Tellus (1986), *38A*, 88–91.

SHORT CONTRIBUTION

A beta-control of buoyancy-driven geostrophic flows

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(Manuscript received January 21; in final form May 8, 1985)

Gill (1982) has recently reviewed the history of theoretical studies of small-amplitude stationary baroclinic planetary waves in the atmosphere induced by topography, surface friction and buoyancy forcing. Inasmuch as we are considering buoyancy forcing, Smagorinsky's early (1953) study particularly comes to mind. In the subpolar gyres of the ocean, there is considerable buoyancy loss (Luyten et al., 1985) with accompanying deep convection and unlike the atmospheric counterpart the scale of the appropriate waves appears to be in the non-dispersive range. Moreover, the process may be driven toward finite amplitude control, thus restricting the possible structures of the zonal flow itself. It seems plausible that the horizontal spreading of the North Atlantic Drift and some features of front formation in it may be dynamically associated with the planetary flow pattern caused by the buoyancy loss.

We present some results for a Goldsbrough (Stommel, 1984) type circulation driven by a cross-interface flux at the upper of two density interfaces separating three layers, the bottom of which is at rest.

For two layers moving over a deep one at rest, the equations of mass-flux divergence are

$$-h_x(D+h)_y + h_y(D+h)_x = (\beta/f)\,h(D+h)_x = (f/g')\,w_1, \tag{1}$$

$$-(D-h)_x D_y + (D-h)_y D_x - (\beta/f)(D-h)D_x = -(f/g')\,w_1, \tag{2}$$

where h is the depth of the upper interface and D the depth of the lower interface, and both are taken to have the same reduced gravity g'. The quantity w_1 on the right-hand side is the forced flux (not necessarily vertical) across the upper interface. We choose a Goldsbrough form $w_1 = w_0 \cos\theta$ within a circulation area of radius a. Outside the circle we choose $w_1 = 0$. Thus in the western half of the circle the model implies a steady heat loss, and in the eastern half an equal heat gain. Outside the circle, there is no buoyancy flux applied. Addition of the two equations yields

$$DD_x + hh_x = 0 \tag{3}$$

or

$$D^2 + h^2 = G(y), \tag{4}$$

where $G(y)$ is an arbitrary function of y and can be chosen freely: its derivative in y, G_y, defines the structure of the zonal current through the system. With $G_y < 0$ the zonal flow is eastward.

Recognizing that

$$D_x = -hh_x/D \tag{5}$$

and

$$D_y = G_y/2D - hh_y/D, \tag{6}$$

we can now write the equations of the lower layer entirely in terms of D as dependent variable and p as independent variable

$$[fG_y/2\beta h + (D-h)]\,D_x = f^2 w_1/\beta g', \tag{7}$$

where h is known in terms of D from eq. (4). This equation can be integrated numerically along latitude circles $y = \text{constant}$. The main complication in the calculation is the possibility that the two terms in the brackets on the left-hand side of eq. (3) may cancel somewhere in the field. We can define a non-dimensional number Z as follows:

$$Z \equiv -fG_y/2\beta h(D-h), \tag{8}$$

noting that when $Z = 1$, the local depth averaged eastward zonal current

$$(u_1 h + u_2(D - h))/D$$

just balances the local westward velocity of propagation of an internal non-dispersive Rossby wave

$$(\beta g'/f^2)\, h(D - h)/D.$$

It will be noted that even within the circle the depth averaged zonal transport is independent of longitude, but the velocity of the Rossby wave is not. We expect then that eastward zonal currents imposed on the system will favor the fluid's finding a place within the circle where the critical condition $Z = 1$ occurs. This phenomenon is analogous to that of the Bernoulli control that occurs in ordinary non-rotating hydraulic channels, but here the wave is a planetary geostrophic one, not a long gravity wave.

Fig. 1 shows five cases starting with a uniform weak westward zonal flow outside of the circle, corresponding to $Z = -0.5$, then with no imposed zonal flow outside the circle, $Z = 0$, and then with three cases of increasing eastward zonal flow: $Z = 0.31$, 1.5, 2.0. These are the values of Z imposed outside of the circle—inside the circle the value of Z must be computed, and we must watch carefully places where it approaches the critical value $Z = 1$.

The panels are arranged so that each of the five rows is a separate case. The four columns represent, in order, the depth, h, of the upper interface, the depth D of the lower interface (this is also a measure of the pressure in the lower layer), the sum $h + D$ which is a measure of pressure in the upper layer, and finally in the fourth column, the quantity Z, so that we can see if the regime is close to critical $Z = 1$ anywhere. In the two center columns, highs and lows of pressure are indicated by conventional symbols H and L, and an indication of the direction of geostrophic flow in the two layers (remembering that the third column, not the first, gives pressure in the top layer) by arrowheads. In the first column we indicate centers where the top interface is deep and shallow by D and S (this use of D in the figure cannot be confused with interfacial depth D elsewhere).

The most striking result shown is the reversal of the sense of direction of flow inside the circle between the third and fourth rows, spanning the gap in applied $Z = 0.31$ to 1.5. For $Z < 0.31$, the upper layer circulates clockwise, the under layer counterclockwise, corresponding to central highs and lows respectively. For $Z > 1.5$, the sense of circulation in the layers and central pressure extrema is reversed. The transition from one case to the other is not smooth and continuous. For example, we can easily see by inspection of the row 4 column 4 that even though the value of $Z = 1.5$ applied outside the circle is well above critical, within the circle it approaches 1.08, and that means that reduction of the applied Z below 1.5 outside the circle will involve encountering criticality inside the circle. Not only are the directions of circulation inside the circle reversed across the critical range of the number Z, but also the magnitude of the circulation (we could call it a recirculation in the language of earlier discussions of planetary flow patterns) increases in the neighborhood of criticality. The transition from one state to the other is therefore through infinity.

It will be noticed that in the upper three rows the sense of circulation in the meridonal plane in the western half of the circle, where buoyancy is being lost, is poleward in the warm layer. This we may call a direct convective cell. In the lower two rows this sense of circulation in the meridional plane is reversed, so this may be called an indirect convective cell, a somewhat bizarre result because the warm water flows equatorwards as it loses buoyancy.

Finally, we call attention to the rotation with depth of the geostrophic velocity from one layer to the other within the circle. Except for the special case with $Z = 0$, the direction of rotation is always counterclockwise in the region of buoyancy loss, and clockwise in the region of buoyancy gain: the solenoids of the pressure and density fields are not parallel inside the circle. It is only in the special case $Z = 0$ (row 2) that the solenoids are parallel, and as a result in this case the interfacial flux w_1 is strictly vertical. In the other cases the two first terms of the Jacobian on the left-hand sides of eqs. (1) and (2) do not cancel each other, and the interfacial flux is more nearly horizontal, and hence does not call up such large amplitude recirculation as might otherwise be expected.

The contours in the figure are drawn by numerical integration. The analytical solution, being transcendental in arc sin $(h/\sqrt{D_0^2 + h_0^2})$ is not convenient to work with except for determining the exact limits of the prohibited range of Z outside the circle. From the analytical form we find this range to be $0.315 < Z < 1.35$. The range $1.35 < Z < 1.5$

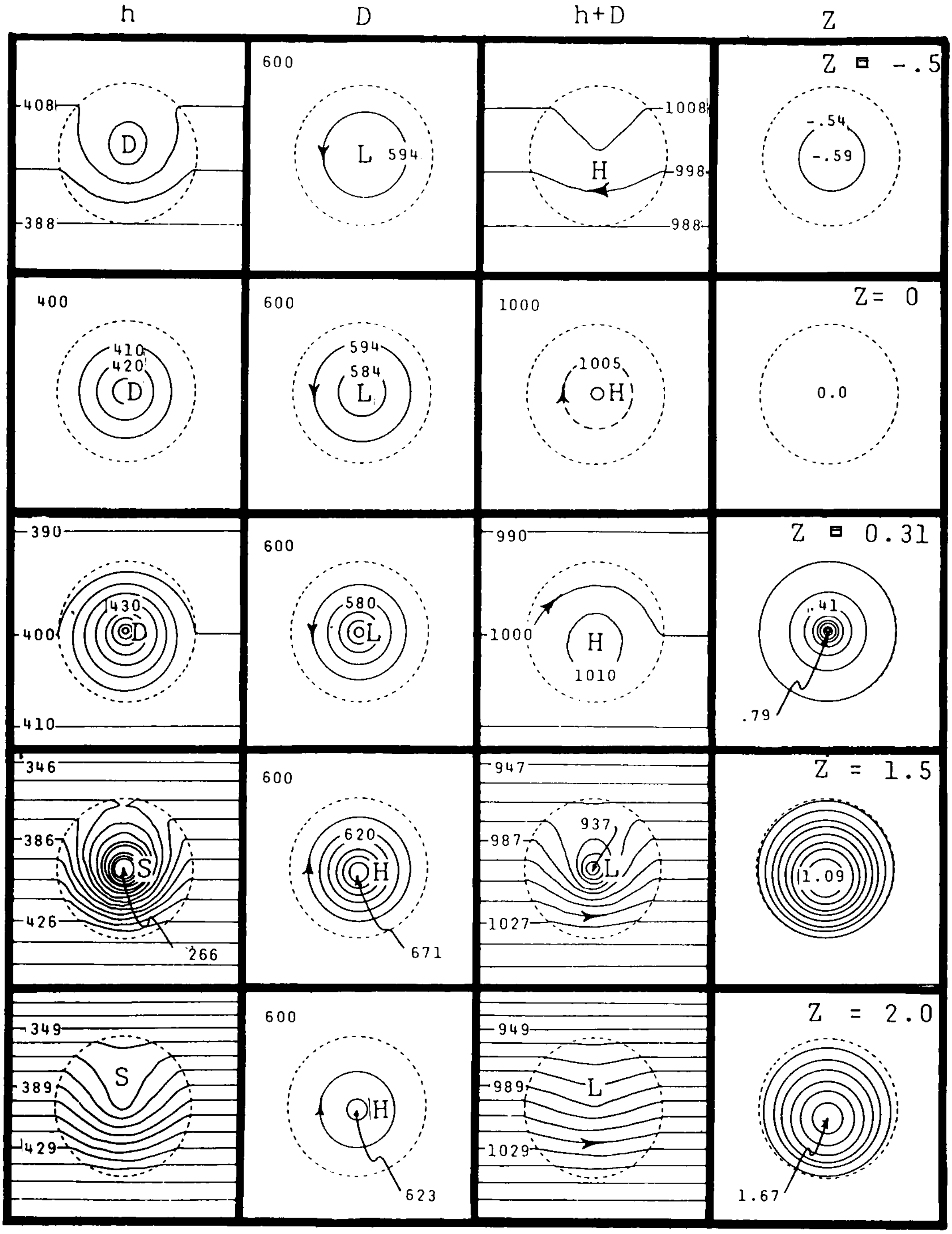

Fig. 1. Each row is a different calculation, for $Z = -0.5, 0, 0.31, 1.5$, and 2.0. The columns are for h, D, $h + D$ and Z. The values of h at mid-latitude, and of D everywhere outside the circle are 400 m and 600 m, respectively. In those cases where Z does not vanish there is a y-slope of h [but not of D] outside the circle. A few labelled contours indicate the direction of slope. In column (1) the letters D and S indicate centers where the upper layer is anomalously deep or shallow, respectively. In columns (2) and (3) the letters L and H indicate centers of low and high pressure in the middle and surface layers respectively. The arrowheads indicate direction of flow parallel to isobars. In column (4) the contoured values of Z are shown as different from the uniform value fixed outside the circle. For $Z > 0$ these central values are closer to critical than the values fixed outside the circle.

was difficult to find by our numerical method because the critical interior Z had shifted to the west of center, at which point the integration scheme became unstable.

Mapping of the critical number Z may be useful for identifying controls in more complicated numerical ocean models.

It seems possible that this beta-control plays a role in determining the position of climatologically important oceanic thermal fronts such as that of the North Atlantic Drift between 40° N and 60° N, where the eastward-directed zonal current is known to broaden and to diminish in speed toward the east.

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