Tellus (1985), *37A*, 403–406, Polar Low Special Issue

A note on CISK in polar air masses

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(Manuscript received October 8, 1984; in final form January 28, 1985)

ABSTRACT

The CISK-mechanism is studied by means of an analytical model which allows for a simple discussion of the effect of different distributions of the diabatic heating. The model indicates that CISK may be of importance in polar air if the heating takes place in a shallow layer at low levels.

1. Introduction

Condensational heating is considered to be an important energy source for meso-scale disturbances in polar air masses. In particular, conditional instability of the second kind (CISK) has been suggested as a primary growth mechanism for polar lows (Rasmussen, 1979; Økland, 1977). In this paper we shall reconsider this possibility by means of an analytical model. The main virtue of the model is its simplicity which allows for an illuminating discussion of the effect of the heating distribution on the growth rate, and on the horizontal scale. We shall also use the model to shed some light on the consequences of a serious simplification which has been used in some recent studies (Rasmussen, 1979; Sardie and Warner, 1983) in the modelling of the horizontal distribution of the heating.

2. The model

We shall start out with a Boussinesq form of the primitive equations linearized about a basic state of rest:

$$\frac{\partial}{\partial t} u = fv - \frac{\partial}{\partial x} \varphi,$$

$$\frac{\partial}{\partial t} v = -fu - \frac{\partial}{\partial y} \varphi,$$

$$0 = \frac{g}{\theta_0} \theta - \frac{\partial}{\partial z} \varphi,$$

$$\frac{\partial}{\partial t} \theta = -\frac{\theta_0}{g} N^2 w + \frac{\theta_0}{c_p T_0} Q,$$

$$\frac{\partial}{\partial x} u + \frac{\partial}{\partial y} v + \frac{\partial}{\partial z} w = 0. \tag{1}$$

Here u, v, w, θ and φ are perturbations of velocity, potential temperature and pressure. The basic state is defined by the buoyancy frequency N and the characteristic potential temperature θ_0 (or T_0). Finally f is the Coriolis parameter, g is the acceleration of gravity, c_p is the specific heat and Q is the diabatic heating.

If (1) is solved for w we get

$$\left(\frac{\partial^2}{\partial t^2} + f^2\right) \frac{\partial^2}{\partial z^2} w + N^2 \nabla^2 w = \frac{g}{c_p T_0} \nabla^2 Q, \tag{2}$$

where ∇^2 is the horizontal Laplacian. Clearly, due to ageostrophic curvature effects, a meso-scale system of large amplitude will not satisfy quasi-geostrophic theory. In the small amplitude limit, however, all curvature effects have been neglected and (2) can be reduced to its quasi-geostrophic version by simply neglecting $\partial^2 w/\partial t^2$ as compared to $f^2 w$. It will be verified later that this approximation is quite good for a wide range of cases, and we shall use it in the following. In our search for simple solutions we shall also assume that

$$Q = \bar{Q}(z, t) \cos(kx) \cos(ly).$$

$$w = \bar{w}(z, t) \cos(kx) \cos(ly),$$

so that (2) takes the form

$$\frac{\partial^2}{\partial z^2} \bar{w} - N^2 K^2/f^2 \bar{w} = -\frac{g}{c_p T_0 f^2} K^2 \bar{Q}, \qquad (3)$$

where $K^2 = k^2 + l^2$. N is assumed to be independent of z. Furthermore, we shall assume that $\bar{Q}$ is a constant Q_0 in the layer $H_1 \leqslant z \leqslant H_2$ and zero elsewhere, and that the heat released in a column of air is equal to the latent heat that enters the column through the top of the boundary layer ($z = 0$). This gives

$$\int_0^\infty Q\rho \, dz = \rho_s q_s L w_s$$

or

$$Q_0 = \rho_s q_s L w_s / [\bar{\rho}(H_2 - H_1)], \qquad (4)$$

where the subscript s refers to the top of the boundary layer. L is the latent heat; q_s and ρ_s are the specific humidity and air-density at $z = 0$, and $\bar{\rho}$ is the average air-density in the layer $H_1 \leqslant z \leqslant H_2$.

This parameterization is meant to simulate the heating due to cumulus convection in regions of large-scale rising motion. Having assumed a harmonic horizontal variation of Q, however, we get an unrealistic diabatic cooling in regions of sinking motion (dashed line in Fig. 1). Clearly, in sinking air $Q = 0$ (full line) would be more satisfactory. Some studies like Charney and Eliassen (1964) and Økland (1977) have paid attention to this problem,

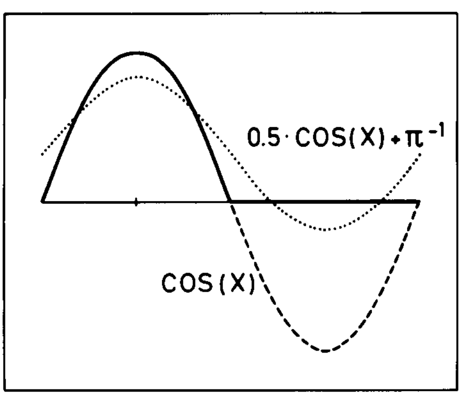

Fig. 1. Horizontal distributions of the diabatic heating.

while others like Rasmussen (1979) and Sardie and Warner (1983) seem to have ignored it. This latter choice certainly leads to a great simplification in the mathematical analysis, but it seems likely that it also implies a serious overestimation of the differential heating term on the right-hand side of (2). In the present work, we shall try to correct for this effect by multiplying Q_0 by a factor γ which will be smaller than one. The use of this device is motivated by Fig. 1 where the dotted line represents the leading terms, $1/\pi + 0.5 \cos x$, of a Fourier approximation to the full line, indicating that $\gamma = 0.5$ may be a reasonable choice.

As an upper boundary condition we shall use

$$w \to 0 \quad \text{as} \quad z \to \infty. \qquad (5)$$

At $z = 0$ (the top of the boundary layer) we shall adopt the commonly used Ekman layer parameterization of Charney and Eliassen (1949):

$$w_s = D\zeta_s,$$

where ζ is the vorticity and D is a constant. By using the vorticity equation, this boundary condition can be expressed as

$$\frac{\partial}{\partial t} w = Df \frac{\partial}{\partial z} w \quad \text{for} \quad z = 0. \qquad (6)$$

Eq. (3) is solved by means of a matching procedure where w and $\partial w/\partial z$ are required to be continuous at $z = H_1$ and $z = H_2$. A solution to the inhomogeneous equation is

$$w_I = \begin{cases} \dfrac{M^2}{2N^2} (e^{\mu H_2} - e^{\mu H_1}) e^{-\mu z} w_s, & H_2 \leqslant z, \\ \dfrac{M^2}{N^2} \{1 - \tfrac{1}{2}[e^{\mu(z-H_2)} + e^{-\mu(z-H_1)}]\} w_s, & H_1 \leqslant z \leqslant H_2, \\ \dfrac{M^2}{2N^2} (e^{-\mu H_1} - e^{-\mu H_2}) e^{\mu z} w_s, & z \leqslant H_1, \end{cases} \qquad (7)$$

where $\mu = NK/f$ and

$$M^2 = \gamma g \rho_s q_s L / [c_p T_0 \bar{\rho}(H_2 - H_1)].$$

In order to satisfy the condition $w = w_s$ at $z = 0$ together with (5), we must add the following solution of the homogeneous equation:

$$w_H = \left[1 - \frac{M^2}{2N^2} (e^{-\mu H_1} - e^{-\mu H_2})\right] e^{-\mu z} w_s. \qquad (8)$$

When the total solution $w = w_I + w_H$ is inserted in (6), we find that the solution grows exponentially in time with the growth rate:

$$\sigma = DNK\left[\frac{M^2}{N^2}(e^{-\mu H_1} - e^{-\mu H_2}) - 1\right]. \qquad (9)$$

Note that the growth rate (9) can be considered as a sum of two different effects:

(i) growth due to latent heat release, represented by the term proportional to M^2;

(ii) frictional dissipation, represented by the remaining negative term.

3. Results

Fig. 2 shows the growth rate as a function of wavelength L for the case $k = l = 2\pi/L$. Five different heating distributions are shown. The heating distributions I, II and III all have their basis at $H_1 = 500$ m, but the depths $(H_2 - H_1)$ are 1000 m, 2000 m and 4000 m, respectively. The distributions IIa and IIb are similar to II but are translated 500 m downwards and upwards respectively. For the rest of the parameters we have used: $N = 0.7 \times 10^{-2}$ s^{-1}, $f = 10^{-4}$ s^{-1}, $D = 200$ m, $q_s = 5 \times 10^{-3}$, $L = 2.8 \times 10^6$ m^2 s^{-2}, $c_p T_0 \bar{\rho}/\rho_s = 2 \times 10^5$ m^2 s^{-2} and $\gamma = 0.5$. These numbers are assumed to represent a very cold air mass of low stability over an ice-free ocean. In good agreement with earlier CISK-studies Fig. 2 shows that the growth rate depends heavily on the vertical distribution of the heating. Significant growth rates are possible, but only if the heating is concentrated in a shallow layer close to the boundary layer. We also note that the maximum growth rate is found for a finite wavelength in most cases (IIa is an exception). Finally we note that $f = 8.64$ day^{-1} and Fig. 2 therefore confirms that the term $\partial^2 w/\partial t^2 = \sigma^2 w$ is much smaller than $f^2 w$ for a wide range of interesting cases.

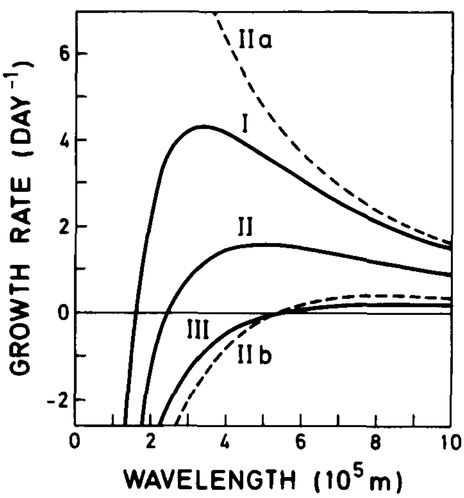

Fig. 2. Growth rates as a function of wavelength for five different vertical distributions of heating. (I) $H_1 = 500$ m, $H_2 = 1500$ m. (II) $H_1 = 500$ m, $H_2 = 2500$ m. (III) $H_1 = 500$ m, $H_2 = 4500$ m. (IIa) $H_1 = 0$ m, $H_2 = 2000$ m. (IIb) $H_1 = 1000$ m, $H_2 = 3000$ m.

Several of the qualitative aspects of Fig. 2 are easily explained. Fast growth of the latent heat release is connected with fast growth of ζ at the top of the boundary layer and this requires strong convergence or vortex stretching at low levels. Strong positive feedback between the heating region and the boundary layer will therefore take place if a certain amount of heat leads to relatively large values of $\partial w/\partial z$ at $z = 0$. This relationship is particularly transparent in (6). An isolated box-shaped heat source will produce a vertical velocity response, w_I, which has a maximum inside the heating region and decays exponentially on both sides. This is illustrated in Fig. 3 which also shows how this response is modified by boundary effects to give the total solution $w = w_I + w_H$ for a case with $\sigma = 0$ ($\partial w/\partial z = 0$ for $z = 0$). Clearly, strong stretching at $z = 0$ is favoured if a large part of the heating takes place at a low level. This fits well with

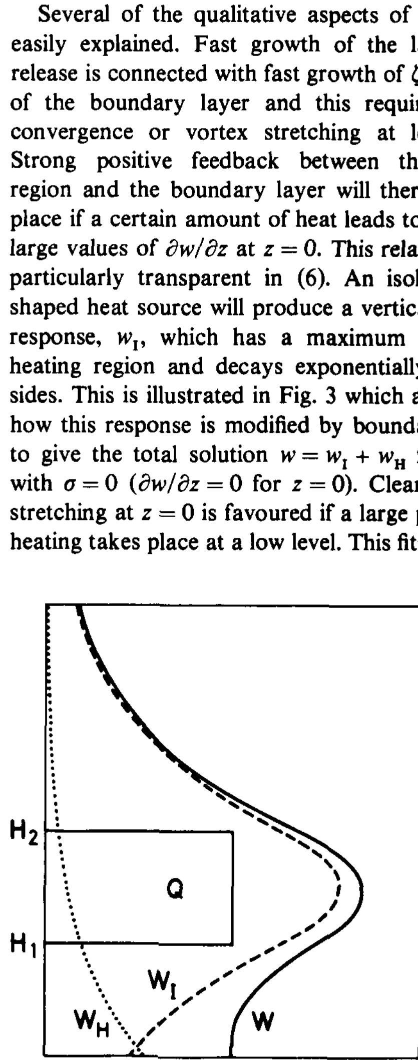

Fig. 3. Example of vertical velocity amplitude as a function of height.

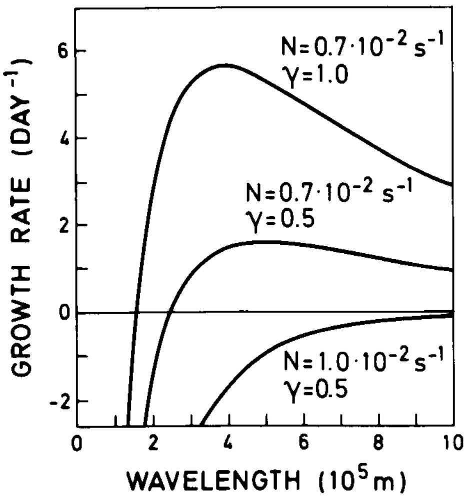

Fig. 4. Growth rate as a function of wavelength in case II (Fig. 2) for different values of N and γ.

the growth rates in Fig. 2. Furthermore the vertical distance needed for the response to decay is proportional to the horizontal wavelength (the ratio of vertical to horizontal scale is f/N). Therefore if the wavelength is sufficiently short, the influence of the heat source will not reach the boundary layer and the positive feedback will not take place. On the other hand it is also clear that very long wavelengths will produce relatively small values of $\partial w/\partial z$. This explains why the fastest growth takes place at intermediate wavelengths in all cases except in case IIa where the "short wave cut-off" fails because the gap between the heating region and the boundary layer is missing. The occurrence of a preferred wavelength is a very satisfactory property of the model because it gives us reason to expect the formation of organized meso-scale systems rather than a mixture of disorganized small scale convection.

Fig. 4 shows the sensitivity with respect to the parameters N and γ for the distribution II. As we would expect the change from the low stability $N = 0.7 \times 10^{-2}$ s^{-1} to the more normal value $N = 1.0 \times 10^{-2}$ s^{-1} has a very strong negative effect on the growth rate. Furthermore an increase of γ from 0.5 to 1.0 gives a substantial increase of the growth rate. This shows that the growth rate is very sensitive to this somewhat dubious parameter, and that the practice of neglecting the adiabatic nature of the sinking air may lead to serious overestimates. It should be noted that the specific humidity in the boundary layer, q_s, occurs in a product with γ in (9). The effect of changing γ is therefore equivalent to a similar change in q_s.

4. Concluding remarks

The main motivation behind the theory outlined above was to make a model of the positive feedback between the boundary layer and the condensational heating which was as simple and transparent as possible. The resulting model is of course quite naïve, and is not likely to be very useful for quantitative predictions. It may, however, prove to be a useful tool in the interpretation of more complicated models based on more complete equations, and it may also be of some value in the teaching of the CISK-concept. In particular the model provides a simple explanation of the response with respect to the vertical distribution of the heating, and of the phenomenon of "short wave cut-off". It is also shown that the estimates made by Rasmussen (1979) and Sardie and Warner (1983) of CISK growth rates are likely to be too high due to their neglection of the adiabatic nature of the sinking air. Nevertheless, as far as its validity goes, the model seems to indicate that CISK in polar air masses is possible if the heating takes place in a shallow layer at low levels.

REFERENCES

Charney, J. and Eliassen, A. 1949. A numerical method for predicting the perturbations of the middle latitude westerlies. *Tellus 1*, 38–54.

Charney, J. and Eliassen, A. 1964. On the growth of the hurricane depression. *J. Atmos. Sci. 21*, 68–75.

Økland, H. 1977. On the intensification of small-scale cyclones formed in very cold air masses heated by the ocean. Institute Report Series no. 26, University of Oslo, Institute of Geophysics.

Rasmussen, E. 1979. The polar low as an extratropical CISK-disturbance. *Q. J. R. Meteorol. Soc. 105*, 531–549.

Sardie, J. M. and Warner, T. T. 1983. On the mechanism for the development of polar lows. *J. Atmos. Sci. 40*, 869–881.