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Multivariate statistical investigations of the northern hemisphere circulation during the El Niño event 1982/83

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ABSTRACT

Using a multivariate statistical testing procedure, the anomalies of the monthly mean northern hemisphere stream function in 1000, 850, 500 and 200 hPa of the period January 1982 to September 1983 are compared to the respective monthly values of the period January 1967 to December 1981, to find out whether or not the El Niño 1982/83 had a statistically significant influence on the northern hemisphere circulation. The multivariate test utilizes the parametric Hotelling T^2 statistics, whereby the stream functions are projected on a low-order set of global orthogonal functions (spherical harmonics) which reduce the number of spatial parameters to be tested drastically while still retaining most of the spatial variance. To clarify which special wave amplitude or combination of wave amplitudes leads to a significant circulation anomaly, a multivariate test hierarchy is defined. The results show clearly and well-defined significant anomalies at 500 hPa between July 1982 and September 1983, where the dominant anomaly is in wave amplitude $\psi_l^m (m = 1, l = 2)$. In 200 hPa, a clear signal is detected only during the winter season 1983 (January–March). At the two lower levels, a signal is not so clear. Using the anomalous ψ_2^1 amplitude at 500 hPa, three distinct periods for this wave can be distinguished: a build-up phase July 1982 – December 1982, a mature phase January 1983 – March 1983 and a decay period from April 1983 – September 1983.

1. Introduction

The El Niño event 1982/83 in the equatorial Pacific was by far the most intense event of this kind during the last 100 years. Its meteorological, oceanographic and biological features in the equatorial region have been well observed and documented. Besides the tropical anomalies, Arkin et al. (1983) provide maps of the circulation anomalies for different pressure levels, seasonally averaged between 40° N and 40° S. Their connection with the tropical events is discussed, e.g., by Rasmussen (1984). For the extratropical parts of the atmosphere details of atmospheric circulation anomalies are discussed by Quiroz (1983). But in all these documentations of the anomalous circulation patterns, no attempt is made to evaluate if all the atmospheric anomalies during 1982/83 are a statistically significant departure from the observations prior to 1982, including also effects of weak El Niño events, or if they are more or less part of the atmospheric internal fluctuations ("noise"). Deciding if there is a significant deviation ("signal") from a reference period and defining which component of the circulation (e.g. which grid point, wave or EOF amplitudes) contains this signal, could be of use for model purposes (Déqué and Royer, 1984).

Using a multivariate statistical test, only von Storch (1984) showed for selected atmospheric statistics of the 500 hPa geopotential height, that January 1983 is in some parts a significantly different month compared to the Januaries 1967–81, while January 1982 is not. Quite recently in a paper by von Storch and Kruse (1985), the methodical aspects of assessing the multivariate statistical significance of difference patterns was discussed in detail and again applied to the 500 hPa January anomaly of 1983 as well as to simulations of El Niño induced anomalies in GCM's. The purpose of this paper will be a statistical investigation of the northern hemisphere extratropical circulation to find out which monthly means between January 1982 and September 1983 are significantly different from the observations of the respective month during 1967 to 1981, and furthermore, to document the resulting significant anomalies also with respect to the seasonal cycle.

The type of these deviations may also give hints on the physical connection with tropical anomalies e.g. by using the proposed mechanism of Hoskins and Karoly (1982) and Simmons et al. (1983). For deriving the significance a multivariate statistical approach is used for two reasons.

(a) We are not interested in the significance of local deviations but in the significance of all (global) deviations, since an El Niño event is a quasi-global phenomenon.

(b) We are dealing with anomalies of the quasi-stationary waves, which exhibit, due to their planetary scales, large correlations in space. Applying simultaneously very many univariate Student t-tests (Chervin and Schneider, 1976) can yield large areas of significance only due to the high spatial correlation. These spatial degrees of freedom, as they show up, e.g., in an EOF analysis, cannot be incorporated in an univariate test as the Student t-test. Therefore, multivariate tests have to be applied.

The organization of this paper is therefore the following: in Section 2 we will describe the data material used and the necessary data manipulations. Section 3 will give the multivariate statistical theory and its implications for the practical application. In Section 4 the results of the application of this testing procedure on the data are given.

2. Data material and manipulation

The basis data used for this investigation are daily analyses (00 UTC) of surface pressure, 850 hPa, 500 hPa and 200 hPa geopotential height fields on a polar stereographic octagon grid with grid distance of 381 km at 60° N from January 1st, 1967 till September 30th, 1983. They are the result of the routine objective analysis scheme of the German Weather Service (DWD) (Kaestner, 1971). Using a 16-point bicubic interpolation scheme these daily data are interpolated to a lat/long grid with 5° spacing in N/S direction between 90° N and 15° N and 10° spacing in E/W direction, giving a total of 576 grid points. From these daily data, monthly means have been calculated, which will be the basic data for the statistical investigation.

Surface pressure is transformed into geopotential height of the 1000 hPa using eq. (1)

$$\phi_{1000} = 0.8\,(p_0 - 1000)$$

$$p_0 \text{ in hPa}, \phi_{1000} \text{ in gpdm.} \tag{1}$$

As will be shown below, it is comfortable for the statistics to project the height fields on a suitable set of global orthogonal functions. We chose spherical harmonics with zonal wave number truncation $M = 11$ and meridional node number truncation $J = 17$. This truncation defines a Gaussian grid which is nearly identical with the 5° × 10° degrees lat/long grid. After spline interpolation in meridional direction from the geographical to the Gaussian grid, a smooth extrapolation of the heights at 20° and 15° N to 10° and 5° and assuming symmetry around the equator, the spherical harmonics amplitude of the geopotential height fields are readily calculated, using standard procedures (Haltiner and Williams, 1980). Utilizing the linear balance equation in spherical harmonics (Merilees, 1968) the geopotential amplitudes are converted to spherical harmonics amplitudes of the monthly mean stream function of the respective pressure level. Stream function is calculated mainly because for two reasons: firstly to facilitate a better comparison with the observation of Arkin et al., 1983 and secondly to use a more natural variable from the modelling point of view. Compared to grid point space, less than one half of the number is necessary to retain the full spatial variance (576 grid points vs. 208 wave amplitudes (real and imagery parts together)). The monthly means of the period January 1967 – December 1981 will serve as the references to which the monthly means from January 1982 till September 1983 will be compared.

3. Statistical theory and its consequences

Recently, von Storch and Kruse (1985) discussed in detail the methodical framework of multivariate testing procedures. Within their notation, this paper (Subsection 3.1) will deal with an extension of their problem I ("significant El Niño anomalies in the atmosphere") using their method B ("spectral expansion of spatial structure") for data compression (compare Subsection 3.2.1) applying a similar test hierarchy (compare Subsection 3.2.2) but utilizing a different test statistics (Subsection 3.2.1).

3.1. *Null/alternative hypothesis*

Since we are interested in the question if the *whole* circulation patterns during the period are different from the respective monthly patterns during the reference period, we define a phase state vector $\boldsymbol{x}$ of dimension p which contains all p components of the stream function representation of a certain month (e.g., grid point values, spherical harmonics amplitudes).

Let us define $\boldsymbol{x}_0$ as the mean value of $\boldsymbol{x}$ during the reference period different for all 12 calendar months. Then the null/alternative hypothesis reads:

H_0: $\boldsymbol{x}$ from a month of the test period is not significantly different from $\boldsymbol{x}_0$ for the same calendar month;

H_1: $\boldsymbol{x}$ from a month of the test period is significantly different from $\boldsymbol{x}_0$ for the same calendar month.

3.2. *Test statistics*

3.2.1. Hotellings T^2. Assuming $\boldsymbol{x}$ to be a multivariate normally distributed random variable, which is approximately true since $\boldsymbol{x}$ is a monthly mean and the central limit theorem holds, the following variable can be computed (Hotelling, 1931; Morrison, 1976)

$$T^2 = (\boldsymbol{x} - \boldsymbol{x}_0)^t \underline{\underline{S}}^{-1} [\boldsymbol{x} - \boldsymbol{x}_0), \qquad (2)$$

where $\underline{\underline{S}}$ is an estimate of the covariance matrix (from the reference period) and t means the transposed vector.

If $\underline{\underline{S}}$ is computed from the standard sum-of-product formula, it is a Wishart-distributed random matrix with $(N-1)$ degrees of freedom (N sample size) and statistically independent from the random variable $(\boldsymbol{x} - \boldsymbol{x}_0)$. In that case and under the null-hypothesis H_0 the random variable T^2 is said to be Hotelling distributed with p parameters and $(N-1)$ degrees of freedom.

The Hotelling distribution is related to the F-distribution via

$$F = T^2 \frac{N-p}{p(N-1)}, \qquad (3)$$

where the random number F is distributed under H_0 as Fisher-F with p and $N-p$ degrees of freedom.

Hannoschöck and Frankignoul (1985) used the T^2-statistics to derive a significance test between a model control and a perturbation run. The T^2 statistics differs from the χ^2-distributed test variable used e.g., by Barnett and Hasselmann (1979) or by von Storch and Kruse (1985) in so far that the sampling distribution of the covariance matrix estimate is taken into account.

Eq. (3) has one severe limitation, which was also noted by Chervin (1981): because F and T^2 are positive semidefinite numbers, the number of parameters to be tested (p) is restricted to be less than the number of statistical degrees of freedom (N). Thus, it is not possible to apply the test to the full grid point values ($p = 576$) or even to the spherical harmonics amplitudes ($p = 208$). Using present day field data sets with N at most 100 Chervin (1981) and also Livezey and Chen (1983) concluded from this that a practical application of the T^2-statistics is not possible. This is not true since one can define a suitable subspace of the vector space containing $\boldsymbol{x}$ which accounts for the largest parts of the possible signal (Hasselmann, 1979), and which will reduce p drastically. The subspace may be defined, for example, by the lowest components of the spherical harmonics expansion (e.g., Hannoschöck and Frankignoul, 1985) if one is dealing with problems of the quasi-stationary planetary waves or also through EOF expansion of the fields (von Storch and Roeckner, 1983).

3.2.2. Multivariate test hierarchy. Applying the T^2 statistics with fixed parameter numbers and degrees of freedom yields a global yes/no answer for the rejection of the null hypothesis but no hint which parameter or combination of parameters is significantly different from the counterpart during the reference period. To circumvent this problem Hasselmann (1979) devised the method of defining a test hierarchy. Changing the hierarchy parameter, that is, e.g., increasing the number of tested parameters, will give indication at which parameter a significant T^2 (a "signal") shows up and/or at which parameter the signal fades away. For the present purpose the hierarchy is defined by decreasing the spatial scale of the spherical harmonics due to a suitable numbering in wave number space (Fig. 1) until the maximum allowable number of parameter is reached.

3.2.3. Univariate testing. Another possibility for

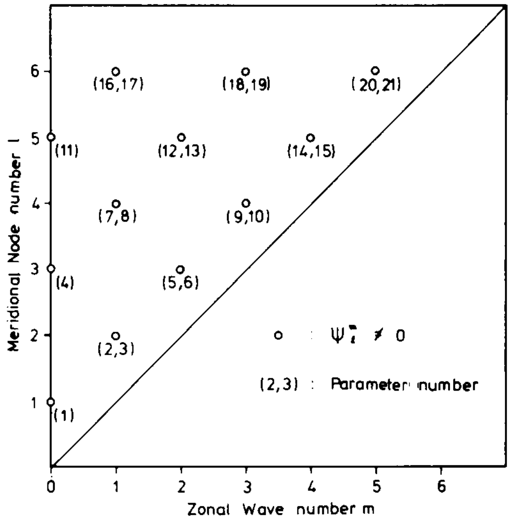

Fig. 1. Schematic description of the multivariate test hierarchy in wave number space of spherical harmonics.

finding the significant parameter *after* a rejection of the multivariate null hypothesis is to test the significant components separately. If, during the multivariate testing, a combination of parameter is found to be significant, this testing will clarify if the whole combination is anomalous or if only one special parameter contains the signal. Since multivariate testing will be done in wave number space, the univariate testing will be done also on the spherical harmonics amplitudes. For amplitudes ψ_l^m with zonal wave number $m = 0$ this will be the standard Student t-test

$$t_l^0 = \frac{\psi_l^0 - \bar{\psi}_l^0\,(\text{ref})}{s(\psi_l^0\,(\text{ref}))}, \tag{3}$$

where $\bar{\psi}_l^0$ (ref) and s^2 (ψ_l^0 (ref)) are the mean and the variance of the respective wave amplitude during the reference period.

A Student t-test is also applied to the first component ψ_l^0 of the multivariate test hierarchy. For amplitudes ψ_l^m with $m \neq 0$ it is necessary to test two numbers together (amplitude and phase, real and imaginary part). This will be done using a bivariate version of the Hotelling T^2 statistics.

$$(T^2)_l^m = (\psi_l^m - \bar{\psi}_l^m)^t\,(\underline{\underline{S}}_l^m)^{-1}\,(\psi_l^m - \bar{\psi}_l^m)$$

$$\underline{\underline{S}}_l^m = \begin{pmatrix} s^2\,(\text{Re}\,\psi_l^m) & \text{cov}\,(\text{Re}\,\psi_l^m, \text{Im}\,\psi_l^m) \\ \text{cov}\,(\text{Re}\,\psi_l^m, \text{Im}\,\psi_l^m) & s^2\,(\text{Im}\,\psi_l^m) \end{pmatrix} \tag{4}$$

3.3. *Statistical estimates*

The mean values for the reference period are estimated as sample means for each calendar month from the monthly means between January 1967 and December 1981, thus the sample size for each month is $N = 15$. The necessary variances and covariances are estimated using the standard sum-of-product formula. Then the covariance matrix $\underline{\underline{S}}$ (eq. (2)) and $\underline{\underline{S}}_l^m$ (4) are Wishart distributed random matrices with $N - 1 = 14$ degrees of freedom. To enhance the number of degrees of freedom of the matrix $\underline{\underline{S}}$ the so-called "pooled" covariance matrix $\underline{\underline{S}}$ will be computed

$$\underline{\underline{S}}_p\,(\text{month}) = \tfrac{1}{3}\,(\underline{\underline{S}}\,(\text{month} - 1) + \underline{\underline{S}}\,(\text{month}) + \underline{\underline{S}}\,(\text{month} + 1)) \tag{5}$$

$\underline{\underline{S}}_p$ is Wishart-distributed with $3(N - 1) = 42$ degrees of freedom (Morrison, 1976) if successive monthly means are independent. Since this is in general not the case one should use instead of N an effective sample size N_{eff}. The effective sample size may be evaluated for the univariate case in a straight forward way, but no general way is known for the multivariate approach. Using a somewhat heuristic approach (see Appendix) we estimated the effective degrees of freedom to be of the order of 30, meaning an effective sample size of 11 or equivalently 3 consecutive months forming a sample of two independent means.

4. Results

4.1. *Performance of the multivariate test hierarchy*

To indicate how the multivariate test hierarchy works and how the interpretations of the results in the Subsection 4.2 have been derived, we present the results for November 1982, which is a characteristic example for all possibilities that occurred (Fig. 2). As covariance estimate the monthly covariance matrix $\underline{\underline{S}}$ with 14 degrees of freedom is used. The test sequence at 200 hPa shows T^2 values always well below the standard significance levels 99%, 95% and 90%, thus one can conclude that there is no signal in the data.

The test hierarchy for the 500 hPa level shows a well-developed signal ($T^2 \gg 99\%$ level) with inclusion of the parameter combination (2, 3), which is the spherical harmonics wave $m = 1, l = 2$ (see Fig. 1). The inclusion of more amplitudes in the testing results only in a small increase of T^2,

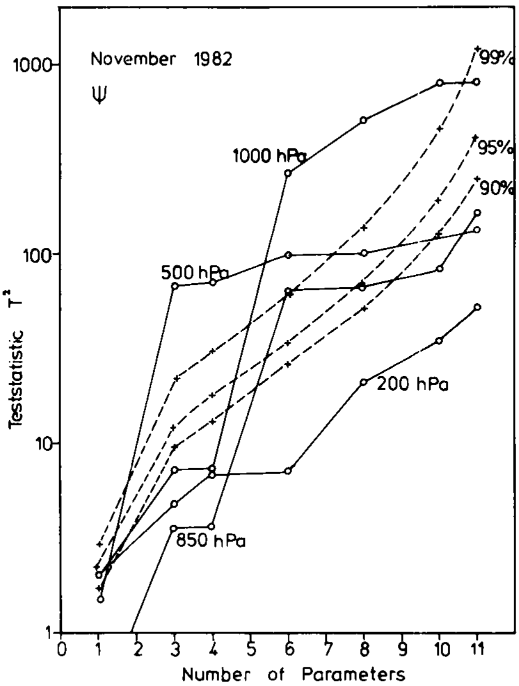

Fig. 2. Resulting T^2 values of the multivariate testing scheme (full lines) of the monthly mean stream function at the 4 pressure levels of November 1982, compared to the 90%, 95% and 99% quantiles of the Hotelling distribution (dashed lines).

obviously this combination depends solely on ψ_l^2 and the higher modes do not add any signal. The decision that the three parameter ψ_0^1 and ψ_1^2 contain the significant signal corresponds with the selection rule D ("maximal significance") of Barnett et al. (1981).

At 850 hPa, only one component exceeds the 95% significance level. This could be by mere chance. The 1000 hPa test sequence shows a similar behaviour as the 500 hPa sequence but with a signal at higher scales (primarily through adding the component $(5, 6) = \psi_3^2$).

4.2. *Time sequence of the multivariate test*

When applying the test to a sequence of l months (here January 1982 to September 1983, $l = 21$) one has to keep in mind that under the null hypothesis, none of the l months being significantly different from the respective months of the reference sample, the number of rejections k of the null hypothesis at the $(1 - \alpha)$. 100% level is a random number with a binomial distribution (Livezey and Chen, 1983):

$$p_k^l(\alpha) = \binom{l}{k} \alpha^k (1-\alpha)^{l-k}, \tag{6}$$

with expectation $E(k) = \alpha \cdot N$.

Thus to interpret a rejection as a signal we will use three arguments: (a) the number of rejections should be well above expectation or/and (b) there is a coherent rate of rejections in time or/and (c) a sequence of rejections of H_0 is found with the increasing number of parameters which corresponds to the selection rule C ("non-sequential fixed significance level") of Barnett et al. (1981). Using the monthly estimates of the covariance matrix $\underline{\underline{s}}$ and a maximum number of parameters of 11, Table 1a–d summarizes the results for the 4 pressure levels.

In 200 hPa, one can see a signal, using argument (c) from above, in April 1982. Geb and Naujokat (1982) showed for 100, 50 and 30 hPa an earlier than normal stratospheric spring warming, the temperature at these levels in the western part of the northern hemisphere during April 1982 reaching almost summer time values with rests of cold air over Asia. Thus the signal in 200 hPa at the border between stratosphere and troposphere could be connected with these stratospheric events. We may speculate if there is a connection with the El Niño event starting about 1 month later. One could think either of an initiation of the event but also of some kind of preparation of the upper tropospheric circulation.

A second significant deviation is found during winter 1983 (February through April), where already the first component ψ_1^0 is significantly different from the reference period. Because ψ_1^0 is proportional to the mean angular momentum of the atmosphere this indicates a significantly different angular momentum which is consistent with the results of Rosen et al. (1984).

In 500 hPa, one sees that there is a rejection during April 1982. This may be by chance but equally, when compared with 200 hPa, be a real signal from the stratosphere. Starting in July 1982, that is about 1 month after the onset of the El Niño event, a well defined sequence of rejections lasting until September 1983 is found. Also the number of rejections is well above expectation, thus we can say that the El Niño event 1982/3 has exhibited a

Table 1. *Results of the multivariate testing scheme applied to the sequence of monthly mean stream functions between January 1982 and September 1983. The horizontal axis shows the hierarchy parameter that is the number of wave amplitudes to be tested. The vertical axis indicates the time January 1982 to September 1983. A "9" in the table shows that for the respective parameter combination the null hypothesis is rejected at the 99% significance level, a "5" at the 95% and a "0" at the 90% level. At the bottom, the sum of rejections, at least at the 95% level, are given; these may be compared with the expectation $E(k) = 0.05 \cdot 21 = 1$ or the probability of occurring with chance P as computed by eq. (6); monthly covariance matrix used (a) 200 hPa, (b) 500 hPa, (c) 850 hPa, (d) 1000 hPa*

	200 hPa																
	1	3	4	6	8	10	11	13	15	17	19	21	22	24	26	28	30
Jan 82																	
Feb																	
Mar																	
Apr	5	0	0		5	0											
May																	
Jun																	
Jul																	
Aug																	
Sep																	
Oct																	
Nov	0																
Dec																	
Jan 83	0																
Feb	9	9	9	5	0	5											
Mar	5	0			0												
Apr	9	9	9	9	5												
May	5			0													
Jun	0																
Jul																	
Aug																	
Sep																	
95% rejections	5	2	2	2	2	1											
Expectation	1	1	1	1	1	1											
Bin probability %	0	20	20	20	20	38											

strong effect on the middle troposphere at the northern hemisphere extratropics.

The main signal sets in by adding the parameter (2, 3) which is (Fig. 1) wave amplitude ψ_2^1.

In 850 hPa and 1000 hPa, a signal cannot be defined equally well as at 500 hPa. The signal is not so coherent in time as in the middle troposphere. Additionally it is also not very coherent in height between 1000 and 850 hPa. The reason may be that the signal is disturbed by the surface influences, or that it is confined to higher scales which are not properly resolved with the present type of testing. The indication for this is that a signal sets in after adding parameter combination (5, 6) which is wave amplitude ψ_3^2.

4.3. *Comparison with the "pooled" covariance matrix* $\underline{\underline{S}}$

Kendall (1975) proposed for multivariate testing not only the necessary condition $N > p$ but also $N > 10p$. Obviously this cannot be achieved with the monthly covariance matrix. But using the pooled estimate $\underline{\underline{S}}_p$ (eq. 5)) it is possible to achieve this for the lowest 4–6 parameters, of course having in mind the restriction through the effective sample size. Table 2 gives the summarizing result for 500 hPa which may be compared to Table 1b. Up to component 6 the number of rejections of H_0 is very similar, the level of significance being slightly higher when using the $\underline{\underline{S}}_p$ estimate due to the higher degrees of freedom. The same holds also for the three other

Table 1b

	500 hPa																
	1	3	4	6	8	10	11	13	15	17	19	21	22	24	26	28	30
Jan 82																	
Feb																	
Mar																	
Apr		5	0														
May																	
Jun																	
Jul		5	5	5	0												
Aug	5	5	5	0	9	5	0										
Sep		9	9	5	5	5											
Oct		9	9	9	9	9	0										
Nov		9	9	9	5												
Dec		9	9	9	9	0											
Jan 83		5	5	0													
Feb	9	9	9	9	9	9	5										
Mar	0	9	9	9	5												
Apr	0	9	5	9	9	5	5										
May		9	9	9	9	5	0										
Jun		9	9	5	0	0											
Jul		5	5	5													
Aug		5	5	0	9	5	0										
Sep		9	5	0													
95% rejections	2	16	15	11	10	7	2										
Expectation	1	1	1	1	1	1											
Bin probability %	20	0	0	0	0	0	20										

layers. Thus Kendall's proposal seems to be not very important, at least for the present data set up to components 4–6. It may be of importance if one is interested in signals confined to higher scales as in 850 or 1000 hPa. But in view of a possible "mixing" of the anomalies between adjacent months when using the pooled estimate the monthly estimates of the covariance matrix are preferred.

4.4. *Univariate testing at 500 hPa*

In Subsection 4.2, it was shown that the 500 hPa stream function is an ideal candidate for studying the time sequence of the El Niño event in the northern hemisphere troposphere due to the clear definition of the signal. Furthermore, after the onset of the El Niño there is a clear evidence that the main signal is confined to an anomalous wave amplitude of zonal wave number $m = 1$ and meridional node number $l = 2$. This is consistent with Quiroz (1983) who noted that the $m = 1$ wave at 50° N was extraordinary during fall/winter 1982/83 but in some discrepancy with von Storch (1984), who noted a significant $m = 3$ amplitude anomaly. The difference may be partly explained by the use of different basis functions for the data compression (spherical harmonics versus EOF's).

Applying the univariate test (eq. (4)) to this amplitude, we found that the anomaly is significantly different from zero during April 1982 at the 95% level and again after July 1982 till September 1983, at least at the 95% level. Fig. 3 shows in the complex plane the amplitude of ψ_2^1 as a function of time. The April anomaly shows up to be 180° out of phase from the anomalies July and later. These anomalies (Fig. 3) may be grouped into 3 periods.

(a) A build-up period in amplitude with roughly fixed spatial phase: July 1982 – December 1982. This corresponds to the period in which the region of anomalous convection, as depicted by the anomaly in outgoing long wave radiation, in the equatorial Pacific moved from 140° E to about 140° W while intensifying (Rasmussen, 1984).

(b) A mature stage with roughly fixed amplitude

Table 1c

	850 hPa																
	1	3	4	6	8	10	11	13	15	17	19	21	22	24	26	28	30
Jan 82																	
Feb																	
Mar																	
Apr																	
May		5	0		0												
Jun					0												
Jul		5	0			5											
Aug																	
Sept	0		5														
Oct																	
Nov				9	0												
Dec																	
Jan 83				5	9	5	0										
Feb	5			5	0												
Mar		0	0	9	9	5	0										
Apr	5			0													
May																	
Jun		0															
Jul		0															
Aug					5	0											
Sep																	
95% rejections	2	2	1	4	3	3											
Expectation	1	1	1	1	1	1											
Bin probability %	20	20	38	2	7	7											

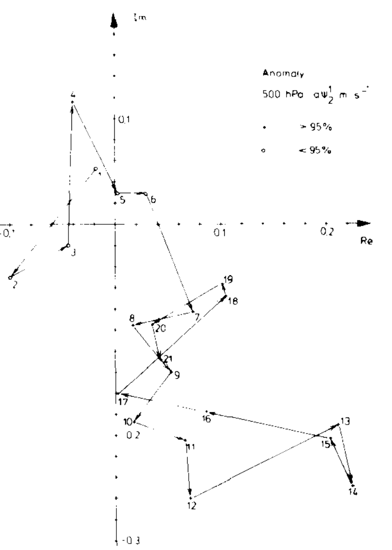

Fig. 3. Complex plane representation of the monthly mean anomaly of the wave amplitude ψ_2^1 between January 1982 and September 1983. Anomalies significantly different from zero at the 95% significance level are indicated by crosses. Numerical values of ψ_2^1 are multiplied by the radius of the earth a, 1–12 indicate January to December 1982, 13–21 January to September 1983.

but a spatial phase shift of about 30° to the east: January 1983 – March 1983. This is also the peak period of the anomalous convection at about 140° W, connected with the largest extension of the convective area (between 160° E and 80° W) and the strongest subsidence in the Indonesian region at 140° E (also Rasmussen, 1984).

(c) A decay stage with amplitudes going down with varying spatial phase position: April 1983 – September 1983. For the equatorial convection in the Pacific, this is about the period of the return-to-normal.

The total amplitude (anomaly plus annual cycle) for ψ_2^1 compared with the mean annual cycle of ψ_2^1 can be seen in Fig. 4a for the amplitude and Fig. 4b for the spatial phase. It shows that the anomaly works as an amplification of the annual cycle

Table 1d

	1000 hPa																
	1	3	4	6	8	10	11	13	15	17	19	21	22	24	26	28	30
Jan 82																	
Feb																	
Mar																	
Apr	0																
May					0												
Jun																	
Jul	5	9	5	5													
Aug	5																
Sep	9	5	9	5													
Oct																	
Nov	0		9	9	9	5											
Dec																	
Jan 83					5	0											
Feb	5			5	0												
Mar		0	0	5	5												
Apr						0											
May		0		5	9	9	5										
Jun		5	0														
Jul	5	5	5	5				0									
Aug																	
Sept																	
95% rejections	5	4	3	7	4	2	2										
Expectation	1	1	1	1	1	1											
Bin probability %	0	2	7	0	2	20	20										

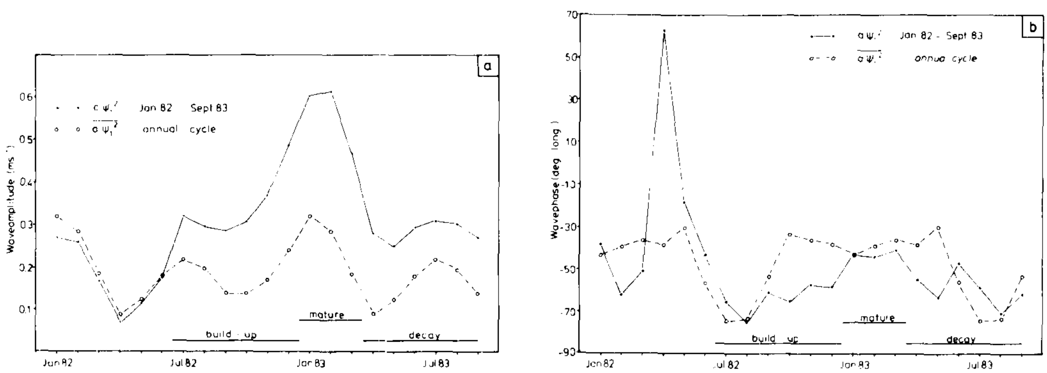

Fig. 4. Amplitude (a) and spatial phase (b) of monthly mean of wave ψ_2^1 between January 1982 and September 1983 (full line) and the mean annual cycle in amplitude and phase of the same wave (dashed line); spatial phase equals the position of the ridge relative to the Greenwich meridian.

during summer 1982 and 1983 and winter 1983, the phases being similar to the mean but the amplitude being strongly enhanced. For fall 1982 and spring 1983 there is an amplification but also a spatial phase shift of about 30° to the west of ψ_2^1 compared to the regular annual cycle.

4.5. *Grid point representation of the anomalies*

The obvious advantage of having used spherical harmonics has been shown above, but they are not very vivid. For evaluating the local character we will project the amplitudes back into the grid point space. This will be done for all anomalies of the

Table 2. *Same as Table 1b, but with the pooled covariance matrix estimate*

	500 hPa																
	1	3	4	6	8	10	11	13	15	17	19	21	22	24	26	28	30
Jan 82																	
Feb																	
Mar																	
Apr		5	5														
May																	
Jun																	
Jul		9	9	5	9	5	5	5	0	0							
Aug	0	9	5	0	9	9	9	9	5	5	9	5	0				
Sep		9	9	9	9	9	9	9	9	9	9	5	5	0			
Oct		9	9	9	9	9	9	9	5	0							
Nov		9	9	9	9	9	9	9	9	5	5	0					
Dec		9	9	9	9	9	9	9	5	0							
Jan 83		9	9	9	9	9	9	5	5	0							
Feb	9	9	9	9	9	9	9	9	5	5							
Mar	5	9	9	9	9	9	9	9	9	9	5	0	5				
Apr		9	9	9	9	9	9	9	9	5	5	0	0				
May		9	9	9	9	9	9	9	9	9	9	5	5				
Jun		9	9	9	5	9	5	5	0								
Jul		9	9	5	9	9	5	5	5	5		0	0				
Aug		5	9	5	9	9	9	9	5	9	9	5	0				
Sep		9	9	9	9	9	9	5	5	5	0						
95% rejections	2	16	16	14	15	15	15	15	13	10	7	4	3				
Expectation	1	1	1	1	1	1	1	1	1	1	1	1	1				
Bin probability %	20	0	0	0	0	0	0	0	0	0	0	2	7				

wave amplitudes yielding the monthly mean anomalies of stream function for the respective month, but also for all amplitudes of the highest parameter number which are significantly different from the reference period, at least at the 95% level. Since the local character of the back projection is not the significant *local* pattern, which is of course due to the result that the T^2 statistics could not be applied in grid point space, we will refer to the representation of significant wave amplitude in grid point space as the back projection of the significant wave amplitude combination or as the statistically filtered fields.

Depending on the period found for the 500 hPa level, the ψ_2^1 amplitude anomalies and filtered anomalies will be presented for the following period and pressure levels.

Mean for October–December 1982 and January to March 1983 for 1000 and 500 hPa. The significant combinations used for the filtered fields present between 65 and 80% of the spatial variance of the total anomaly. For the last period, no grid point representation has been calculated due to the variation in spatial phase.

Fig. 5 gives the maps for the build-up period at 1000 hPa (a) and 500 hPa (b). Obviously here are differences between the lower and the middle troposphere (see below and Subsection 4.6).

Fig. 6 presents the maps for the mature period and may be compared to Fig. 5. For the filtered fields the 1000 hPa maps show a very strong ridge in the Atlantic and western Pacific, a strong trough in the eastern Pacific and compared to Fig. 5a a clear phase shift to the east at about 30° in higher latitudes. 500 hPa exhibits, as one may expect, the characteristics of the dominant ψ_2^1 wave, amplified and phase shifted compared to Fig. 5b, but not differing in character.

4.6. Vertical structure of the anomaly

From Subsection 4.5, one can see that the stream

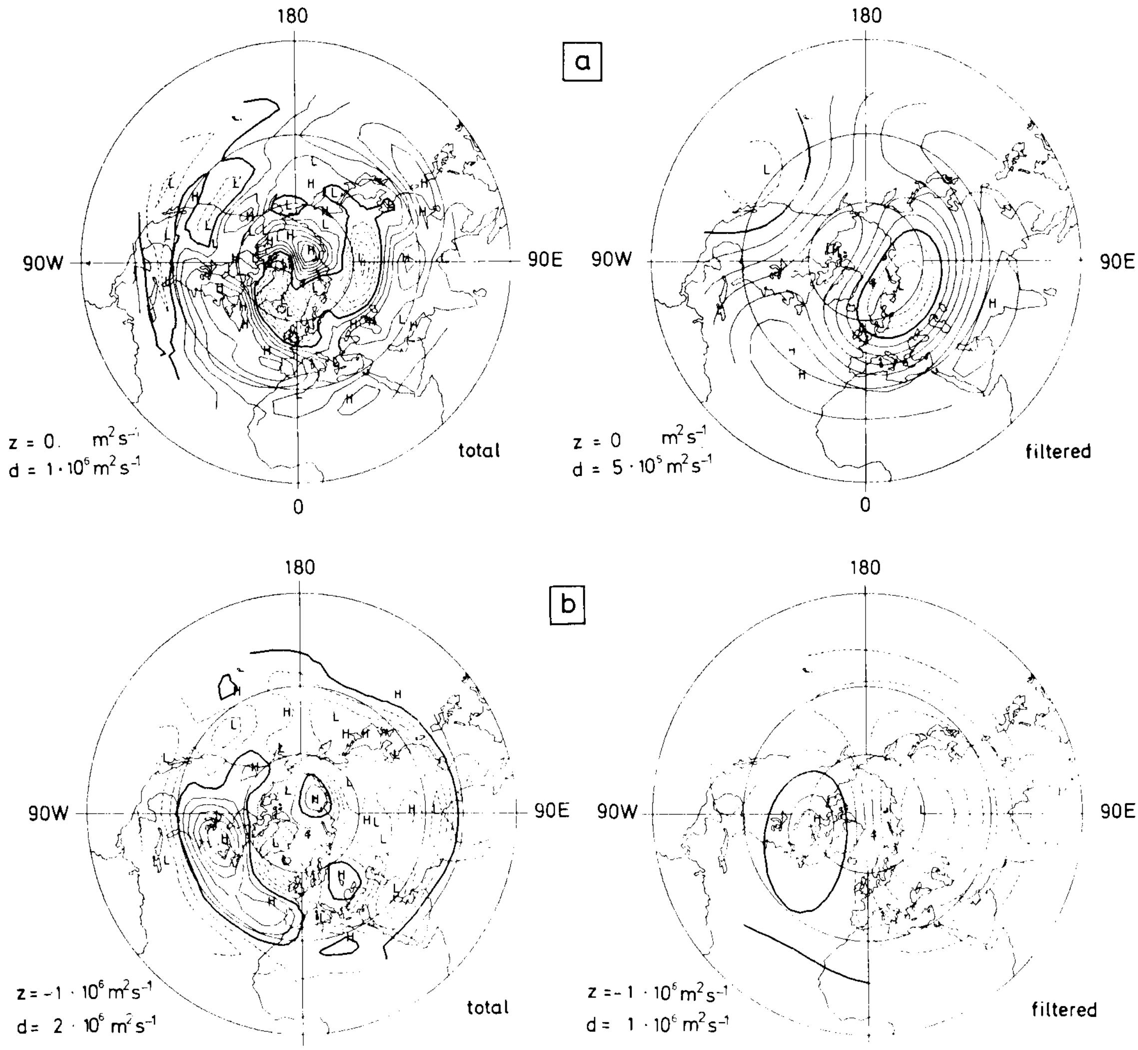

Fig. 5. Observed deviation of the mean stream function ($m^2\ s^{-1}$) for the period October to December 1982 from the mean October to December 1967–81 (left) compared to the statistically filtered stream function for the same period calculated from the significant wave amplitudes (right). The value of the central line (bold line) is indicated by z; contour interval is indicated by d; contour lines less than z dashed and larger than z full; local extremes are indicated by H and L. (a) 1000 hPa. (b) 500 hPa.

function amplitude ψ_2^1 is the dominant anomalous wave number during the El Niño 1982/83, especially in 500 and 200 hPa, but also for some time in 1000 and 850 hPa. Subsection 4.5 gives some evidence of differences between the lower and the middle tropospheres. Therefore, the vertical structure of the ψ_2^1 anomaly will be investigated in more detail.

Figs. 7a,b,c give for the same period as in Subsection 4.5 and for April 1982 the spatial phase for the anomalous amplitude of stream function wave ψ_2^1. The vertical structure during April 1982 (Fig. 7a) is almost barotropic, the phase differences between the different levels being of the order of 10° which is the resolution in the zonal direction.

In contrast with that, the vertical structures

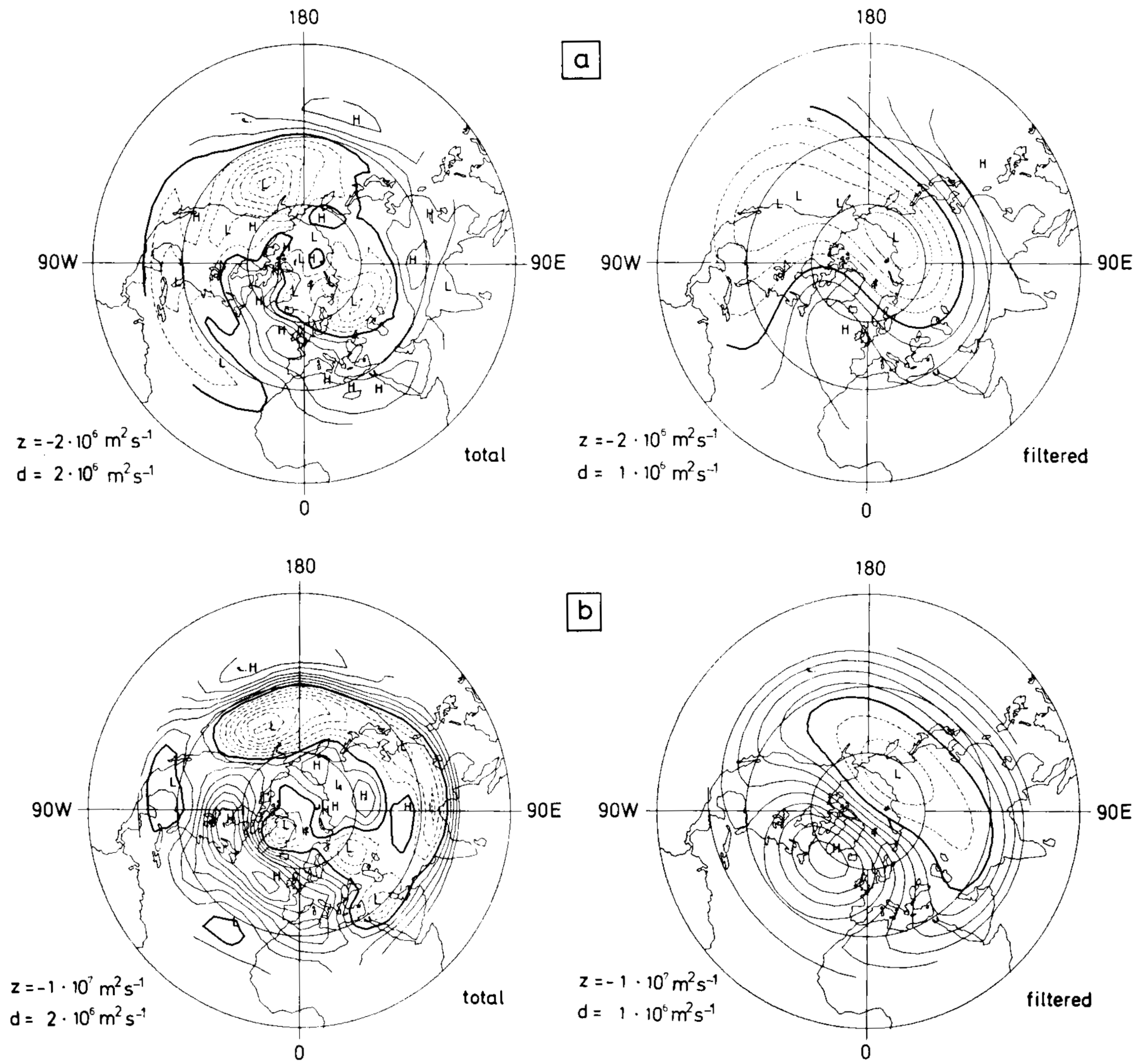

Fig. 6. Same as Fig. 5 but for the period January 1983 to March 1983. (a) 1000 hPa. (b) 500 hPa.

during the build-up and mature periods are different. The lower troposhere (1000 and 850 hPa) shows an almost 90° out-of-phase relationship compared to the upper levels (500 and 200 hPa), with only small phase differences between 1000 and 850 hPa or 500 and 200 hPa. The anomaly seems to exhibit a baroclinic structure at least during winter 1983. Estimating from the variance and covariance of the real and imaginary part of ψ_2^1 a 95% confidence interval of the phase, gives roughly $\pm 60°$ at all pressure levels. But this does not account for the vertical correlations. Thus, the question of whether the observed baroclinic structure is *significantly* different from a barotropic structure cannot be answered within the present statistical framework. Furthermore, the vertical tilt seems not to be continuous as one may expect: the data indicate more or less a two-layer structure. Overall, a more detailed investigation using data of higher vertical resolution and putting emphasis on significant vertical structures seems to be worthwhile.

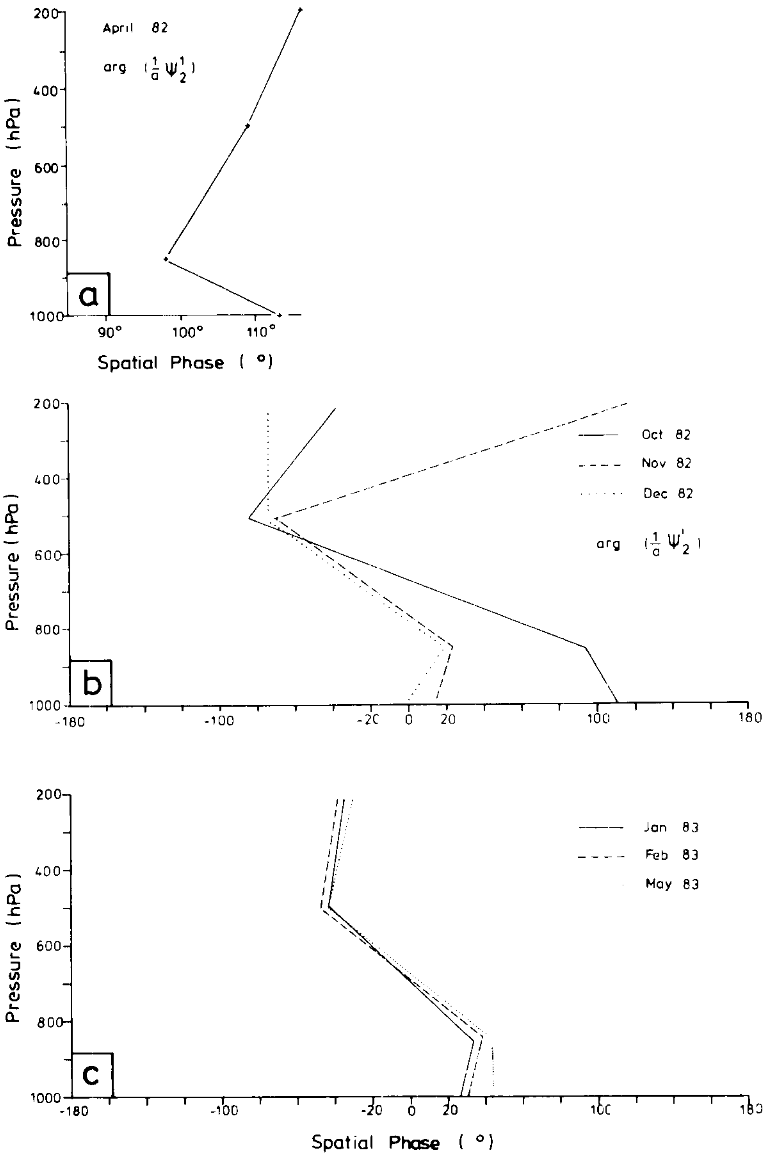

Fig. 7. Spatial phase of the anomaly of the wave amplitude ψ_2^1 as function of pressure level; spatial phase equals the position of the ridge relative to the Greenwich meridian. (a) For April 1982. (b) For October, November and December 1982; note the change of the scale of the abscissa compared to (a). (c) For January, February and March 1983; note the change of the scale of the abscissa compared to (a).

5. Summary and conclusions

The application of the multivariate testing scheme (similar to Hannoschöck and Frankignoul, 1985 and von Storch and Roeckner, 1983) to a data/data comparison using the El Niño event 1982/83 as a perturbation "run" of the real atmosphere and the period 1967–81 as a control "run" is described: of course this testing scheme may equally well be applied to a model/model or model/data comparison (e.g., a simulation of the effects of the El Niño event on the atmosphere).

The problems with the suitable use of the multivariate test statistics (number of statistical degrees of freedom larger than the number of parameters to be tested) are circumvented by a projection of the atmospheric field in a low-order set of global orthogonal functions (here spherical harmonics). This retains most of the spatial variance but reduces the number of parameters drastically. The problem with deciding which parameter or parameter combination is responsible for a rejection of the null hypothesis, that is which parameter combination contains a signal, is solved through the use of a multivariate test hierarchy and an "a posteriori univariate" testing of the single parameters. The necessary statistical estimates of mean and covariance matrix are calculated from the monthly mean stream function amplitudes of the period 1967 till 1981 (reference period) for each calendar month. Thus, applying the multivariate testing scheme to the period January 1982 to September 1983 (test period) will yield the answer if a particular wave amplitude combination of the test period is significantly different from the respective combination of the reference period.

The result is that in 500 hPa, about 1 month after the onset of the El Niño event in June 1982, a clear and well-defined signal can be identified lasting at least until September 1983. The signal is clearly dominated by an anomalous amplitude of the wave with zonal wave number $m = 1$ and meridional node number $l = 2$. Some indication for a signal is found at 1000 and 850 hPa, but it is not as coherent in time as it is at 500 hPa. The reason for this may be that surface effects disturb the signal or that in the El Niño signal for the lower troposphere, smaller scales than at 500 hPa are involved.

At the tropopause level at 200 hPa, a signal of the El Niño event is found only during winter/spring 1983 (January–March). This suggests two interpretations. Either only during the winter period the energy of the anomaly may have propagated vertically into the stratosphere, as is predicted by linear planetary wave models (Dickinson, 1968 or Derome, 1980), or that only during the peak phase of the El Niño (~4–6 K anomaly in SST) over vast areas of the equatorial Pacific (Arkin et. al., 1983) convection in the equatorial region was large enough to affect the stratospheric circulation in the northern hemisphere. During January till April 1983, not only the ψ_2^1 amplitude is significantly affected but also the stream function amplitude ψ_1^0, which is proportional to the mean angular momentum of the atmosphere at the respective level. Comparing the variations in the length of the day with the mean angular momentum budget of the atmosphere, Rosen et al. (1984) described the same phenomenon. The time sequence of the dominant wave amplitude $\psi_2^1(t)$ helped to classify 3 stages of the atmospheric signal on the El Niño event: an amplifying period July–December 1982, where the spatial phase of the anomaly is roughly fixed but the amplitude increases, a mature stage January–March/April 1983, where the December amplitude is roughly fixed but the spatial phase is shifted by about 30° to the east and a decay stage April/May 1983 – September 1983, where the amplitude decreases and the spatial phase is varying much stronger than in the two earlier periods. These stages also correspond to the time of the SST stages at the equatorial Pacific between 80° W and 140° E (Rasmussen, 1984).

Comparison of the amplitude of ψ_2^1 during the test period with the mean annual cycle of the same wave shows, as one would of course expect, a strong interaction with the normal annual cycle. During the winter and summer months 1982 and 1983, ψ_2^1 is spatially in phase with a strong amplification of the annual cycle, but during fall and spring 1982 and 1983, ψ_2^1 is not only different in amplitude but also out-of-phase spatially with the annual cycle. The statistical investigation does not provide an explanation for the time behaviour of the ($m = 1$, $l = 2$) wave amplitude but it shows the necessity of research into processes which determine $\partial\psi_2^1/\partial t$, e.g., by using a barotropic model.

Another possibility for finding a more dynamical interpretation of the El Niño anomalies can be offered by the choice of the a priori basis functions. The spherical harmonics used for the present study have been chosen from a practical point of view, but a multivariate test based, e.g., on a normal mode expansion (following e.g. Simmons et al., 1983) could provide a better basis for the dynamical interpretation. This could also help to clarify the problem of the vertical structure.

One major question of course still remains and cannot be answered by the present statistical analysis: are the observed anomalies in the extra-tropics *caused* by El Niño? Following von Storch and Kruse (1985), who compared GCM-generated El Niño anomalies and observed anomalies in midlatitudes for January and who found a statistically significant equality between them, one may conclude that at least the extratropical anomalies during winter are induced by the equatorial events, but it remains open whether this is true for the summer anomalies as well.

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7. Appendix. Fairness of the multivariate test

One of the questions that has not been asked so far is the following. Could the reference period 1967–1981 be a representative period for estimates of the statistical parameter mean and covariance matrix? This will be tested by applying the multivariate test scheme to the sequence of months between January 1967 and December 1981, using the monthly mean and the monthly estimate of the covariance matrix. The month in question will be removed from the respective estimate; thus the statistical degrees of freedom are reduced by one. The number of rejections of the null hypothesis will be compared with expectation, including its binomial distribution. If the number of rejections at a certain parameter combination appears to be well above or below the number to be expected, one could conclude that the mean and covariance matrix estimates from the reference period possess some systematic bias for the T^2 statistics and thus do not provide a fair test. Table A1 gives for each calendar month (15 different tests) and for the total of all months (180 different tests) the number of the rejections of H_0 for the test hierarchy for 500 hPa, as an example, the layers at 1000 and 850 hPa being similar, utilizing the monthly covariance matrix estimates with 13 degrees of freedom. As one can see, the reference period seems to provide a fair basis for the T^2 statistics, especially as the total number of rejections are close to expectation. Some rejections, more than one can expect, are found for

Table A1. *Same as Table 1b, but for the months of the reference period*

	500 kPa No. of rejections at the 95% level Jan. 67 – Dec. 81																		
	1	3	4	6	8	10	11	13	15	17	19	21	22	24	26	28	30	Expec	P(2)	P(0)
Jan																		1	13	46%
Feb	1	1	1		1															
Mar	2			1																
Apr	1	2	2	1	1	1														
May	1	2	1																	
Jun	2	1			1															
Jul	1	2	1	1			1													
Aug	1	1	1	1		1	1													
Sep	1			1	2	1	1													
Oct	1	1	1			2	1													
Nov			1	1	1															
Dec	1	1				1	1													
Total	12	11	8	6	6	6	5													
	9	9	9	9	9	9	9											Expectation		
	7	10	13	9	9	9	6											*p* (Total) %		

parameters (2, 3 and 4) in 200 hPa (not shown). Since we found a series of rejections for these parameters also in the test period, this means that the anomalies during 1982/83 must be even larger than those from the reference period which are already larger than one expects from pure noise. Applying the same procedure to the data of this reference period but utilizing the pooled covariance matrix estimate with assumed 42 degrees of freedom, led to a very high number of rejections of H_0 for parameter 10 and larger. Assuming 30 degrees of freedom in the pooled covariance matrix removed this deficiency for the data of the reference period but still retained the number of rejections for the parameters 1–10 at a level comparable to Table A1.

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