

Systematic errors in the baroclinic waves of the ECMWF model

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ABSTRACT

The space-time spectral analysis technique is used to investigate the forecast errors of the baroclinic waves in the ECMWF forecast model. The study concentrates on two groups of disturbances, the medium-frequency baroclinic waves (MFBW: wavenumber 4 to 7, period 3.6 to 10.3 days) and the high-frequency baroclinic waves (HFBW: wavenumber 5 to 9, period 2.4 to 3.5 days).

Both mid-latitude wave types suffer a considerable weakening in the early stages of the forecast. The geographical distribution of this error shows the largest decrease over North America and the North Atlantic. A close connection between the forecast changes in the baroclinic waves and the time-mean state is indicated by the decrease of the baroclinicity of the mean flow over North America. In the Pacific region, an increased baroclinicity seems to be responsible for increased wave activity in the high-frequency range.

An increase of the SW to NE phase tilt of the MFBW's at low latitudes seems to be due to problems in the interaction of deep baroclinic waves with the tropical flow. In the vertical, the forward tilt of the temperature waves increases during forecast.

1. Introduction

Space-time spectral analysis has become a widely used tool for studying the large-scale travelling disturbances for both the observed and the simulated general circulation of the atmosphere. A review of the methods and applications by Hayashi (1982) demonstrates the power of this analysis to investigate the characteristics, structure and energetics of travelling waves in a wide range of time and space scales.

The application of the space-time decomposition technique to general circulation model output (Hayashi, 1974; Hayashi and Golder, 1977) and to observations or to both (e.g. Pratt, 1977 and 1979; Böttger and Fraedrich, 1980; Strauss and Shukla, 1981) reveals some differences in the transient wave statistics between models and the real atmosphere. However, wave statistics from the

general circulation experiments may depart from those observed due to large changes in the mean zonal flow of the forecast as well as through differences in the treatment of the transient waves themselves. Compared to long integrations, the short- to medium-range forecast can be seen as a much closer realization of real atmospheric wave activity. At the beginning of the forecast when the changes of the mean zonal flow are still rather small, it will be easier to understand the mechanisms which cause the differences in the wave statistics between the analysis and the forecast. To investigate these differences we will compare the space-time spectrum of forecast ensembles with the spectrum calculated from the corresponding analyses. We will spectrally analyse the ensembles of 1-day forecasts, 2-day forecasts etc; each of these ensembles can be thought of as a realization of a model atmosphere, provided the forecasts are sufficiently accurate to give reasonable time continuity.

In Section 2 we describe the data for the study while Section 3 discusses the methodology. The

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time-mean state of the atmosphere and the changes in the forecast are discussed in Section 4. The results are presented in Section 5–7. In Section 5 we examine the inconsistencies in successive forecasts. In Section 6, 7 the forecast errors for two wavenumber frequency groups (medium frequency baroclinic waves and high frequency baroclinic waves) are discussed in detail.

2. Data

The data used in this study was extracted from the ECMWF analysis and forecast archive. The observational data set for the space-time decomposition consists of 112 initialized analyses valid for 12 GMT starting from 20 November and ending at 11 March for the two winter seasons 1980/81 and 1981/82. Similar calculations have been carried out for the succeeding winter seasons until 1983/84. The initialized analysis is the final product of the ECMWF data assimilation system (Bengtsson et al. 1982). From this analysis, the 10-day forecast is run operationally on a daily basis; major features of the forecast system are described in Hollingsworth et al. (1980). From the 10-day forecast sets, equal time steps verifying on the same analysis ensemble element number were selected to form a forecast ensemble of the same length as the analysis time series. The wavenumber-frequency analysis was carried out on 9 pressure levels (1000, 850, 700, 500, 400, 300, 200, 100, 50 mb) and on a latitude band from 25° N to 76° N in 3° intervals. This space domain will be sufficient to look at major features of the mid-latitude baroclinic waves in the northern hemisphere.

The data set for the space-time decomposition is not uniform throughout the period of investigation. A number of changes in the data assimilation system and in the forecast model have been made within the two winters and in the period which separates them. The effect of single changes in the data assimilation or forecast system on the space-time analysis is difficult to assess, but from the results we will see that the major model errors for the baroclinic waves are very similar in the two winters.

3. Methodology

As in many other investigations (e.g. Hayashi, 1974; Hayashi and Golder, 1977; Fraedrich and

Böttger, 1978; Pratt, 1979; Straus and Shukla, 1981; Mechoso and Hartmann, 1982), we will use the space and time decomposition technique proposed by Hayashi (1971). In this, the disturbances, as defined by deviations from the zonal mean, are decomposed into progressive and retrogressive waves

$$W(x, t) = \sum_k \sum_{\omega} R_{k, \pm \omega} \cos(kx \pm \omega t + \phi_{k, \pm \omega}),$$

where k is the zonal wavenumber and ω the frequency; $R_{k, -\omega}$, $\phi_{k, -\omega}$ and $R_{k, +\omega}$, $\phi_{k, +\omega}$ are the amplitude and phase angle of the progressive and retrogressive waves, respectively. A brief description of the wavenumber-frequency analysis technique is given in Appendix A. The interpretation of the amplitudes of progressive and retrogressive waves becomes difficult when a large standing part is present in the transient wave activity (Pratt, 1976). This affects mainly the ultra-long waves in the low-frequency range. For the medium frequency and high frequency baroclinic waves discussed here, this problem is negligible as the standing part of the power spectrum is very small compared to the travelling part.

Before we begin the detailed discussion of the ensemble spectral analysis, it is important to consider what kind of forecast errors we can detect when we compare the spectrum of one forecast ensemble with the spectrum of the initialized analysis. During the forecast the baroclinic waves may experience a systematic change in their amplitude, spatial phase structure or phase speed. Systematic amplitude changes will be seen as changes in the variance for particular wavenumber-frequency bands, whilst changes in the spatial phase structure can be detected by level to level cross-spectral analysis (see Appendix A) or by latitude to latitude cross-spectral analysis. If the waves are generally too slow or too fast we will see a phase difference in the cross-spectral analysis between the forecast ensembles (D + X) and analysis ensembles.

Apart from the systematic errors, we can expect that the amplitudes and phases will undergo random changes in different forecasts. When the prediction range increases the forecasts diverge from each other and irregularities in the wave phases and amplitudes will enter the time series of the forecast ensemble. This will increase the noise

level of the wavenumber-frequency spectrum and will have a smoothing effect where significant peaks are present in the spectrum of the analysis sample. Generally the spectrum will have a trend towards a white noise spectrum with advancing forecast time.

4. Time mean state

The development of baroclinic waves is closely related to the state of the mean zonal flow, as many theoretical investigations and numerical integrations have shown. This will be discussed in connection with the presentation of the results in Section 5. Therefore, when we describe the baroclinic disturbances in terms of deviations from the zonal mean, it is useful to know the time-mean and zonal-mean state for the period for which the spectral analysis has been carried out. In the discussion of the wavenumber-frequency analysis we can then refer to the structure of the mean state. We will be interested in the position of the disturbances relative to the main baroclinic zones of the mean field. Furthermore it will be important to see if changes of the baroclinic waves in the forecast are directly connected to changes in the mean fields. Our description of the mean state will be restricted to the temperature and the zonal component of the wind field. The zonal and time averaged fields in the initialized analysis are very similar in the two winters 1980/81 and 1981/82. As we also found a close agreement in the forecast errors of the mean fields for the two winters, we will present only the results for one winter. The longitudinal variation of the time-mean flow, however, shows differences in the two winters which are worth showing as they reveal connections with the geographical distribution of the variance of the baroclinic waves (see Sections 6 and 7).

In the zonal mean temperature field (Fig. 1) we find a strong baroclinic zone around 30°N and so the thermal wind equation implies a large vertical increase of the mean zonal flow in this region. The tropopause is well marked and shows the typical downward slope with increasing latitude. Examination of the difference pattern ($D + 3$ minus $D + 0$) reveals that there is mid-tropospheric cooling and warming near the surface. The mid-tropospheric cooling has a clear minimum at latitudes around 36° to 40°N and the largest

temperature changes take place near the low latitude tropopause. Strong cooling in the stratosphere and warming of the upper troposphere indicates that the forecast shifts the tropopause slightly upwards; it also reduces the sharpness of the tropopause. The mid-tropospheric cooling by day 3 leads to a destabilization of the lower troposphere and a more stable upper troposphere. The stratospheric cooling has a destabilizing effect as well.

The configuration of the mean zonal flow is closely connected to the location and structure of the baroclinic waves. Fig. 2 shows that the subtropical jet has a maximum above the major baroclinic zone at latitudes around 30° and the axis of the wind maximum tilts equatorward with height.

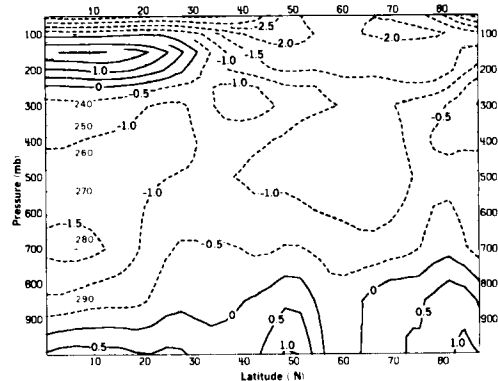


Fig. 1. Latitude-height plot of the zonal and time mean temperature in the initialized analysis (dotted lines) and the difference between the forecast and the analyses ($D + 3 - D + 0$, thicker lines). Winter 1980/81, time averaging period 112 days. Units: K.

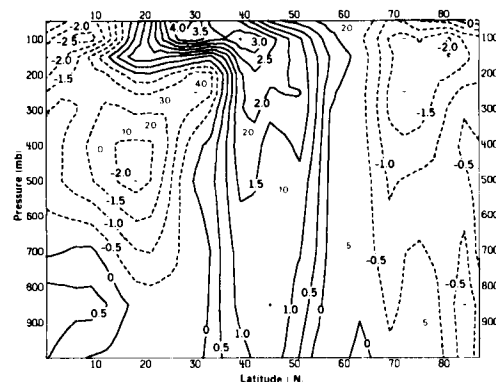


Fig. 2. As Fig. 1 except for the mean zonal wind. Units: m s^{-1} .

The pattern of the difference between the forecast and the analysis indicates a poleward and upward shift of the subtropical jet. The geostrophic mean zonal flow has the same difference pattern indicating that the major changes of the zonal flow are coupled with the changes of the thermal structure via the thermal wind equation. The change of the mean zonal flow at middle latitudes has a barotropic part of nearly 1 m s^{-1} due to changes of the

surface pressure (Arpe, 1983). The baroclinic changes correspond to an increase of the baroclinicity at middle latitudes and a decrease at lower latitudes.

The mean flow in the middle troposphere is illustrated by the time-mean geopotential height field at 500 mb (Figs. 3, 4). The major troughs over the east coasts of North America and Asia can be found in both winters. Over the west coast of North

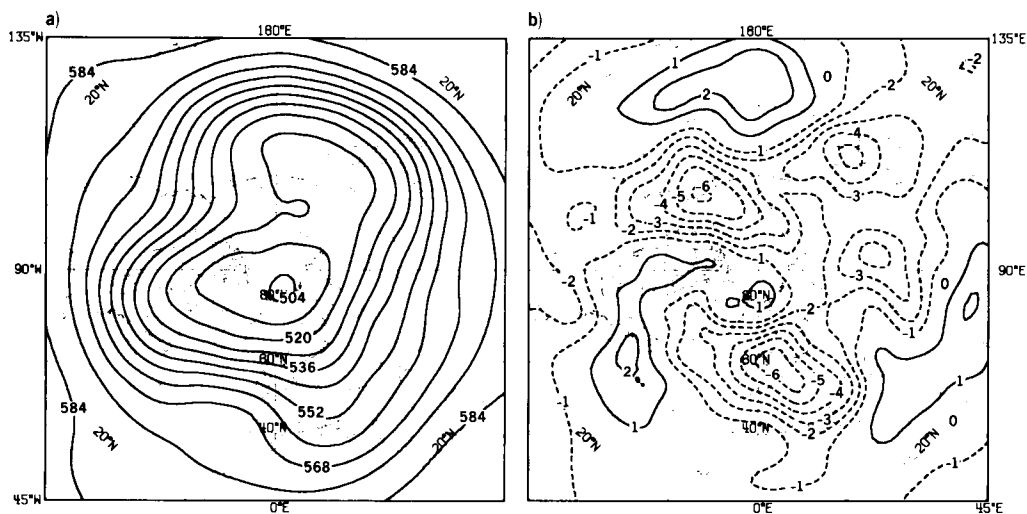


Fig. 3. Time mean of the 500 mb height field in the Northern Hemisphere for the winter 1980/81. (a) Analysis, (b) D + 3 forecast-analysis. Units: 10 m.

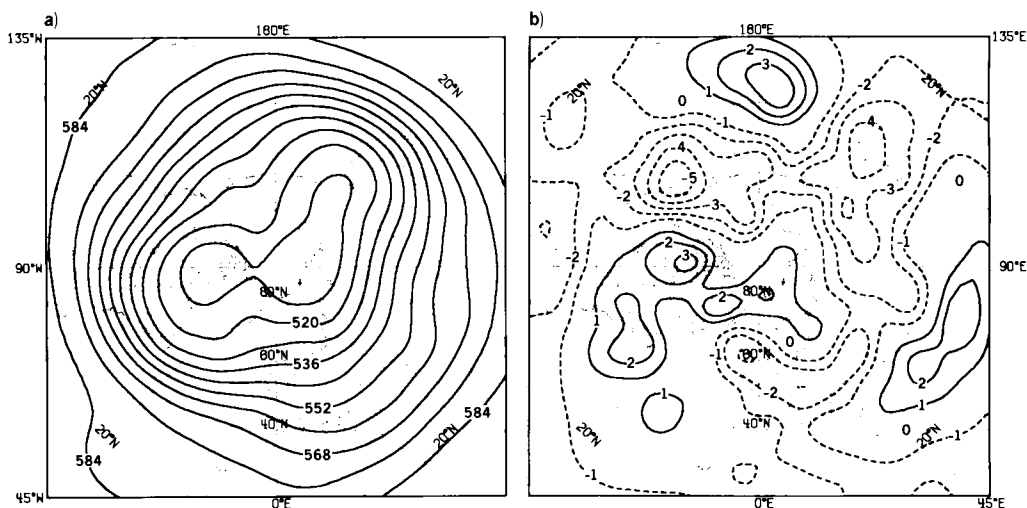


Fig. 4. The same as Fig. 3 except for the winter 1981/82.

America the flow pattern is more zonal in the winter 1981/82 than in the previous winter, whereas over the Norwegian Sea a ridge is present in the winter 1981/82 whilst in 1980/81 the flow is more zonal in that region.

The pattern of the forecast error in the height field at 500 mb (Figs. 3, 4) has not changed since it was first reported by Hollingsworth et al. in 1980. Only the magnitude of this error has decreased due to improvements in the forecasting system (Arpe, 1983). The negative bias of the forecast over Alaska and the positive one over the Pacific extends the west Pacific frontal zone eastwards. The second persistent forecast error occurs over western Europe where we find another negative bias. Due to this change of the height field the flow directed into southern Europe is intensified. This is a particular feature of the early part of the forecast, since in the later stages the mean circulation changes to a more zonal flow.

5. The growth of the inconsistency of successive forecasts

In this section, we will investigate the wavenumber-frequency spectrum of forecast ensembles to examine the growth of the noise level due to the inconsistency of successive forecasts. The two-sided spectrum is calculated for the initialized analysis and for three forecast ensembles ($D + 3$, $D + 5$, $D + 7$). The spectral values are calculated for three latitudes 45° , 50° , and 55° N. In the figures a cosine weighted mean for this latitude band is shown.

The wavenumber-frequency spectrum of the kinetic energy in the initialized analysis shows a distribution (Fig. 5) similar to those found in other investigations (e.g. Hayashi and Golder, 1977; Böttger and Fraedrich, 1980). The maximum values of the spectral density are arranged along a line corresponding to the dispersion line determined from the Rossby formula $c = \bar{U} - L^2/\beta^2$ (Rossby, 1939) for a mean zonal wind between 15 and 20 m/s^{-1} . In the winter of 1980/81, well separated peaks in the spectrum represent different types of wave motions. The ultra-long waves with wavenumbers 1 to 4 have their maximum variance in the low-frequency region. At wavenumbers 5 and 6, two peaks appear in the medium frequency spectrum of the eastward travelling waves. The first

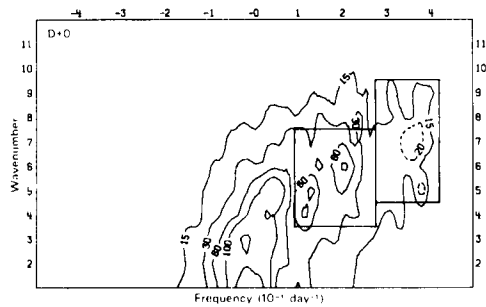


Fig. 5. Wavenumber-frequency spectrum of the kinetic energy at 500 mb in the initialized analysis. Power spectrum density values are averaged over three latitudes, 45° , 50° and 55° N. Winter 1980/81. Units: $\text{m}^2 \text{s}^{-1} \text{day}^{-1}$. The two boxes show the integration area for the medium frequency and the high frequency baroclinic waves.

maximum representing slowly moving baroclinic waves have a period of 12 days and the second maximum at higher frequencies represents the well-developed cyclones with a period of around 5 days. The more rapidly moving cyclones are seen as a variance peak at a period of 2.7 days.

By integrating the spectrum over wavenumber and frequency as indicated in Fig. 5, we will concentrate on two wave classes: the medium frequency baroclinic waves (MFBW's—wavenumbers 4 to 7 and periods from 3.6 to 10.2 days) and the high frequency baroclinic waves (HFBW's wavenumbers 5 to 9 and periods from 2.4 to 3.5 days). A Hovmöller diagram (Fig. 6) may help to understand the typical differences of the two wave types investigated here. In the beginning of February 1981 an intense upper air trough is developing over the eastern parts of North America and travels slowly to the east. The phase speed of this trough (assuming wavenumbers 4 to 5) would fall into the medium frequency range. The intensity of this baroclinic wave is underlined by its large meridional extent from 30° N– 70° N. About ten days later the cyclone activity is enhanced and a number of storms travel eastward over the eastern parts of the US and the West Atlantic. In the Hovmöller diagram this can be seen as disturbances with larger wavenumbers moving faster than the trough in the beginning of the month. With the observed phase speed and assuming wavenumber 6 or 7 these waves would fall into the high frequency range group.

The increase of the noise in the time series becomes very clear when we investigate the spectrum integrated over the range of baroclinic waves which are of interest here (wavenumbers 4 to 9). In Fig. 7 we have plotted the natural logarithm of the wavenumber integrated power spectrum. A small increase for the westward moving waves from $D + 0$ to $D + 3$, a larger one from $D + 3$ to $D + 5$

and an equal increase from $D + 5$ to $D + 7$ indicates an exponential growth of the noise due to the inconsistency in the forecast ensemble. The wavenumber integrated power spectrum also shows that the change in the low frequency region throughout the forecast period up to $D + 7$ is rather small; the major decrease is found in the medium frequency band. These results indicate good consistency of the forecasts out to day 3.

In the following two sections we will focus our attention on changes which take place within the first 3 days of the forecast. For this forecast range, the noise due to inconsistencies in the time series is low enough to see systematic changes in the structure of the waves. As the errors of the baroclinic waves are generally similar in the winters examined so far we will concentrate our description on the winter 1980/81. We will show the winter 1981/82 results only where differences in the variance distribution occur.

6. Systematic forecast errors of the medium frequency baroclinic waves

To investigate the medium frequency baroclinic waves (MFBW's: wavenumbers 4 to 7 and periods from 3.6 to 10.2 days), we begin by selecting one wavenumber out of this group and we describe its vertical structure. We can thereby compare the results from the initialized analysis with similar investigations using analysed data or data taken from numerical models. A comparison with the wave structure found in theoretical models will also be made. We will then investigate the changes in the phase structure during the forecast, whereas changes in the amplitudes will be examined in the second part where we discuss the power spectrum integrated over the wavenumber-frequency group containing wavenumbers 4 to 7. We will focus our attention on the changes in the early part of the forecast where systematic changes seem to be larger than non-systematic ones.

6.1. Vertical structure of wavenumber 6 in the zonal plane

In this section we will discuss the vertical structure of the wave with respect to geopotential height, temperature, zonal wind, meridional wind, and vertical velocity. The vertical phase lines of each variable and the phase difference between

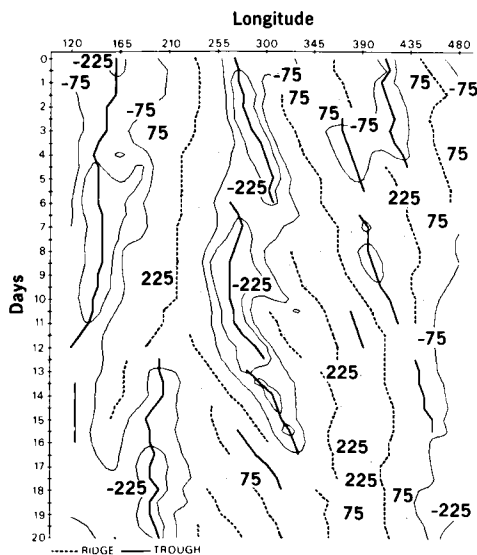


Fig. 6. Trough-ridge diagram for wavenumbers 1-9 of the observed 500 mb height at 55°N for the first 20 days of February 1981. The contour interval is 100 m with contours starting from ± 75 m. Heavy solid lines indicate trough lines, heavy dashed show ridge lines.

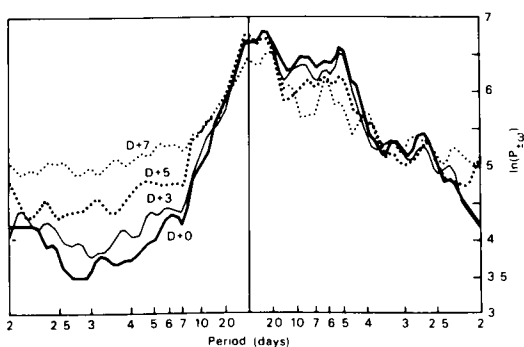


Fig. 7. Frequency spectrum of the kinetic energy integrated over wavenumbers 4 to 9 and averaged over 3 latitudes (45° , 50° , and 55°N). Spectral values are plotted on a logarithmic scale. The four graphs show the spectrum for the analysis (thick line) and the forecast ensembles $D + 3$ (thin line), $D + 5$ (thick dotted line) and $D + 7$ (thin dotted line). Winter 1980/81.

them will be investigated. For a better understanding of the relation between the variables we will make use of simple diagnostic equations we have derived in Appendix B.

In Fig. 8 (winter 1980/81), we show the vertical amplitude and phase structure for the geopotential height wave (wavenumber 6) as a mean over the latitude band from 43°N to 64°N . The vertical phase structure is determined by first calculating the phase difference between the temperature and each other variable at 500 mb. Then for all variables the vertical phase lines are continued upward and downward from 500 mb by adding the calculated phase differences between two adjacent levels. The quality of the phase calculation can be measured in terms of coherence. At 500 mb the coherence is a measure of reliability for the phase difference between the temperature and the other variables. At all other levels the coherence indicates the accuracy of the level to level phase increments for that variable.

Some general wave characteristics shown here for wavenumber 6 agree quite well with the structure derived from the linear theory of baroclinic waves (Charney, 1947; Eady, 1949; Green, 1960). Assuming a phase speed of 10 ms^{-1} (which corresponds to a wavenumber 6 disturbance travelling eastward with a period of 5.3 days at 45°N), the steering level where the phase speed of the wave matches the mean zonal flow would be approximately at 600 mb. The β effect acting on baroclinic waves distorts the vertical symmetry which exists for $\beta = 0$. With respect to the steering

level, the amplitude of the temperature wave is biased to lower levels and the vertical velocity wave to upper levels. The geopotential has a maximum at the lower boundary and at the tropopause (which acts as an imperfect upper lid to the baroclinic waves below). There is also a good qualitative agreement between observation and theory in the vertical phase structure. The geopotential tilts westward with height, and below 400 mb the temperature tilts forward with height. The vertical tilt of the vertical velocity is in the same direction as the tilt of the meridional wind. Though the tilt of the vertical velocity is smaller, we can expect some similarities in the poleward and upward eddy fluxes of heat.

The main discrepancy between linear theory and observation is that the theoretically predicted upper level wave maximum is too weak. Though linear theory shows that a sharp tropopause can enhance the amplitude of the baroclinic waves at upper levels (Simmons and Hoskins, 1976), non-linear processes are required to explain upper level wave maxima which are larger than the surface maximum by a factor of at least 2 (Simmons and Hoskins, 1978). The relatively larger enhancement of upper level disturbances is also a function of the lifetime of the baroclinic waves. The mean zonal flow can be modified by a growing baroclinic wave in such a way that the growth rate near the surface decreases, thereby allowing the upper level part of the wave to attain larger amplitudes before all growth ceases at the mature stage (Gall, 1976). It seems that the well developed baroclinic waves with

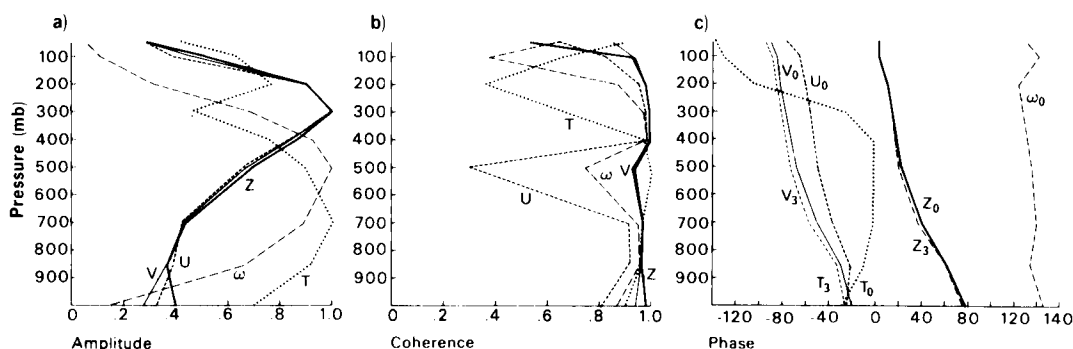


Fig. 8. Vertical structure of wavenumber 6 in the medium frequency band (mean period 5.3 days). Profiles of amplitude (a), normalized by the maximum value, coherence (b) and phase difference with respect to temperature at 500 mb (c). Spectral values are averaged over a latitude band from 43°N to 64°N . The graphs show the vertical structure of the geopotential height (Z), temperature (T), zonal wind (U), meridional wind (V) and vertical velocity (ω) for the initialized analysis. The phase structure is shown for the forecast ensemble D + 3 as well. Winter 1980/81.

large amplitudes aloft are the major contributors to the variance in this wavenumber-frequency group.

The temperature wave has its major maximum at 700 mb and a minor one above the tropopause. The phase difference between the temperature and the geopotential is smallest in the middle troposphere, and a large phase change of the temperature wave occurs through the tropopause. The change of the amplitude of the geopotential height wave in the vertical is consistent with the vertical phase structure of the temperature wave and geopotential height wave for hydrostatic flow (see Appendix B).

The close agreement of the vertical amplitude structure of the meridional wind and that of the geopotential indicates that the variance of the meridional wind is dominated by the geostrophic part. It is only near the surface that the structures (normalized by the maximum amplitude of Z and v) depart from each other; here friction has decreased the amplitude of the meridional wind. The large decrease of the meridional wind amplitude with height in the lower stratosphere is again in good agreement with the vertical phase structure. As expected from the thermal wind equation for waves (diagnostic formula B2 in Appendix B), the meridional wind leads the temperature wave above the wind maximum.

The vertical profile of wavenumber 6 is very similar to the profiles found by Hayashi and Golder (1977) in the GFDL general circulation model. Though they calculated the phases as differences with respect to temperature for all parameters and levels, instead of adding up phase differences from level to level, their phase profiles are almost the same as found here. We also have a good agreement between the wave structure in our analysis and studies which are based on observational data (Hartmann, 1974; Böttger and Fraedrich, 1980).

The phase structure of the D + 3 forecast ensemble was calculated in the same way as for the analysis. Additionally the phase difference between the D + 3 disturbances and the disturbances in the analysis was determined by calculating the co-spectrum and quadrature spectrum between the travelling disturbances of the two ensembles. For the geopotential height this calculation shows that the wave in the forecast ensemble is slightly too slow (Fig. 8). For other wavenumbers in the medium frequency range which are not shown here, the phase lag was around 150 km after 3 days and

300 km after 5 days. We have checked the reliability of the calculation of the phase differences between the forecast and the analysis by an artificial change of the forecast ensemble. We repeated the calculation with an unchanged analysis ensemble but with a forecast ensemble starting and ending one day later. The phase difference is exactly as large as we would theoretically expect for a wave with a period of 5.3 days travelling eastward for one day.

An important feature of all baroclinic waves in the forecast is that the forward tilt of the temperature wave with height increases during the forecast. This error shown for wavenumber 6 is representative of the other wavenumbers as well. The phase change increases the poleward heat flux as the phase difference between the meridional wind and temperature becomes smaller in the lower troposphere. The phase difference between the geopotential and temperature becomes larger giving a more pronounced decrease of the geopotential wave amplitude with height near the surface.

6.2. Forecast errors in the medium-frequency baroclinic waves: integrated results

In this section, the variance of single variables and the cospectrum of pairs of variables, both integrated over wavenumbers 4 to 7 and frequencies from 1/10.2 to 1/5.2 (1/days), are investigated. By integrating over a region in the wavenumber-frequency domain we can improve the significance of our results (see Appendix A). We will describe the latitude-height distribution of the variance for the initialized analysis and look at changes which arise during the forecast. Attention will be concentrated on the differences between the variance of the D + 3 forecast ensemble and the variance of the initialized analyses.

6.2.1. Geopotential and temperature. The vertical profile of the variance (Fig. 9) is the same as we have shown in the single wave description of wavenumber 6 in the previous section (see Fig. 8); a major maximum of the geopotential height variance is found at the tropopause and a minor one near the surface. The large decrease of the variance with height above the tropopause indicates that the MFBW's are not able to penetrate into the stratosphere. The distribution of the variance in the latitude-height plane is nearly the same in the two winters.

The thermal structure of the MFBW group (Fig.

10) reveals one temperature maximum in the lower troposphere and one in the lower stratosphere. As

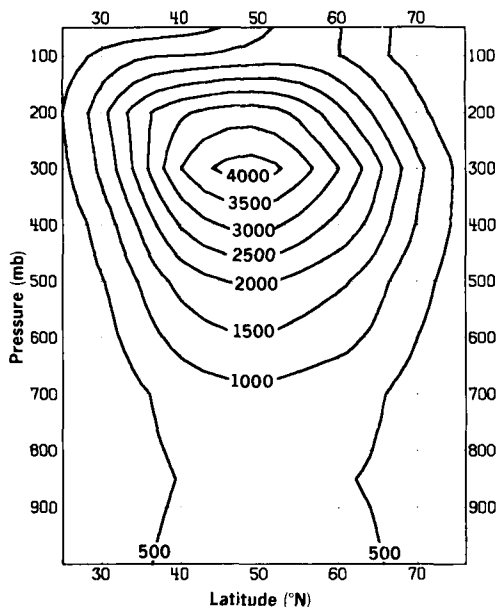


Fig. 9. Latitude-height section of the geopotential height variance for the MFBW's in the initialized analysis. Units: m^2 . Winter 1980/81.

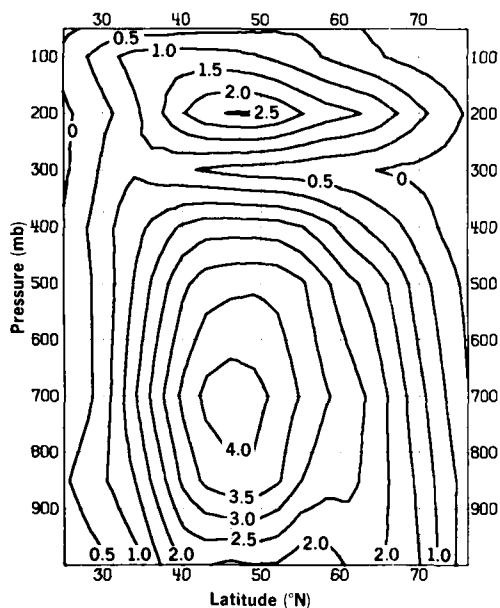


Fig. 10. Latitude-height section of the temperature variance for the MFBW's in the initialized analysis. Units: K^2 . Winter 1980/81.

we have shown for the vertical phase structure of wavenumber 6 (see Fig. 8), the temperature wave and the geopotential height wave are nearly in phase in the middle troposphere. However, they are out of phase by more than 90 degrees above the variance maximum of the geopotential height wave. As we know from the hydrostatic equation for waves (diagnostic formula (B1) in Appendix B) this phase difference is necessary to explain the vertical decrease of the geopotential wave with height. In this sense the lower stratospheric temperature waves compensate the tropospheric temperature waves which account for the vertical increase of the geopotential height wave.

During the first 3 days of the forecast large changes in the vertical structure of the MFBW's take place (Fig. 11). A marked weakening of the upper level maximum is the dominant feature in both winters, with the largest reduction of the variance on the poleward side of the observed variance maximum. Up to $D + 3$ the upper level maximum has decreased by 15%. In the same forecast period the near surface layer shows an increase in the variance which has almost the same relative magnitude. This positive deviation at low latitudes extends into upper levels as well. The

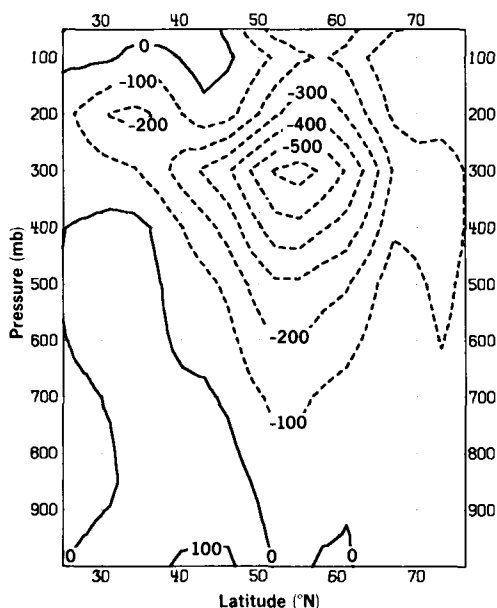


Fig. 11. Latitude-height section of the geopotential height variance error for the MFBW's. Forecast ($D + 3$)-analysis. Units: m^2 . Winter 1980/81.

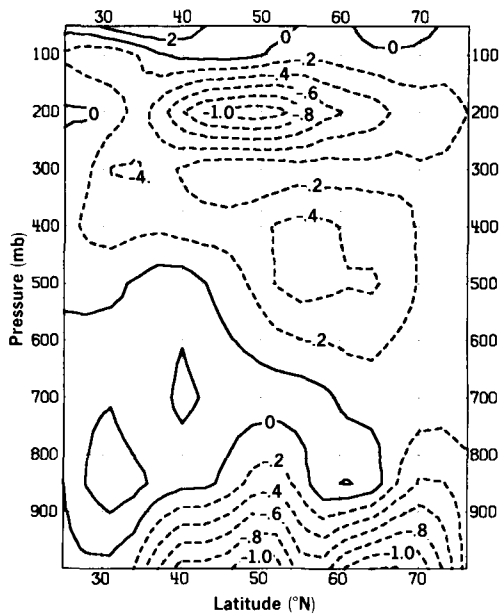


Fig. 12. Latitude-height section of the temperature variance error for the MFBW's. Forecast (D + 3)-analysis. Units: K. Winter 1980/81.

weakening of the upper level maximum of the geopotential height wave continues up to D + 5 (not shown here) with a further decrease of 15%. The decrease has also spread to lower levels and only a weak increase remains near the surface at middle latitudes. The hydrostatic equation (Appendix B) relates forecast errors of height waves to changes in the thermal structure of the waves. We find a decrease of the temperature variance in the middle troposphere and a weakening of the compensating temperature waves in the lower stratosphere as well (Fig. 12). These two changes in the variance of the temperature are closely coupled with the decrease of the geopotential height variance and should not be seen as separate processes.

The general tendency of the MFBW's in the ECMWF forecast model to have an upper level maximum which becomes too weak relative to the surface maximum seems to be a common feature for other numerical integrations. Simmons and Hoskins (1978) performed some non-linear experiments to simulate the life cycle of baroclinic waves and found that the ratio of the upper level kinetic energy to the surface values was 5 to 1, whereas we found here a ratio of 8 to 1 in the initialized

analyses. Too weak baroclinic waves in numerical integrations were also identified by Pratt (1979) when he compared observed variances with simulations from the NCAR and GFDL general circulation models. The underestimation of baroclinic wave amplitudes at upper levels was larger for the NCAR model than for the GFDL model. Pratt assumed that the differences could largely be attributed to the higher resolution of the GFDL model. The GFDL model had, however, a higher resolution in the vertical as well as in the horizontal, with five layers below 300 mb instead of three layers as in the NCAR model. As mentioned before, linear theory (Simmons and Hoskins, 1976) shows that the sharpness of the jet at the tropopause in the model can influence the upper level maximum of the baroclinic waves and we notice a reduced sharpness of the tropopause during the forecast (see Fig. 1). Compared to the NCAR and GFDL model the GLAS model is more successful in simulating the observed variance of the baroclinic waves (Straus and Shukla, 1981). To what extent the ECMWF model error in the MFBW's is dependent on the model resolution is a question which could be answered by appropriate sensitivity experiments with higher vertical and/or horizontal resolution.

The changes of the MFBW's are consistent with the changes of the stability of the zonal mean state for the period under investigation. Below the tropopause a stabilization of the mean state could be responsible for smaller growth rates at upper levels. However, the stabilization seems to be rather small. The destabilization in the lower part of the troposphere would have the effect of enhancing the growth of baroclinic waves. The explanation for the overdevelopment near the surface is, however, not as straightforward as it appears. The longitudinal contributions for the too high zonal mean temperatures come from large positive forecast errors at the lee side of the mountains (Arpe, 1983). In the discussion of the geographical distribution of the forecast errors for the MFBW's, we will notice large longitudinal variation of the variance errors as well.

6.2.2. Horizontal eddy momentum flux. The height-latitude distribution of the integrated horizontal momentum flux is very similar in the two winters (Fig. 13). The maximum poleward flux is located north of the subtropical jet which agrees with calculations by Hollingsworth et al. (1976)

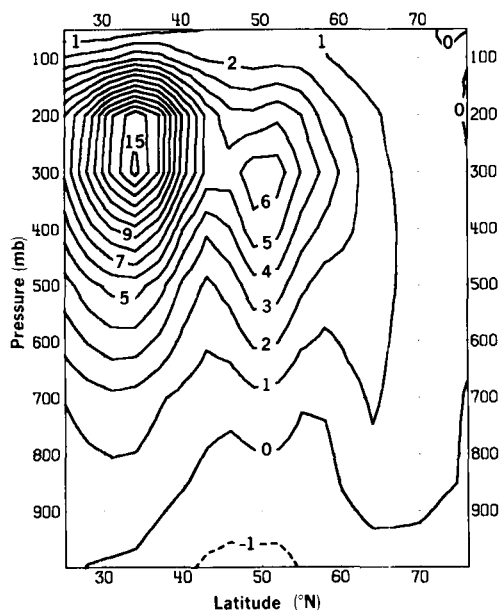


Fig. 13. Latitude-height section of the eddy momentum flux for the MFBW's in the initialized analysis. Units: $\text{m}^2 \text{s}^{-2}$. Winter 1980/81.

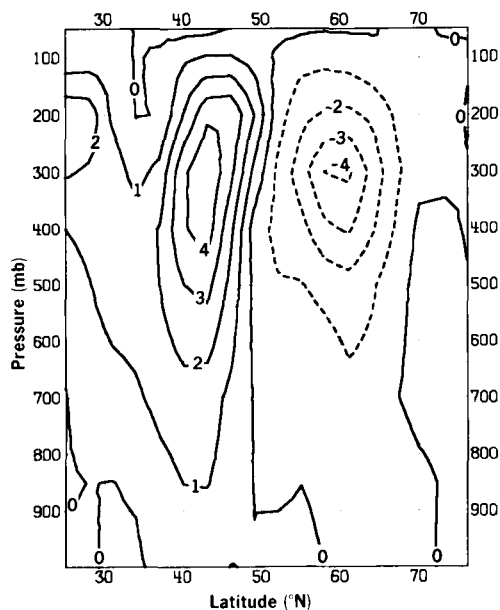


Fig. 14. Latitude-height section of the eddy momentum flux error for the MFBW's. Forecasts (D + 3)-analysis. Units: $\text{m}^2 \text{s}^{-1}$. Winter 1980/81.

and with simulations of baroclinic waves by Simmons and Hoskins (1978).

The changes which arise in the forecast in the first 72 h (Fig. 14) are very different in the two winter seasons. Only at low latitudes do the two winters show the same kind of change in the D + 3 forecast ensemble. Examining the contributions from single wavenumbers to the change shown by the integrated values, we find similar changes in the two winters for wavenumbers 4 and 6, whereas wavenumbers 5 and 7 are changed differently.

The horizontal momentum flux calculated from geostrophic winds (relation (B4) in Appendix B) is a function of the amplitude and the horizontal phase tilt of the geopotential height wave, giving a poleward momentum flux for a wave with a SW to NE tilt. We find good confirmation of this geostrophic relation in the analysis of the horizontal phase structure at 300 mb (Fig. 15). Wavenumber 6 has a horizontal tilt which is largest (and directed from SW to NE) at low latitudes. For baroclinic waves with different wavenumbers the phase structure is not as consistent from one winter to the next as shown here for wavenumber 6. This is so especially in middle latitudes where the horizontal phase tilt for the single wavenumbers is

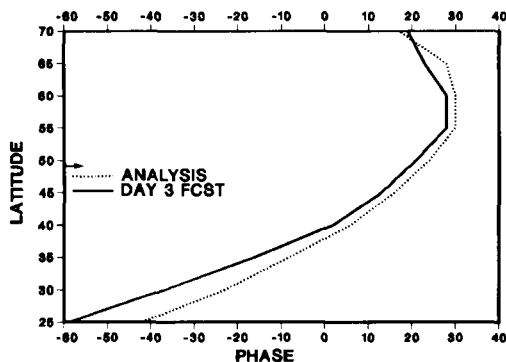


Fig. 15. Latitude plot of the geopotential height phase structure of wavenumber 6 in the medium frequency band (mean period 5.3 days) at 300 mb, dashed line: initialized analysis, solid line: forecast D + 3. Units: degrees. Winter 1980/81. (†: Reference point of phase calculation).

often different in different winters. The variability of the horizontal phase makes a comparison with theoretical studies very difficult. Simmons and Hoskins (1976) noticed that, in contrast to the uniformity of the eddy heat flux in all their experiments, horizontal momentum fluxes were sensitive to the mean zonal flow profile. As the

horizontal eddy heat and eddy momentum fluxes are, to a good approximation, proportional to the vertical and horizontal tilt of the geopotential wave (see formulae (B4) and (B5) of Appendix B), their results are reflected in the varying horizontal phase tilt of observed baroclinic waves. However, to the south of the variance maximum all baroclinic waves in the medium frequency range show a large SW/NE tilt which is consistent with both the theoretical result of Hollingsworth et al. (1976) for linear waves, and the more general results discussed by Edmon et al. (1980).

The same diagnostic formula also provides us with some information with which to understand the structural changes during the forecast. At low latitudes where the eddy momentum flux increases, we find an increase of the SW/NE tilt as well (Fig. 15). This change of the horizontal phase tilt is not only present for wavenumber 6 but for the other wavenumbers as well. One indication of an interaction process between the eddies and the zonal mean flow can be seen in the change of the mean zonal flow which has decreased at low latitudes and increased at middle latitudes (see Fig. 2). On the other hand there seems to be a close connection between the horizontal tilt and the interaction between the baroclinic waves at middle latitudes with the flow in the tropics. This suggests that the forecast errors in the tropics, which grow faster than at middle latitudes, can have an important effect on the interaction of the two regions. Haseler

(1982) has shown in case studies that errors arising from the interaction of deep mid-latitude troughs with the tropics can have a significant impact on the subsequent evolution of the flow in extra-tropical regions. Haseler shows that initially the SW/NE tilt of the baroclinic waves increases too much, a result which agrees with the structural changes found here with the wavenumber-frequency analysis. The incorrect orientation of the troughs then leads to a spurious conversion from eddy to zonal kinetic energy. In other words the increased phase tilt may lead to a stronger barotropic damping of the baroclinic waves.

6.3. Geographical distribution of the forecast errors in the medium frequency baroclinic waves

The geographical distribution of the variance for the MFBW's shows two maxima of wave activity in the winter 1980/81 (Fig. 16). Both of them are located downstream of the two major troughs on the eastern coasts of North America and Asia. The MFBW's are more intense in the Atlantic than in the Pacific. A pronounced ridge is present in the time-mean height field over the western parts of North America and over the eastern Pacific (see Fig. 3) so that a clear minimum separates the Atlantic wave activity region from the one in the western Pacific. In the following winter (1981/82) the time-mean flow over the eastern Pacific and western parts of North America is

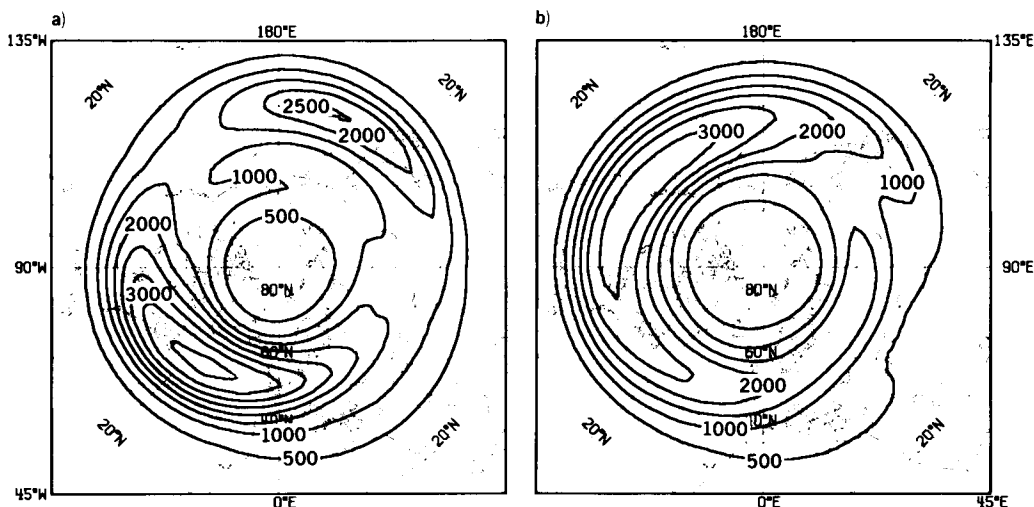


Fig. 16. Geographical distribution of the geopotential height variance for the MFBW's at 500 mb. Units: m^2 . (a) Winter 1980/81, (b) Winter 1981/82.

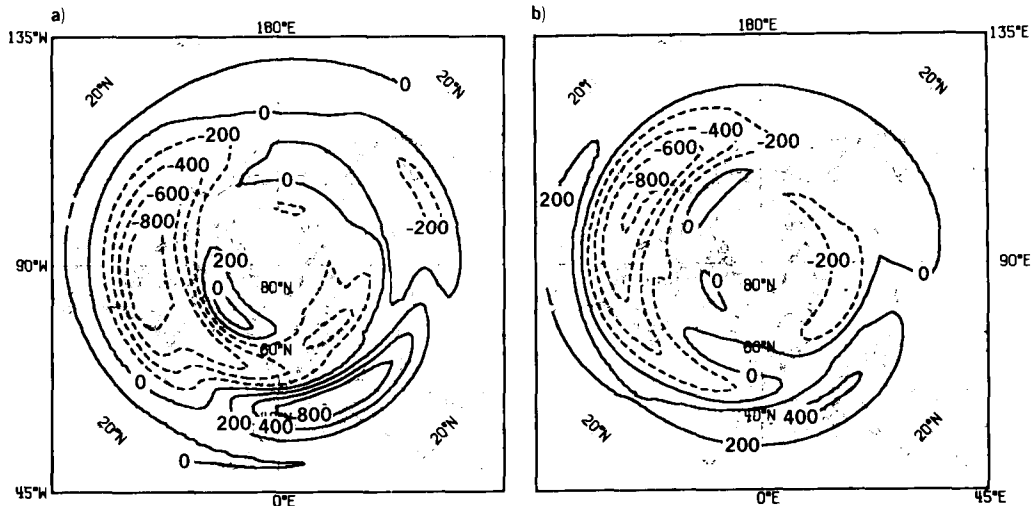


Fig. 17. Geographical distribution of the geopotential height variance error. ($D + 3 - D + 0$) for the MFBW's at 500 mb. Units: m^2 . (a) Winter 1980/81, (b) Winter 1981/82.

much more zonal (see Fig. 4). In this situation the two regions of baroclinic wave activity are merged into one centred on the west coast of North America. In the North Atlantic the MFBW's are much weaker in this winter than in the previous one. Apart from the two winters examined in detail here, the cross-spectral analysis has also been calculated for the winter 1982/83 and 1983/84. Comparing all winters, the geographical distribution of the MFBW's have a larger annual variation than the high frequency group which corresponds more to the band passed filtered variance shown by Blackmon and Lau (1980).

Though the geographical distribution of the variance of the medium frequency baroclinic waves is different in the two winters, the forecast errors are very similar in both cases (Fig. 17). In general the MFBW's are weakened in the forecast over North America and enhanced over SE Europe. When we compare the variance of other forecast ensembles ($D + 1$ and $D + 5$ not shown here) with the variance of the analyses we see that the geographically fixed error pattern is already established by day one. The negative error in the Atlantic region then increases very rapidly until $D + 5$, whereas the positive error over SE Europe has a much smaller growth rate. The difference maps of the variance for other levels (not presented here) indicate increased negative deviation of the forecast variance from the analysed variance with height. A

corresponding error in the temperature variance suggests the baroclinic nature of the forecast error over North America.

The changes of the time-mean state in the forecast over North America are quite remarkable in this context. A warming over large areas of the North American continent and a cooling over Central America and the Caribbean has led to a decrease of the north-south temperature gradient at middle latitudes. The location of the largest reduction of the temperature gradient occurs in the eastern part of North America in the winter 1980/81 and in the central parts in the winter 1981/82. A vertical cross-section for the corresponding longitudes (Fig. 18) shows the difference between the time mean potential temperature of the forecast ($D + 3$) and the analysis. A warming of low levels at high latitudes and a cooling of upper levels at low and middle latitudes has led to a decrease of the baroclinicity at middle latitudes where we find the decrease of the wave activity of the MFBW's. This warming of the lower levels is consistent with the error in the mean height field (Fig. 3) which produces too strong southerlies over North America by day 3.

For a zonal flow in thermal wind balance, the linearized equations for the potential temperature (θ) and quasi-geostrophic potential vorticity equation give an equation for the normal modes which relates the growth rate (σ) of the baroclinic

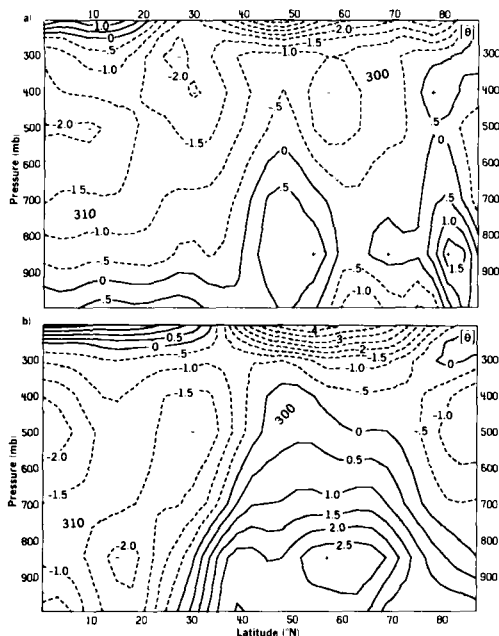


Fig. 18. Latitude-height plot of the mean time potential temperature in the initial analysis (dotted lines) and the difference between the forecast and the analysis ($D + 3 - D + 0$, thicker lines). Time averaging period 112 days. Units: K. (a) Winter 1980/81 zonally averaged from 90°W to 60°W , (b) Winter 1981/82 zonally averaged from 117°W to 90°W .

waves to the meridional slope of the mean potential temperature surfaces ($-\bar{\theta}_y/\bar{\theta}_z$) and to the phase slope (ϕ_x) of the potential temperature wave ($-\theta'_x/\theta'_z$) $\sim \phi_z$ (Hoskins and Heckley, 1981).

$$\sigma = \frac{fk^2}{k^2 + l^2} \cdot \frac{-\bar{\theta}_y/\bar{\theta}_z}{-\theta'_x/\theta'_z},$$

where k and l are the zonal and meridional wavenumbers, and x , y and z are the derivative indices. This relation indicates a decrease of the growth rate of the baroclinic waves when the vertical slope of the mean potential temperature surfaces decreases and the slope of the temperature wave does not change. The decrease of the vertical phase slope of the temperature wave (see Fig. 8) would increase the growth rate, but this change is only present at lower levels.

Over SE Europe, the variance increase of the MFBW's during the forecast has a larger barotropic part suggested by a smaller variation with height. The European forecast error pattern in the

two winters compared here is also found in the winter 1982/83 and 1983/84. Rather noticeable is the fact that the enhancement of the MFBW's in the winter 1980/81 is nearly double the size found in the following winters. In the winter 1980/81 when the forecast error in the mean flow was largest, the increase of the MFBW's was double the size found in the following winter. It is worth mentioning that in April 1981 the heights of the mountains in the model were increased, which may have contributed to an increased blocking effect over the Alps. But still we find that in many winter cases that almost stationary troughs over SE Europe start to travel eastward in the forecasts. The forecast error in the mean flow over the west coast of North America has a different relationship with the flow pattern; there we do not find such a significant increase of the flow over the mountains.

7. Systematic forecast errors of high-frequency baroclinic waves

The second wavenumber-frequency band to be studied contains the high frequency baroclinic waves (HFBW's; wavenumber 5 to 9 in a period range from 2.4 to 3.5 days). The vertical structure of the waves in this class is very similar to the structure of the medium frequency baroclinic waves. Therefore we will not discuss the details of the single wavenumber structure, but concentrate on the description of the wavenumber and frequency integrated variances and covariances.

7.1 Structure of the forecast error in the meridional plane

The meridional structure of the geopotential height variance shows that the latitudinal distribution of these waves can differ very much from one winter to the next (Fig. 19). While the main baroclinic wave activity in the winter 1981/82 is concentrated in middle latitudes, we see two latitude bands of maximum variance in the preceding winter. The ratio of the upper level and surface variance maxima is around 5 to 1 which is clearly smaller than that for the MFBW's where it was 8 to 1. The tendency for the higher wavenumbers (which are faster moving) to have smaller upper level maxima relative to the surface values has already been predicted by the linear theory of

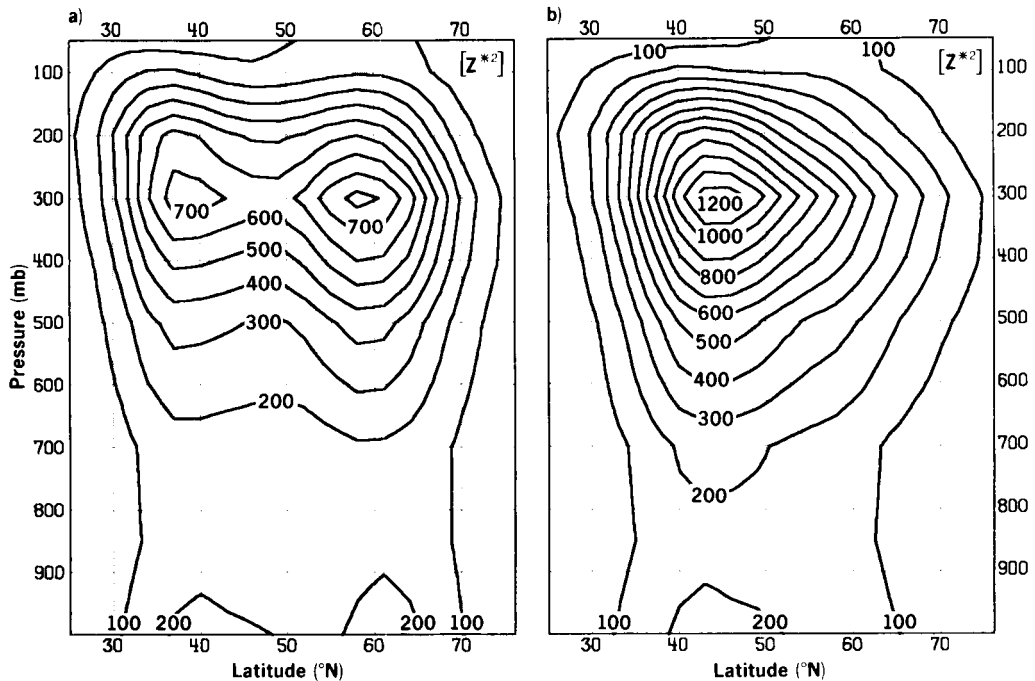


Fig. 19. Latitude-height section of the geopotential height variance for the HFBW's in the initialized analysis. Units: m^2 . (a) Winter 1980/81, (b) Winter 1981/82.

baroclinic waves (Gall, 1976). The large decrease of the variance with height above the 300 mb indicates that these waves are confined to the troposphere. This is also evident from the cross-spectral analysis of different levels. The level to level coherence of the disturbances drops sharply above 300 mb.

Though the two winters have a different latitudinal distribution of baroclinic disturbances, the changes occurring in the forecast within the first 72 h are remarkably similar. In both winters the geopotential height variance decreases at high and low latitudes (Fig. 20). In middle latitudes a small band of enhanced variance can be found which extends from the surface to the upper troposphere. This latitude dependent picture of changes during the forecast contrasts with the MFBW's which are weakened in a broad latitude band. The vertical structure of the differences indicates that this positive deviation has a large barotropic part.

7.2. Geographical distribution of the forecast errors in the high frequencies

In the winter 1980/81, the HFBW's have

maximum variance in the North Atlantic around 58°N and in the Pacific at a much lower latitude of 36°N (Fig. 21). In this geographical distribution the high frequency baroclinic waves coincide with the two major storm tracks connected with the baroclinic zones on the eastern sides of the continents. In connection with the Atlantic storm track, some disturbances develop over the eastern parts of North America and some penetrate deep into northern parts of Russia. In the following winter, the mean frontal zone (see Figs. 3 and 4) is further to the south in the Atlantic region and the high-frequency baroclinic waves are found at much lower latitudes. A ridge in the mean flow over northern Europe seems to have blocked the migration of the high frequency waves into the northern parts of Russia for this winter. In the Pacific the baroclinic waves are weaker in 1981/82 than in 1980/81, coinciding with a weaker baroclinicity in the later winter. The geographical distribution of the HFBW's variance depicts the major storm tracks over the oceans. The variance pattern is similar to the bandpass filtered variance in Blackmon and Lau (1980), who considered a

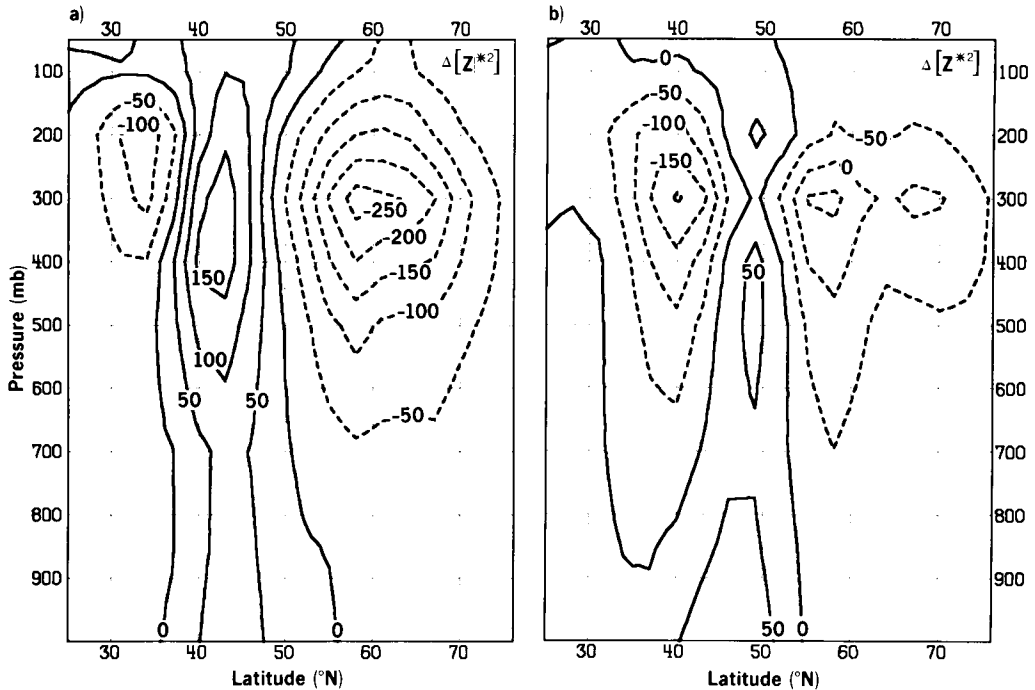


Fig. 20. Latitude-height section of the geopotential height variance error for the HFBW's. Forecast ($D + 3$)-analysis. Units: m^2 . (a) Winter 1980/81, (b) Winter 1981/82.

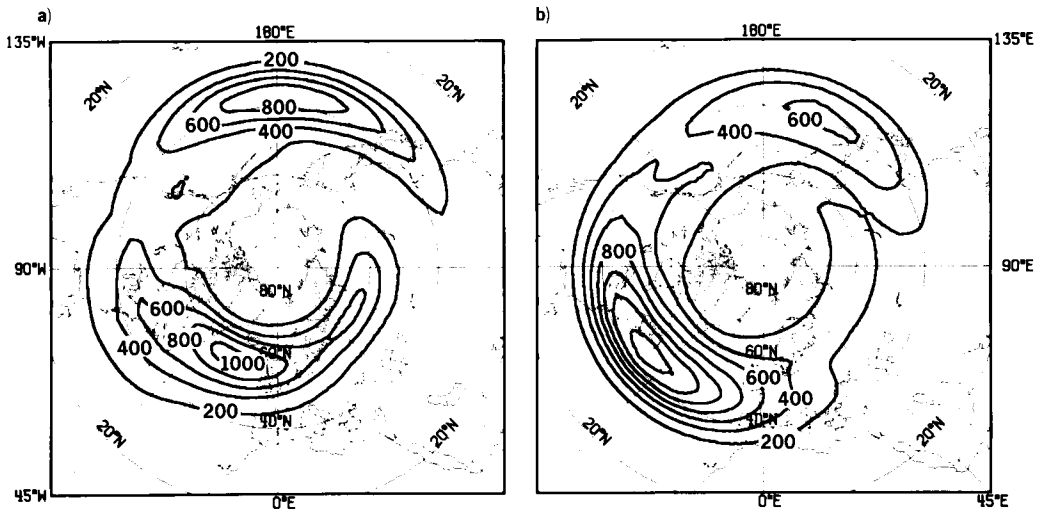


Fig. 21. Geographical distribution of the geopotential height variance for the HFBW's at 500 mb. Units: m^2 . (a) Winter 1980/81, (b) Winter 1981/82.

period from 2.5 to 6.0 days compared to 2.4 to 3.5 days used here.

In both winters, a weakening of the HFBW's in the forecast occurs downstream of the continental regions (Fig. 22). The baroclinicity of the time-

mean flow on the eastern coast of both Asia and North America becomes weaker in the forecast. This effect is most pronounced downstream of the North American continent where we found the largest decrease in the baroclinicity (see Fig. 18).

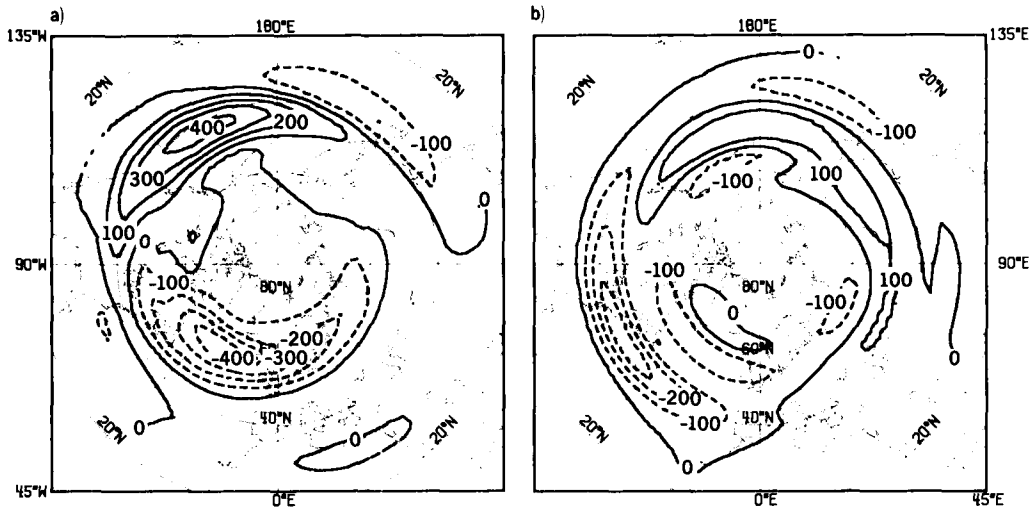


Fig. 22. Geographical distribution of the geopotential height variance error for the HFBW's at 500 mb. Forecast (D + 3)—analysis. Units: m^2 . (a) Winter 1980/81, (b) Winter 1981/82.

The North American error pattern is already established in the difference between the D + 1 forecast and the analysis. The latitudinal structure of the time-mean temperature change in the forecast over North America is closely connected to the changes on the HFBW's in the forecast further downstream.

In the winter 1980/81 the baroclinicity in the forecast is weakening over a broad latitude band from 30°N to 60°N over North America. In this case, the HFBW's become weaker in the forecast where we found the maximum variance in the analyses to the SE of Greenland. In the following winter, the major decrease in the horizontal temperature gradient over North America occurs around 36°N (see Fig. 18). The weakening of the HFBW's is split into two regions on both sides of the analysed variance maximum at 45°N whereby the largest decrease occurs downstream of the large change in the baroclinicity.

8. Summary and conclusions

Space-time decomposition has been applied to time series of forecast ensembles and of initialized analyses. The calculated power spectrum was then integrated over two regions in the wavenumber-frequency domain which represent the strong baroclinic waves moving eastward with a mean period of 5.3 days (MFBW) and the high frequency

baroclinic waves with a period of 2.8 days (HFBW). The three-dimensional structure of these waves was discussed in detail and compared with theory and model simulations.

The main aim of this investigation is to identify the forecast errors which appear as changes of the travelling baroclinic waves. The MFBW's undergo a considerable weakening which amounts to 15% by D + 3 and 30% by D + 5. The maximum decrease occurs at 300 mb where these waves have maximum amplitudes. The under-forecasting of the upper level wave maximum seems to be a problem for other models as well. Linear theory shows that this might be the consequence of a low resolution near the tropopause. Other contributions to the weakening of these waves at upper levels may come from the mid-tropospheric cooling, which leads to a stabilisation of the upper troposphere, and secondly from an enhanced barotropic damping associated with inaccurate meridional phase tilts. It seems desirable to test these hypotheses by suitable sensitivity experiments.

The influence of the time-mean flow can be seen very clearly when we compare the geographical distribution of the time-mean forecast error with the forecast errors of the travelling baroclinic waves. The main centre of weakening of these waves is found over North America, where the baroclinicity of the time-mean flow decreases due to low level warming in high latitudes and upper level cooling in middle latitudes. The second persistent forecast

error for the MFBW's occurs over SE Europe, where a significant increase in the variance during the forecast has taken place in four successive winters so far. Stationary troughs over SE Europe in the analyses tend to propagate slowly eastward in the forecast. This error seems to be dependent on the height of the Alps in the model as the error was largest in 1980/81 when the ECMWF model had only a very smooth representation of the mountains.

The three dimensional phase structure of the baroclinic waves is changed in two important ways during the forecast. In the horizontal the MFBW's increase their SW to NE tilt at low latitudes. Consequently the poleward eddy momentum flux increases in these latitudes and lead to a spurious conversion from eddy kinetic energy to zonal kinetic energy. The interaction of the MFBW's with the tropical flow seems to be responsible for this error. In the vertical the forward tilt of the temperature wave is increased at low levels. This is a forecast error we found for all baroclinic waves.

Cross-spectral analysis between the forecast ensemble and the analysis shows that the baroclinic waves are generally too slow. This can be seen by a growth of the phase lag between forecast and analysis with increasing forecast range. The MFBW's are lagging by around 150 km at day 3 and by 300 km at day 5. Spectral computations with recent datasets have shown that the phase error has been significantly reduced with the introduction of the spectral model into operation in April 1983.

The high frequency baroclinic waves with a mean period of 2.8 days are weakened at higher and lower latitudes, whereas at middle latitudes a region of over-development in the forecast is seen in the two winters examined here and in the later winters as well. The geographical distribution of this forecast error is likely to be closely connected to the changes in the mean flow. As these waves are faster than the MFBW's their main weakening occurs further downstream from the region where we found a decrease of the baroclinicity. The largest reduction in the variance of the HFBW's during the forecast occurs over the North Atlantic.

A clearly persistent error for all winters is the east-west dipole error structure in the Pacific. A less baroclinic flow near the east Asian coast leads to a weakening of the downstream developments in the west Pacific. Increased baroclinicity over the central Pacific has the effect of a much stronger development of the high frequency baroclinic waves

in the eastern part of the Pacific. When these waves decelerate in a later stage of their life cycle they seem to contribute to the negative deviation of the time-mean height of the analysis over Alaska. The forecast error in the time-mean flow contributes to an erroneously large poleward flux of heat over North America. This could explain the decrease in the baroclinicity of the flow over North America, where we find smaller growth rates of the baroclinic waves. However, errors in the diabatic or orographic forcing are also very likely to cause that kind of deviation in the mean fields.

The results show a close connection between the errors in the time-mean flow and the errors in the travelling baroclinic waves. However it is still difficult to be sure which error is the primary one. From our analysis it seems that the different evolution of the baroclinicity over land and ocean is probably the primary error which then causes changes in the baroclinic waves.

The forecast error pattern of the baroclinic waves has only a small inter-annual variation in the northern hemisphere. The largest differences are closely connected to the two main frontal zones and the major storm tracks. The main features of these errors are very similar to the four winters since 1980/81. In this respect the transient forecast errors behave similarly to the time mean errors. Though the amplitude has been reduced, the main structure of the systematic error has even survived some major changes of the ECMWF forecast system such as the increase of mountain heights in the model and the introduction of the spectral forecast techniques. So far the detailed examination of forecast errors for the baroclinic waves has been restricted to the northern hemisphere. The time-spectral computations, however, have been done for the southern hemisphere as well. There we found a much higher inter-annual variability in the geographical distribution of the forecast errors in the scales of baroclinic waves compared to those in the northern hemisphere. In the zonal mean statistics the baroclinic waves of the southern hemisphere are weakened even more than in the northern hemisphere.

9. Acknowledgements

The wavenumber-frequency analysis of forecast ensembles was proposed by A. Hollingsworth. His helpful suggestions and comments throughout this

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10. Appendix A

Space-time spectral analysis

The travelling disturbances on latitude circles can be expressed in terms of amplitudes and phases of progressive and retrogressive waves

$$W(x, t) = \sum_k W_k(x, t),$$

$$W_k(x, t) = \sum_{\omega} R_{k, \pm \omega} \cos(kx \pm \omega t + \phi_{k, \pm \omega}), \quad (A1)$$

where k is the wavenumber, ω frequency, $R_{k, \pm \omega}$ amplitude, $\phi_{k, \pm \omega}$ phase of the progressive ($-\omega$) and retrogressive ($+\omega$) waves. The last equation can be rewritten as a sum of space sine and cosine contributions with time dependent coefficients:

$$W_k(x, t) = C_k(t) \cos kx + S_k(t) \sin kx. \quad (A2)$$

Following Hayashi (1971), the two-sided space-time power spectrum is then

$$P_{\pm \omega} = \frac{1}{2} R_{k, \pm \omega}^2 \\ = \frac{1}{4} \{P_{\omega}(C_k) + P_{\omega}(S_k) \pm 2Q_{\omega}(C_k, S_k)\}, \quad (A3)$$

where $P_{\omega}(C_k)$ and $P_{\omega}(S_k)$ are the power spectra of the cosine and sine coefficients, respectively, and $Q_{\omega}(C_k, S_k)$ is the quadrature spectrum of the cosine and sine coefficients.

For the computation of the eddy fluxes of heat and momentum we will use the cospectrum between travelling waves of two different parameters like the zonal and meridional wind component for the horizontal eddy momentum flux. According to Hayashi (1971), the cospectrum between two different kinds of disturbances (W and W') can be calculated from the cospectrum and quadrature spectrum of the cosine and sine coefficients:

$$K_{\pm \omega}(W_k, W'_k) = \frac{1}{2} R_{k, \pm \omega} R'_{k, \pm \omega} \cos(\phi'_{k, \pm \omega} - \phi_{k, \pm \omega}) \\ = \frac{1}{4} \{K_{\omega}(C_k, C'_k) + K_{\omega}(S_k, S'_k) \\ \pm Q_{\omega}(C_k, S'_k) \mp Q_{\omega}(S_k, C'_k)\}, \quad (A4)$$

and the quadrature spectrum by

$$Q_{\pm \omega}(W_k, W'_k) = \frac{1}{2} R_{k, \pm \omega} R'_{k, \pm \omega} \sin(\phi'_{k, \pm \omega} - \phi_{k, \pm \omega}) \\ = \frac{1}{4} \{\pm Q_{\omega}(C_k, C'_k) \pm Q_{\omega}(S_k, S'_k) \\ - K_{\omega}(C_k, S'_k) + K_{\omega}(S_k, C'_k)\}. \quad (A5)$$

With the knowledge of the cospectrum and quadrature spectrum between the two travelling waves, we can compute their phase difference.

$$\phi'_{k, \pm \omega} - \phi_{k, \pm \omega} \\ = \tan^{-1} \{Q_{\pm \omega}(W_k, W'_k) / K_{\pm \omega}(W_k, W'_k)\}. \quad (A6)$$

The quality of our phase calculation can be measured in terms of coherence between the two waves:

$$\text{coh}_{\pm \omega}^2(W_k, W'_k) = \frac{K_{\pm \omega}^2(W_k, W'_k) + Q_{\pm \omega}^2(W_k, W'_k)}{P_{\pm \omega}(W_k) P_{\pm \omega}(W'_k)} \quad (A7)$$

A value close to 1 for the coherence indicates a reliable determination of the phase difference.

In practice, the time spectra were calculated by the Fast Fourier Transform method. The basic time-spectral interval for the analysis and forecast ensemble dataset is 1/112 days. For the discussion of the baroclinic waves the spectrum was integrated over two spectral bands. The medium frequency band contains 21 spectral intervals and the high frequency band 17 intervals. The limiting coherence to reject a null hypothesis of zero coherence at the 95 % level can be calculated by a formula proposed by Julian (1975).

$$\beta^2 = 1 - (1 - p)^{1/(n-1)}. \quad (A8)$$

When we assume that the actual coherence is zero we will find with a given probability p that a coherence of β or larger will be found depending on how many effective Fourier coefficients n (number of degrees of freedom) are present in the spectral band. The coherence between two disturbances in the medium frequency band should be above 0.37 and in the high frequency band above 0.41 in order to have significant spectral estimates. When the vertical structure of the baroclinic waves is looked at in a latitude band, a further integration over 8 latitude circles is carried out. This increases the number of effective Fourier harmonics by a factor of 8 and the necessary coherence for significant results drops to 0.13 in the medium frequency and 0.15 in the high frequency band. Goodman (1957) showed that an estimate of the possible phase error

can be calculated from

$$\sin^2(\Delta\phi) = \frac{1 - \text{coh}^2}{\text{coh}^2} \{ (1 - p)^{-2/n} - 1 \} \quad (\text{A9})$$

For an assumed coherence of 0.95 this calculation gives phase errors of 11 to 12 degrees in the frequency integrated bands and about 4 degrees in the case where an additional averaging over 8 latitudes has been carried out. However, these error estimates seem to be too high, as we found a close agreement between the fluxes calculated from the mean amplitude and mean phase difference of two waves (first part of eq (A4)) and from the sum of the co- and quadrature spectra (second part of eq (A4)). An exception was found for the computation of the horizontal eddy momentum flux where a very low coherence between the zonal and meridional wind component makes it impossible to determine the phase difference between the two variables.

The spatial variation of the time amplitude of transient disturbances can be calculated by using a method proposed by Hayashi (1979). The space-time series of the data are filtered in space by a zonal Fourier decomposition. For the wavenumber range of interest, the spatial values on the selected latitudes are calculated:

$$\begin{aligned} C(x, t) &= \sum_k C_k(t) \cos kx + S_k(t) \sin kx \\ &= \sum_k A_k(t) \cos [-kx + \phi_k(t)], \end{aligned} \quad (\text{A10})$$

$$\begin{aligned} S(x, t) &= \sum_k -C_k(t) \sin kx + S_k(t) \cos kx \\ &= \sum_k A_k(t) \sin [-kx + \phi_k(t)], \end{aligned}$$

and a time cross-spectral analysis is carried out using the Fast Fourier Transform method at each gridpoint (x) on that latitude.

11. Appendix B

Diagnostic formulae

For the discussion of the vertical structure of the baroclinic waves some very useful diagnostic

formulae have been derived which show the hydrostatic and geostrophic relationship of some variables. Assuming that the wave structure for a fixed time is given by

$$\psi(x, y, p) = \psi_0(y, p) \cos [kx - \phi_\psi(y, p)],$$

where $\psi_0(y, p)$ is the amplitude and $\phi_\psi(y, p)$ is the phase of either of the variables Z , T , u , v or ω . Substituting the wave form of the geopotential height and the temperature into the hydrostatic equation we get an equation

$$\frac{\partial Z_0}{\partial(\ln p)} = \frac{R}{g} T_0 \cos(\phi_T - \phi_Z) \quad (\text{B1})$$

which relates the vertical change of the geopotential height wave to the amplitude of the temperature wave and to the cosine of the phase difference between Z and T .

From the thermal wind equation, we find that the vertical change of the meridional wind amplitude is proportional to the amplitude of the temperature wave and the sine of the phase difference between T and v_g .

$$f \frac{\partial V_{0,g}}{\partial(\ln p)} = -RT_0 \sin(\phi_T - \phi_{v,g}). \quad (\text{B2})$$

Any form of horizontal or vertical eddy flux can be written as

$$\overline{\beta' \gamma'} = \frac{1}{2} \beta_0 r_0 \cos(\phi_\beta - \phi_r). \quad (\text{B3})$$

Using the geostrophic wind approximation, we find that the horizontal eddy momentum flux

$$\overline{v'_g u'_g} = \frac{kg^2}{2f^2} Z_0^2 \frac{\partial \phi_z}{\partial y} \quad (\text{B4})$$

depends on the horizontal phase tilt of the geopotential wave and the horizontal eddy heat flux

$$\overline{v'_g T'} = \frac{kg^2}{2fR} Z_0^2 \frac{\partial \phi_z}{\partial(\ln p)} \quad (\text{B5})$$

depends on the vertical tilt of the geopotential height wave.

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