*Tellus* (1986), *38A*, 378–379

LETTER TO THE EDITOR

# Comments on "A note on CISK in polar air masses"

By XIANG-YU HUANG and ERLAND KÄLLÉN, *Department of Meteorology, University of Stockholm, Arrhenius Laboratory, S-106 91 Stockholm, Sweden*

(Manuscript received December 2, 1985)

In a recent study, Bratseth (1985) presented a model to study the CISK problem. He starts out with a linearized, Boussinesq form of the primitive equations, proceeds with separation of variables and obtains the governing equation which only contains the variation in the vertical direction. By solving the differential equation separately in different vertical regions and matching the solutions at inner boundaries, the amplitude of the vertical velocity as a function of height is obtained and then the growth rate of the solution is computed. In the model, cloud base and cloud top are free parameters. The results indicate that CISK may be important if the diabatic heating takes place in a shallow layer at low levels.

The horizontal assymetry of the diabatic heating has also been incorporated into the model, according to the author. We do not think that this property of the diabatic heating is incorporated properly. In trying to model the absence of evaporational cooling in the region of sinking motion, Bratseth suggests the use of a function $1/\pi + \gamma \cos x$ to approximate the function

$$\begin{cases} \cos x & \text{for } -\frac{1}{2}\pi < x < \frac{1}{2}\pi \\ 0 & \text{for } \frac{1}{2}\pi < x < \frac{3}{2}\pi, \end{cases}$$

which describes the asymmetric horizontal diabatic heating field. (The two functions with $\gamma = 0.5$ are shown in Fig. 1 in Bratseth (1985).) When inserting this function into the governing equation of the model, Bratseth disregards the first term of the expansion given above (the term $1/\pi$). For a one-dimensional model, this is permissible as it is the Laplacian of the heating field which enters the governing equation. In a two-dimensional case, this is no longer true; here the term $1/\pi$ will give a contribution to the inhomogeneous part of the equation. In the following, we will show this in more detail.

By using an inviscid Boussinesq model and linearizing it about a state of test, Bratseth obtains a partial differential equation for the vertical velocity $w$:

$$f^2 \frac{\partial^2 w}{\partial z^2} + N^2 \nabla^2 w = \frac{g}{c_p T_0} \nabla^2 Q, \tag{1}$$

where $f$ is the Coriolis parameter, $g$ is the acceleration of gravity, $c_p$ is the specific heat of air, $N$ is the buoyancy frequency, $T_0$ is the characteristic temperature, $Q$ is the diabatic heating.

Simple structures are assumed for $w$ and $Q$:

$$w = \bar{w}(z,t) \cos(kx) \cos(ly), \tag{2}$$

$$Q = \bar{Q}(z,t)(1/\pi + \gamma \cos(kx))(1/\pi + \gamma \cos(ly)), \tag{3}$$

and for $\bar{Q}(z,t)$, the following vertical parameterization scheme is used:

$$\bar{Q}(z,t) = \begin{cases} \rho_s q_s L_v w_s / (\bar{\rho}(H_2 - H_1)) & \text{for } H_1 < z < H_2 \\ 0 & \text{otherwise,} \end{cases} \tag{4}$$

where $\rho_s$, $q_s$ and $w_s$ are the air density, specific humidity and vertical velocity at $z = 0$, $\bar{\rho}$ is the mean value of the density in the layer $H_1 < z < H_2$, $L_v$ is the latent heat, $H_1$ is the height of cloud base and $H_2$ is the height of the cloud top.

By inserting (2), (3) and (4) into eq. (1), we obtain for $H_1 < z < H_2$,

$$\frac{\partial^2 \bar{w}}{\partial z^2} \cos(kx)\cos(ly)$$

$$-\frac{N^2}{f^2}(k^2+l^2)\bar{w}\cos(kx)\cos(ly)$$

$$= -\frac{\gamma^2 g}{c_p T_0 f^2}(k^2+l^2)\bar{Q}\cos(kx)\cos ly$$

$$-\frac{\gamma g \bar{Q}}{\pi c_p T_0 f^2}[k^2\cos(kx)+l^2\cos(ly)]. \quad (5)$$

By only taking terms into account which vary horizontally according to $\cos(kx)\cos(ly)$, Bratseth obtains an equation for the amplitude of the vertical velocity, $\bar{w}$.

From eq. (5), we can clearly see that the last term gives no contribution after the projection, because it is orthogonal to the other wave component. Therefore, we may simply use a function having the form $\gamma \cos x$ to replace $(1/\pi + \gamma \cos x)$ and the model behaviour remains the same.

The parameter $\gamma$ is the one which Bratseth has chosen to study the effect of horizontal asymmetry of the abiabatic heating. After showing the sensitivity of the model to $\gamma$, he commented that "the practice of neglecting the adiabatic nature of the sinking air may lead to serious overestimates" (of the growth rate of the solution). However, if we use a horizontal parameterization scheme with a harmonic horizontal variation of $Q \sim \gamma \cos x \cos y$, i.e., with a cooling in the region of sinking motion, the asymmetric property of the diabatic heating cannot be captured. Because $\gamma$ is a scaling parameter for the diabatic heating $Q$, it certainly influences the growth rate of the solution, but as the model already contains a number of free parameters, $\gamma$ is unnecessary (see eq. (4)). The conclusions, reached by Bratseth, about $\gamma$ and their physical interpretations are thus questionable.

## REFERENCE

Bratseth, A. M. 1985. A note on CISK in polar air masses. *Tellus 37A*, 403–406.