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On the conjugate behaviour of weak along-shore flows

By ROGER L. HUGHES, *Department of Geology and Geophysics, Yale University, P.O. Box 6666, New Haven, CT 06511, U.S.A.*

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ABSTRACT

Using a simple analytical model containing a step shelf in a rotating ocean, confirmation is found for the idea that unstratified positively directed currents (poleward on western boundaries, equatorward on eastern boundaries) may be arranged in one of two structural forms each of which transports the same flux of each water type. In other words, the flux of water of each potential vorticity (or Bernoulli head) occurring within the current is the same for both structural forms. These structural forms are referred to as conjugate structural forms of the current. In agreement with an earlier study, it is shown that the wider of these two conjugate forms is subcritical to shelf waves and the narrower form is supercritical to the same waves. The present study seeks to understand the behaviour of these structures in the limit of weak flow. It is shown that as the velocity is decreased in a subcritical structure of fixed width, the conjugate supercritical structure continues to exist but with decreased width as well as velocity. At rest, conditions may be obtained by letting the flow tend to zero in either the subcritical structure of fixed width or its conjugate supercritical structure. However, the branch of the dispersion curve that comes to correspond to shelf waves in the at rest state depends on whether this state is approached as the weak flow limit of a subcritical or supercritical flow.

1. Introduction

Hughes (1985) has considered theoretical models of unstratified coastal flows over a sloping continental shelf. In that study, it was found that positive flows (that is poleward western boundary flows and equatorward eastern boundary flows) can often be structured in one of two alternative forms. These forms transport the same flux of water of each water type (where the potential vorticity or Bernoulli head denotes the water type). As an illustration, suppose the structural form of the flow is specified on one coastal section. After passing frictionlessly over some unspecified slowly varying bottom irregularity, it is ambiguous whether the coastal flow returns to its original structural form or its alternative form at some downstream coastal section. The dynamics of these two forms are strongly coupled and we refer to them as conjugate structural forms. The narrower of these conjugate forms has higher velocities than the wider form, enabling the total flow of each water type within the currents to be the same. Hughes argued in general that the narrower of these forms was supercritical to shelf waves (that is, long shelf waves are swept downstream by the flow) and the wider form was subcritical (that is, long shelf waves can propagate against the flow).

Collings and Grimshaw (1980) considered the behaviour of shelf and shear waves in an ocean with positive along-shore flow over a stepped topography. They did not interpret their flows as being single members of conjugate flow pairs. However, Collings and Grimshaw did find that narrow flows were supercritical and wide flows were subcritical. Although supportive, these results do no act as a clear confirmatory example of the criticality of conjugate structures because the current forces considered were not conjugate to each other. It is the purpose of this study to consider a model by Niiler and Mysak (1971) (which is a specific example of one considered by Collings and Grimshaw) with the aims of confirming the possibility of conjugate structural forms.

The study also aims at rationalising the existence of two conjugate structures in an ocean with a weak coastal current with the existence of only one possible structure for an ocean at rest. This latter aim is the resolution of possibly the most fundamental of dilemmas facing the use of hydraulic theory in the modelling of coastal currents. As the flow is weakened does the supercritical structure cease to exist (with an envisaged difficulty in functional continuity for this narrower solution regime) or does the supercritical structure exist down to zero flow (with an envisaged difficulty in its ability to advect waves downstream)? Furthermore, what happens at zero flow where it is known that an ocean at rest can "obviously" only exist in a single state? The resolution of this dilemma is an essential step in the development of an understanding of the role of hydraulic theory in the overall context of coastal currents.

2. Multiple states of the basic flow

Fig. 1 shows the velocity field and underlying bathometry of a flow considered by Niiler and Mysak. It is the aim of this section to show that there are two possible ways of arranging this flow (while maintaining the potential vorticity associated with each water column in the flow). We refer to the velocity field shown in Fig. 1 as the primary structure of the flow. The field to which it may be rearranged is referred to as the secondary structure.

This flow is composed of two water types. Water type (1) is that type of water on the shelf in Fig. 1. This water has a potential vorticity ε_1 and a volume flow of Q_1 where

$$\varepsilon_1 = \frac{f + v_0/x_0}{\lambda H}, \qquad Q_1 = \tfrac{1}{2}\lambda H v_0 x_0. \tag{2.1}$$

Water off the shelf but still within the current is denoted by water type (2). This water has a potential vorticity ε_2 and flow Q_2 of

$$\varepsilon_2 = \frac{f - v_0/x_0}{H}, \qquad Q_2 = \tfrac{1}{2}H v_0 x_0. \tag{2.2}$$

Here v_0, x_0, H and λ are as marked in Fig. 1 and f is the constant Coriolis parameter. Let us attempt to rearrange this flow as shown in Fig. 2. This figure shows water of type (1) within the region $0 \leqslant x < x_{\mathrm{I}}$ and water of type (2) within the region $x_{\mathrm{I}} \leqslant x < x_{\mathrm{s}}$. Notice that the vorticity of water of type (1) is the same in both Fig. 1 and Fig. 2. Similarly, the vorticity of water of type (2) is the same in both figures when $x > x_0$. However, where water of type (2) has moved up the step onto the shelf to occupy the region $x_{\mathrm{I}} \leqslant x < x_0$, the vorticity is changed from its value off the shelf in accordance with conservation of potential vorticity. A convenient way of thinking of the alternative flows in Fig. 1 and 2 is to consider the flow in Fig. 1, used by Niiler and Mysak, as the flow upstream of some topographic irregularity on the continental shelf, and then ask what are the possible structures of the flow downstream of the topographic irregularity assuming there is no dissipation within the flow?

Using Fig. 2, the flow of water of type (1) is

$$Q_1 = \lambda H(V x_{\mathrm{I}} + \tfrac{1}{2} v_0 x_{\mathrm{I}}^2/x_0), \tag{2.3}$$

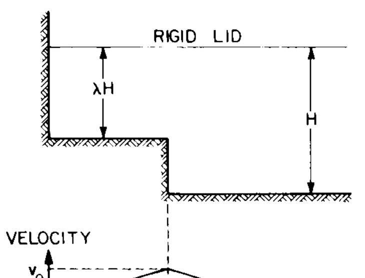

Fig. 1. Diagrammatic representation of the topography and along shore velocity field considered by Niiler and Mysak (1971). Here this velocity structure is referred to as the primary structure. The system rotates with constant Coriolis parameter, f.

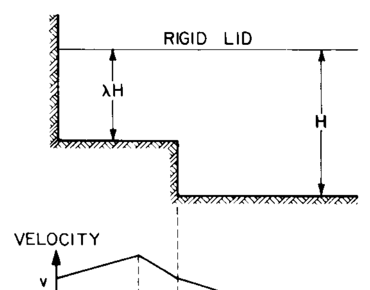

Fig. 2. Diagrammatic representation of the topography and the along-shore velocity field which is alternative to that in Fig. 1. The alternative velocity structure is referred to as the secondary structure.

and the flow of water of type (2) is

$$
\begin{aligned}
Q_2 &= \lambda H(V + v_0 x_1/x_0)(x_0 - x_1) \\
&\quad + \tfrac{1}{2}\lambda H[f(\lambda - 1) - \lambda v_0/x_0] \times (x_0 - x_1)^2 \\
&\quad + \tfrac{1}{2}H[V + v_0 x_1/x_0) \\
&\quad + (f(\lambda - 1) - \lambda v_0/x_0)(x_0 - x_1)]^2 \times x_0/v_0 \qquad (2.4)
\end{aligned}
$$

Equating (2.1) and (2.3), both of which give Q_1, and (2.2) and (2.4) both of which give Q_2, produces two equations for V and x_1. For our purposes, V, the velocity at the coast, is of little interest other than that we require it to be positive for the velocity field to be positive everywhere. This condition is equivalent to restricting attention to structures with $x_I \leqslant x_0$ as assumed in Fig. 2. Thus eliminating V to get a single equation for x_I gives the quartic:

$$
\begin{aligned}
\{x_I^3[-1 + 8F - 4F^2 + 4\Lambda F] \\
+ x_I^2 x_0[-1 - 4F + 4F^2 - 4\Lambda F] \\
+ x_I x_0^2[1 + 4F] + x_0^3\}\,(x_I - x_0) = 0, \qquad (2.5)
\end{aligned}
$$

where

$$\Lambda = \lambda - 1, \qquad F = R_0^{-1}(\lambda - 1), \qquad (2.6)$$

and R_0 is the Rossby number

$$R_0 = \frac{v_0}{f x_0}. \qquad (2.7)$$

For most of what follows, the non-dimensional change in depth, Λ, is a more convenient parameter than the non-dimensional shelf depth, λ.

The four roots of the quartic, (2.5), correspond to the primary structure

$$x_I = x_0, \qquad (2.8)$$

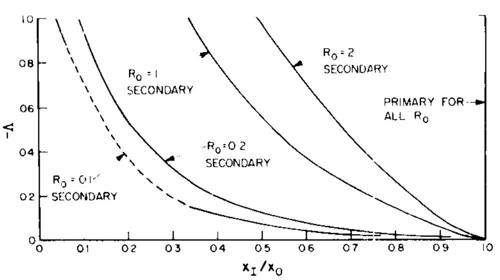

Fig. 3. Plot of the width of the inner water type (1), x_1, as a function of step rise for the primary and secondary flows of different Rossby numbers. The dashed portion of the curves represents cases where the entire current is on the step.

as required by the derivation of the equation, two meaningless roots and the desired secondary structure corresponding to the conjugate values of x_1/x_0 as the non-dimensional step rise $(-\Lambda)$ increases from zero towards unity is shown in Fig. 3 for various values of the Rossby number (2.7). The dashed portions of these curves correspond to the entire flow being on the shelf and they are not produced by (2.5) but are given by

$$x_1 = (S - (S^2 - 1)^{1/2})x_0, \qquad (2.9)$$

where

$$S = (1 - \Lambda R_0^{-1}(\Lambda + 1)^{-1})^{1/2}. \qquad (2.10)$$

For a small step rise Λ is a small negative number. For such a rise (2.5) and (2.8) imply

$$x_I \approx x_0 + 2R_0^{-1}\Lambda \qquad (2.11)$$

for the secondary structure. It follows that as the step rise decreases to zero, the two conjugate structures approach each other in form. In the limit of zero step rise, corresponding to a flat bottom, $\Lambda = 0$ and both conjugate structures become the same with $x_1 = x_0$. The necessity of shelf topography for the existence of distinct conjugate structures was noted by Hughes.

The above calculation has been given in terms of the width of the water type (1). The calculations could equally easily be presented in terms of the width of the total flow, x_s. This latter width, x_s, is related to the former width, x_1, by

$$x_s = x_0 + \frac{1}{2}\frac{x_0^2}{x_I} + \frac{1}{2}x_I - [\Lambda + 1 - R_0^{-1}\Lambda](x_0 - x_1) \qquad (2.12)$$

in regions where (2.5) is applicable (that is $x_s > x_0$) and

$$x_s = x_I + \frac{x_0^2 + x_I^2}{2x_I}[\Lambda + 1 - R_0^{-1}\Lambda]^{-1} \qquad (2.13)$$

in regions where (2.9) is applicable (that is $x_s < x_0$) As the step rise approaches the full depth of the flow, so that Λ tends to -1, the shelf has a negligible influence on the width of the flow x_s. Thus x_s tends to $2x_0$ and (2.12) implies

$$x_I = x_0 \frac{R_0}{R_0 + 2}, \qquad (2.14)$$

in agreement with Fig 3. Even though $x_s \approx 2x_0$ for both conjugate structures when the step rise is

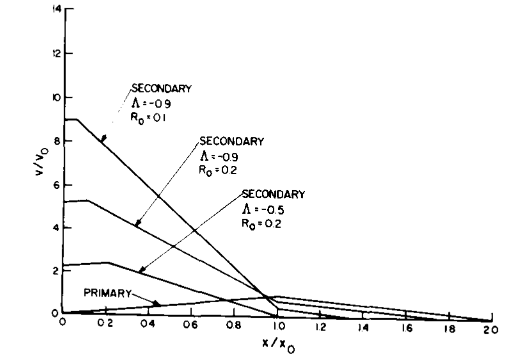

Fig. 4. Examples of the velocity structure of the flow for various step rise and Rossby number.

large, the value of x_I for the secondary structure as given by (2.14) is distinct from the value of $x_I = x_0$ for the primary structure. Thus x_I is possibly a better discriminator of the structure than x_s.

Fig. 4 illustrates the structure of the along-shore velocity for the primary structure and its conjugate secondary structure for three values of step rise and Rossby number. A slow but distinct maximum in the velocity occurs for the wide (primary) structure. A fast but less distinct maximum in the velocity occurs for the narrow (secondary) structure. The maximum occurs closer to the coast for the narrower structure than for the wide structure.

3. The relationship flow versus width

To understand more clearly how the primary and secondary structures are related, it is useful to study a double structured current as in Fig. 1 and 2 with the potential vorticity of both water types (1) and (2) and flow of only the water of type (1) given as before by (2.1) and (2.2); but the flow of type (2) water, Q_2, left unspecified. We ask: What are the possible current widths, x_s, as a function of the flow, $(Q_1 + Q_2)$?

Elementary considerations of the general form used in the previous section imply:

(i) an equation representing specification of the flow of type (1) water, this equation being

$$Vx_I + \tfrac{1}{2}\frac{v_0}{x_0}x_I^2 = \tfrac{1}{2}v_0 x_0; \tag{3.1}$$

(ii) an expression for the flow of type (2) water, Q_2, this expression being

$$\tfrac{1}{2}\frac{V_0}{x_0}(x_s - x_0)^2 + \tfrac{1}{2}\lambda\left[2\frac{v_0}{x_0}(x_s - x_0) - \left(-\lambda\frac{v_0}{x_0} + \lambda f - f\right)(x_0 - x_I)\right]\cdot(x_0 - x_I) = Q_2/H; \tag{3.2}$$

(iii) an equation requiring that the velocity at the outer edge, x_s, be zero, this equation being

$$V + \frac{v_0}{x_0}x_I + \left(-\lambda\frac{v_0}{x_0} + \lambda f - f\right)(x_0 - x_I) - \frac{v_0}{x_0}(x_s - x_0) = 0. \tag{3.3}$$

These equations form a set of three equations for the four variables x_I, x_8, V and Q_2. Eliminating x_I and V gives a single equation relating Q_2 and x_s. (Note that as Q_1 is specified variations in Q_2 represent variations in the total flow $(Q_1 + Q_2)$.) Fig 5 illustrates this relationship for various Rossby numbers and a step rise equal to half the depth, (i.e., $\Lambda = -0.5$). As expected from Section 2, solutions occur in conjugate pairs with a wide and narrow structure for a given value of Q_2. For low Rossby numbers, the locus (or relationship) extends into the region $x_s < x_0$ corresponding to the possibility of the narrow structural form being located entirely on the shelf. As shown by Hughes the wide structural form cannot lie entirely on the flat shelf, for this would require conjugate structures to exist over a flat bottom.

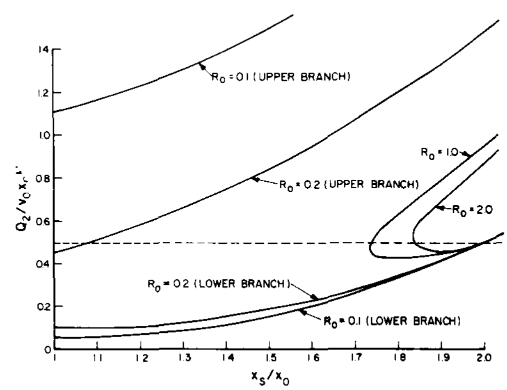

Fig. 5. Loci relating the flow of outer water type (2) to the width of the current with step rise equal to half the depth. Each locus corresponds to a fixed Rossby number and is traced by varying the velocity at the wall, V. The case of $Q_2/v_0x_0 = 0.5$ is that of interest here.

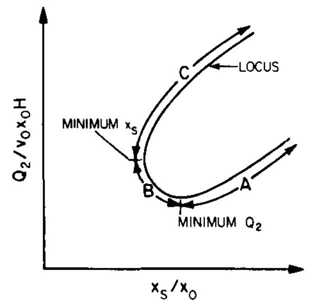

Fig. 6. Idealistic representation a locus from Fig. 5. The locus has been marked into three sections—"A", "B" and "C" for purposes of identifying the character of the dispersion relation at long wavelengths.

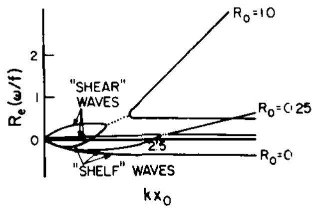

Fig. 7. Dispersion relation for waves in the situation represented in Fig. 1 with $\lambda = 0.5$. Only the real part of the frequency ω is plotted. The dotted portions of the curve represent unstable waves (i.e., ω is complex). R_0 is the Rossby number, v_0/x_0f.

Fig. 6 shows an idealistic representation of a typical locus of the form shown in Fig. 5. The locus has been divided into three parts marked A, B and C. Hughes considered the behaviour of long shelf waves in the presence of currents corresponding to points on the locus in the neighborhood of the junction of parts A and B. For currents in this neighborhood, the part of the locus marked A was identified as being associated with subcritical flows (that is long shelf waves can propagate against the flow) and the part marked B was associated with supercritical flows (that is long shelf waves are swept downstream by the flow). Away from the region of the locus considered by that local analysis, the behaviour of long waves has not been previously considered in items of the locus.

4. The behaviour of long waves

Fig. 7 is a modified version of the dispersion relation found by Niiler and Mysak for the case shown in Fig. 1 with $\Lambda = -0.5$. Consideration of the long shelf wave part of this dispersion relation shows that the current is a subcritical flow. Therefore, according to Hughes the current might be expected to be characterised by a point on the part of the locus marked A. This conclusion is in agreement with Section 2 where it was shown that the background flow shown in Fig. 1, (the primary structure) is the wider of the two conjugate possible flows.

It is instructive to determine examples of the behaviour of small amplitude long waves in all the three parts of the locus marked A, B and C in Fig. 6. Such a calculation will illustrate the way in which the flow influences the long wave dispersion relation away from the limited part of the locus considered by Hughes.

The method of determining the dispersion relation is standard and clearly presented for example by Niiler and Mysak. Here only the resulting form of the dispersion relation for the shelf and shear waves of the system in Fig. 2 is given. Irrespective of position on the locus and length of the wave, this relationship is:

$$\begin{aligned}\omega = R_0 f \cdot [&\sinh kx_s + c_1 \sinh k(x_s - x_1) \\ &+ c_2 \sinh k(x_s - x_0)]/[\exp kx_s \\ &+ c_1 \exp k(x_s - x_1) + c_2 \exp k(x_s - x_0)] \qquad (4.1)\end{aligned}$$

where as in Fig. 7, ω and k are the angular frequency and wavenumber of the wave and c_1 and c_2 are further functions of k, given by

$$c_1 = \frac{R_0 f}{\omega - v_1 k} [(\Lambda + 2) - R_0^{-1} \Lambda] \sinh kx_1, \qquad (4.2)$$

$$c_2 = \frac{\Lambda}{1 - \Lambda} [\cosh kx_0 + c_1 \cosh k(x_0 - x_1)], \qquad (4.3)$$

where v_1 is the value of the along-shore velocity at x_1, given by

$$v_1 = \frac{v_0}{2} \left(\frac{x_0}{x_1} + \frac{x_1}{x_0} \right). \qquad (4.4)$$

As in Fig. 5 we consider the case in which $\Lambda = -0.5$ and restrict attention to the case of $Q_2/v_0x_0H = 0.5$ as studied in Section 2. For this step size Fig. 8 shows four dispersion relation calculations corresponding to waves in the primary

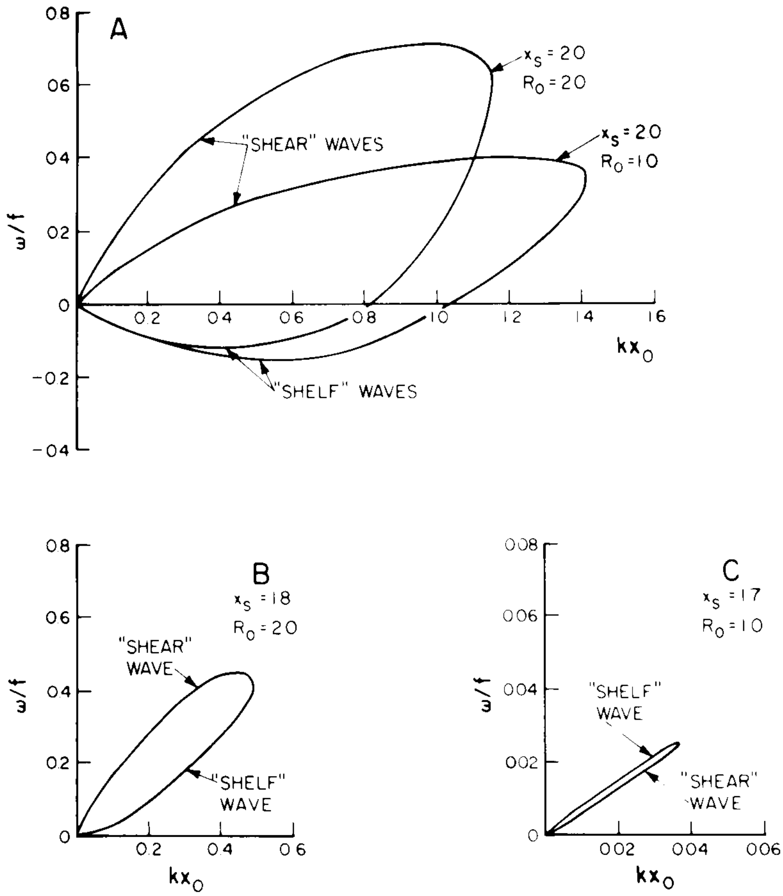

Fig. 8. Examples of the dispersion relating for long waves (up to the lowest unstable wavenumber). The case in Fig. 8A for which $x_s = 2.0$, $R_0 = 1.0$ should be compared with Fig. 7. Here Figs. 8A, B and C represent respectively dispersion relations applicable to basic currents of type "A", "B" and "C" on the locus of Fig. 6. Note the different scale for Fig. 8C.

and secondary structures for Rossby numbers equal to 2 and 1. Only wavenumbers less than that for which the shelf and shear wave modes combine to form an instability as in Fig. 7 are shown. Fig. 8 is divided into parts A, B and C corresponding to the basic current being represented by a point on the locus marked A, B or C. The case of a Rossby number of 2, correponds to a primary structure (the wide current form) of type A which is subcritical and a secondary structure (the narrow form) of type B which is supercritical as expected. In this case the shelf wave frequency ω_H is less than the shear wave frequency ω_c for both flow structures. Calculations at Rossby number greater than 2 show that as the Rossby number becomes large so that by Fig. 5 both the primary and secondary structural forms coalesce with $x_s \approx 2$, the speed of the shelf wave at zero wavenumber approaches zero as required by the local calculation of Hughes.

In the case of a Rossby number of 1, the primary structure (the wide form) is still of type A as expected but the secondary structure (the narrow form) is of type C rather than type B. Again the primary structure is subcritical and the secondary structure is supercritical. Interestingly, however, for the secondary structure the shear wave frequency ω_c is now less than the shelf wave frequency ω_H. Calculation of the dispersion relation for the secondary structure when the Rossby number is 1.5, approximately corresponding to a background current represented by the interchange from B to C, shows instability at exceeding long wavelengths. Thus it appears that at the transition point from type B to C, the corresponding basic flow has a long shear wave speed equal to the long shelf wave speed. This matching of long wave speeds results in the balloon-like structure of the dispersion relation (between the shear and shelf waves) collapsing so that instability may occur at zero wavenumber.

5. Behaviour of the secondary structure of weak flows

Having studied the Niiler and Mysak model (as a special case of the Collings and Grimshaw model) and found that this simple system does indeed behave in a manner similar to that given by Hughes, we are now free to address the dilemma stated in the Introduction. Fig. 3 shows that as the Rossby number appropriate to the primary flow is lowered, the (supercritical) secondary structure narrows and moves entirely onto the shelf, provided Λ does not equal zero or minus unity. Hence using (2.9), (2.10) and (2.13), the width of the flow of water type (1) and the width of the overall flow are given by

$$x_1 = \tfrac{1}{2}\left(\frac{\Lambda + 1}{-\Lambda}\right)^{1/2} R_0^{1/2} x_0, \tag{5.1}$$

$$x_s = \left(1 + \frac{2}{\Lambda + 1}\right) x_1, \tag{5.2}$$

respectively, for the secondary structure at low Rossby number R_0. It can be seen that the width of the flow varies as $R_0^{1/2}$. Thus if the flow is to vary as R_0 as required for compatability with the primary structure, the velocity within the secondary structure must also vary as $R_0^{1/2}$. Of importance here is that the secondary structure continues to exist at small Rossby numbers.

Conservation of potential vorticity arguments require that the shear within the flow of water of type (1) must be $f R_0$. However, within the flow of water of type (2) conservation of potential vorticity requires the shear to be $f\Lambda$ to dominant order in R_0. It can be seen that the shear in this latter water type is of unit order as required for the velocity and width of the flow to both be of order $R_0^{1/2}$. The existence of a unit order shear within the flow sustains the possibility of a strong influence of the current on a wave within the flow. Such a possibility would allow the flow to continue to be supercritical to a shelf wave even at small Rossby numbers when advection of the wave is clearly unimportant.

The locus (as in Section 3) for low Rossby number may be shown to be

$$Q_2 = \tfrac{1}{8}(-\Lambda)(\Lambda + 1) H f x_0^2 \left\{ \frac{x_s}{x_0} \pm \left[\left(\frac{x_s}{x_0}\right)^2 + 2R_0\Lambda^{-1} \right]^{1/2} \right\}^2. \tag{5.3}$$

This locus has a minimum in current width, x_s, of $(-2R_0\Lambda^{-1})^{1/2} x_0$. At this minimum, that separates regions C and B as in Fig. 6, $Q_2 = \tfrac{1}{4}(\Lambda + 1) H f x_0^2 R$. However, for the case of interest to us $Q_2 = \tfrac{1}{2} v_0 x_0 H = \tfrac{1}{2} H f x_0^2 R_0$ which is greater than the above flow separating regions C and B. Hence conditions are characterised by a point on the locus in region C with waves satisfying a dispersion relation of the character of Fig. 8c as expected from the trend within Fig. 5 that occurs as the Rossby number is decreased.

The number of waves in the long wave limit is equal to the number of locations at which either the vorticity or depth shows a discontinuity for a piecewise continuous model as here. For the primary structure discontinuities occur at x_0 and $2x_0$, giving two waves. Calculation of the dispersion relation appropriate to a low Rossby number secondary structure gives three branches in the long wave limit. The two of these branches appropriate to Fig. 8c are

$$\omega = k v_1, \tag{5.4a}$$

$$\omega = k V \tag{5.4b}$$

The third branch of the dispersion relation, which has no counter part in the primary flow as considered, is

$$\omega = k f x_0 \Lambda \tag{5.5}$$

Dispersion relations of the form (5.4) are also obtained if the shelf is neglected, while the dispersion relation (5.5) may be obtained if the current is neglected. Thus (5.4) unmistakably represent shear waves, while (5.5) represents the shelf wave. Interestingly the wave represented by (5.5) has no along-shore velocity reversal even in the presence of the current. Thus it is of the correct structure for the (first and only mode) shelf wave.

For low Rossby numbers, $x_s < x_0$ and a new far field water type is found. This new water type occurs on the shelf and invades the shelf as the Rossby number is decreased. The shelf wave corresponding to (5.5) propagates on the new potential vorticity discontinuity between the two far field water types. Thus in the low Rossby number limit, the shelf waves of the primary structure and the shelf wave of the secondary structure are actually different dispersion curve branches.

As the Rossby number decreases to zero, so does the velocity field of the flow upon which the shear waves propogate. (This decrease occurs as R_0^1 for the primary structure and $R_0^{1/2}$ for the secondary structure.) Thus the velocity amplitude of these waves must also decrease for linear theory to be appropriate. At zero Rossby number the velocity of the flow is zero as must be the amplitude of the shear waves.

6. Conclusions

By considering a simple stepwise shelf topography and a piecewise continuous along-shore velocity field, confirmation has been found for the predictions of conjugate flow behaviour with the wider structure subcritical and the narrower structure supercritical to shelf waves. While such an idea is not new, the present study has extended the previous study (of smoothly varying topographies and flows) by Hughes to behaviour away from conditions at which the structures coalesce and the shelf wave is critical.

It has been shown that both structures of a flow may exist at low Rossby numbers. The narrow supercritical flow adopts a form in which there exists a local shear of the order of the Coriolis parameter. Thus the narrower flow has a Rossby number based on local shear of the order of unity even though the overall Rossby number of the entire system may be small. It has been shown that at zero Rossby number both structures give the same at rest state with the same dispersion relation for the shelf wave within that system. However, the shelf wave appropriate to the zero Rossby number case corresponds to a different branch of the dispersion curve depending on whether the at rest state is reached through a subcritical or supercritical state.

The present model exhibited analytical simplicity. However, the simplicity of this model leaves unresolved puzzles concerning its applicability to predictions of the behaviour of more complicated systems. Although the present study is of little value in modelling realistic situations, it provides an important foundation for interpreting hydraulically based coastal models and hopefully it will increase confidence in such models.

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