average of the 5 Julys between 1979 and 1983 from the NMC analyses*

m	$\langle K \rangle$	$\langle L \rangle$	$\langle M \rangle$	LB
1	12.6	0.6	−0.2	0.1
2	8.4	7.5	4.0	0.1
3	10.1	3.7	7.8	0.0
4	9.0	−0.1	8.4	0.0
5	7.6	−7.4	8.7	0.1
6	7.9	−8.1	11.5	−0.0
7	6.5	−3.0	6.3	0.0
8	3.7	−1.5	2.9	0.0
9	2.6	−0.6	2.3	0.0
10	2.0	−0.5	1.0	−0.0

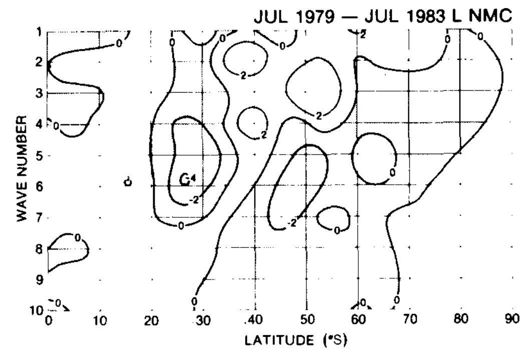

Fig. 13. Same as Fig. 10 except for an average of 5 Julys between 1979 and 1983.

some of the budget quantities for an average of five Julys. The hemispheric results are given in Table 4 and are similar to those for July 1979. The $\langle K(m) \rangle$ maximum is at $m = 1$ with peaks at $m = 3$ and $m = 6$. The $\langle M(m) \rangle$ term shows that $m = 6$ transfers the greatest amount of EKE to ZKE and that $m = 1$ receives a small amount of energy from the ZKE. The wave-wave interaction term, $\langle L(m) \rangle$, is a source of energy for $1 \leqslant m \leqslant 3$ and is a sink of energy for $4 \leqslant m \leqslant 10$.

Fig. 12 gives the EKE storage term, $K(m, \phi)$, for an average of the five Julys. This shows a large amount of EKE in $m = 1$ as well as a secondary maximum at $m = 6$ and 32°S. As was the case for the 5 January average, the spectral peak in the EKE did not occur at the same wavenumber in each of the individual years. For the five Julys, the peak in the EKE occurred at $m = 5$ in 1979, at $m = 6$ in 1980, at $m = 6$ in 1981, at $m = 4$ in 1982, and at $m = 7$ in 1983. The wave-wave interaction term, $L(m, \phi)$, is given in Fig. 13 which shows that at 32°S this term is small at most wavenumbers except $m = 6$ where it is extracting energy which results in a reduction and broadening of the $m = 6$ EKE maximum. The wave-zonal conversion term, $M(m, \phi)$, shown in Fig. 14 is a source of energy at 32°S and is only a weak function of wavenumber. This term is sink of energy both to the north and to the south of 32°S and therefore produces a meridional sharpening of the medium scale EKE maximum.

Price (1975) used the operational analyses of the ABM to compute monthly vertically-integrated APE and KE spectra for the Southern Hemisphere between 10°S and 70°S for the period May 1972 to April 1973. These single year results are similar to the multi-year NMC-based

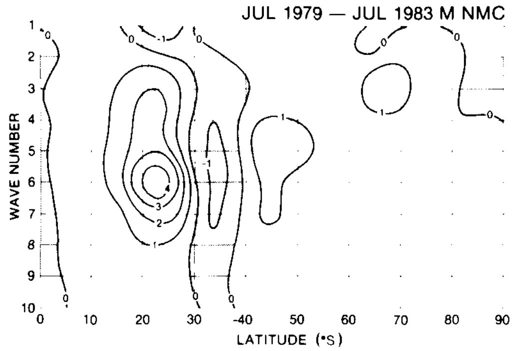

Fig. 14. Same as Fig. 11 except for an average of 5 Julys between 1979 and 1983.

calculations. The ABM analyses for January 1973 showed a mid-scale peak in the EKE spectrum at $m = 5$ but the large scales contained less energy than in the NMC spectrum. The July 1972 results show 2 mid-scale peaks in the spectrum at $m = 5$ and $m = 7$ which suggests that wavenumber 5 was dominant for part of the month and wavenumber seven was dominant for another.

Although the NMC-based wavenumber-latitude results and the corresponding ECMWF-based results are qualitatively similar, it can be seen that the maxima and minima are less intense in the NMC calculations. Price and Nicholls (1982) compared monthly averages of the APE and KE of the Southern Hemisphere from four years of operational analyses of the ABM with the similar result that the levels of energies tended to be higher in the ECMWF analyses. It is not clear whether the lower energies in the operational analyses result from the longer time average used in the calculations or from deficiencies in the analyses. However, the qualitative similarity of the results suggests that operational analyses are able to provide some useful information on the energetics of the Southern Hemisphere.

4. Conclusions

The EKE storage terms, $\langle K(m) \rangle$, for the Southern Hemisphere for January 1979 and July 1979 are similar for $m > 5$ including the occurrence of a spectral peak at $m = 5$. For the band $1 \leqslant m \leqslant 4$, the 2 months are quite different with July containing considerably larger amounts of energy. The wavenumber-latitude results show that the January $m = 5$ maximum is much sharper and about 30% larger than that of July.

Since the time-averaged budget used in this study describes only the changes in the kinetic energy in a given wavenumber with time, one is unable to make a definite statement concerning the contribution of the terms on the right-hand sides of (1) and (2) to the magnitude of the EKE storage term. In spite of this one can say that the major source term in the hemispheric EKE budget for each of the wavenumbers from 1 to 10 is the conversion of EAPE to EKE as described by the $\langle C(m) \rangle$ term which also has a maximum at $m = 5$ in both months. As a result, this term is probably important in explaining the observed spectral peaks at $m = 5$.

The character of the wave-wave interaction term, $\langle L(m) \rangle$, is completely different in the two months. In January 1979, this term was a source of energy for medium wavenumbers and a sink of energy at both higher and lower wavenumbers while in July 1979, the opposite was the case. The wavenumber-latitude results show that this term was sharpening and increasing the $m = 5$ peak in January and in July it was reducing and broadening the $m = 5$ maximum. Consequently, this term also appears to be important in explaining the sharp peak in the EKE spectrum. The importance of the wave-wave interaction term in explaining the growth and decay in each wavenumber is discussed in Tsay and Kao (1978) who showed that time rate of change of EKE is well correlated with the wave-wave interaction term.

The wave-zonal conversion term, $\langle M(m) \rangle$, behaves in a similar fashion in both months. In general, it is a sink of energy and tends to have its extreme value near the wavenumbers where the EKE is largest.

The character of the EKE storage term, the wave-wave interaction term, and the wave-zonal conversion term revealed by the ECMWF analyses for January 1979 and July 1979 was also evident in the results for averages of 5 Januaries and 5 Julys using less reliable operational analyses. This suggests that the results of this study reflect the climatology the Southern Hemisphere rather than being applicable only to a specific year.

5. Appendix

The terms in the energy budget eq. (1) were computed from the following expressions.

(a) The EKE:

$$\hat{K}(m) = U(m)\,U(-m) + V(m)\,V(-m).$$

(b) The EKE production:

$$\hat{C}(m) = -\frac{1}{a\cos\phi}\{U(m)\Phi_\lambda(-m) + U(-m)\Phi_\lambda(m)\} - \frac{1}{a}\{V(m)\Phi_\phi(-m) + V(-m)\Phi_\phi(m)\}.$$

(c) The EKE to ZKE transfer:

$$\hat{M}(m) = \frac{1}{a}\{U_\phi(0)(U(m)\,V(-m) + U(-m)\,V(m)) + V_\phi(0)(V(m)\,V(-m) + V(-m)\,V(m))\}$$
$$+ U_p(0)(U(m)\Omega(-m) + U(-m)\Omega(m)) + V_p(0)(V(m)\Omega(-m) + V(-m)\Omega(m))$$
$$+ U(0)\frac{\tan\phi}{a}(V(m)\,U(-m) + V(-m)\,U(m)) - V(0)\frac{\tan\phi}{a}(U(m)\,U(-m) + U(-m)\,U(m)).$$

(d) The wave-wave interaction:

$$\hat{L}(m) = -\sum_{\substack{l=-M \\ l\neq 0}}^{M}\Bigg(\frac{1}{a\cos\phi}[U_\lambda(l)\{U(m)\,U(-m-l) + U(-m)\,U(m-l)\}$$
$$+ V_\lambda(l)\{V(m)\,U(-m-l) + V(-m)\,U(m-l)\}]$$
$$+ \frac{1}{a}[U_\phi(l)\{U(m)\,V(-m-l) + U(-m)\,V(m-l)\} + V_\phi(l)\{V(m)\,V(-m-l) + V(-m)\,V(m-l)\}]$$
$$+ U_p(l)\{U(m)\Omega(-m-l) + U(-m)\Omega(m-l)\} + V_p(l)\{V(m)\Omega(-m-l) + V(-m)\Omega(m-l)\}$$
$$+ U(l)\frac{\tan\phi}{a}\{V(m)\,U(-m-l) + V(-m)\,U(m-l)\}$$
$$- V(l)\frac{\tan\phi}{a}\{U(m)\,U(-m-l) + U(-m)\,U(m-l)\}\Bigg),$$

where U is the Fourier coefficient of the zonal wind, V is the coefficient of the meridional wind, Ω is the coefficient of the vertical motion, and Φ is the coefficient of the geopotential and

$$F_x(m) \equiv \frac{\partial}{\partial x}F(m).$$

The following operator is applied to the above terms to produce vertically-integrated, time averaged, and area-weighted values.

$$(\quad) = \int_p \overline{(\hat{\ })}\cos\phi\,\frac{\mathrm{d}p}{g},$$

where the overbar is a time average.

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