

Introduction to translation of V. W. Ekman's “Outline of a Unified Ocean Current Theory”

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Vagn Walfrid Ekman's classical paper on the “spiral” (Ekman, 1905) is well-known to the oceanographers, but his other contributions to the theory of the general oceanic circulation, several of which were written in German and Swedish, have escaped notice even of some experts in this field. The paper “Über Horizontalzirkulation bei winderzeugten Meeresströmungen” (Ekman, 1923) gives, for the first time, a complete formulation of the problem of the steady wind-driven circulation in a homogeneous sea, when variable topography, bottom friction, and the β -effect are included. The final equation, when written in terms of a transport stream function, takes essentially the same form as the model equations of the oceanic circulation given later by Stommel¹ (1948), which lead to the explanation of the westward intensification of ocean currents. Ekman's equation is somewhat more complex, because of the use of his vertical laminar-type friction, and because topography is included.

Why did Ekman, who generally worked out simple examples to demonstrate his theories, not try to give some solutions for a bounded sea using his basic equation, a task in which he certainly would have succeeded, and which in all likelihood would have led him to the discovery of the nature of the Gulf Stream phenomenon a quarter of a century before Stommel? One reason may be Ekman's reluctance to make approximations, other than such which resulted from properly taken limiting cases of the set of Navier-Stokes equations on a rotating earth, from which he started. He did not use the β -plane approximation, and he included terms which, although of formal importance, would be considered practically negligible in modern theories (e.g., the extra terms arising from the change of the Ekman bottom layer depth with latitude). Another possible explanation

is that Ekman started to lose interest in this theory after 1923. He may have been affected by the tremendous surge in new, fundamental physics, which was seen at that time, and felt that his theoretical attempts belonged to a bygone era in physics (he was actually professor of physics in Lund, Sweden, not professor of oceanography). Anyway, Ekman turned his sights to other, and mainly experimental, problems after his 1923 work.

However, one paper which carried the 1923 theory further was actually produced by Ekman. It was published in 1924 in Swedish, in a separate printing. It was actually an invited paper which he was asked to deliver, together with his commencement address, when, as “promoter” at the University of Lund, Sweden in 1924, he was giving the insignia to the new Doctors of Philosophy. This paper is listed in the bibliography by Kullenberg², but I had not been able to find copies of it except in Ekman's own reprint collection in the Oceanographic Institute at Göteborg.

The main aim of the 1924 paper is to present a generalized form of the 1923 theory, by adding stratification, in the form of a layer model. The paper begins with a discussion of the underlying, general assumptions, and Ekman then proceeds to introduce the “solenoidal field”, and write down the formal equations and some particular solutions for the “Ekman spiral”, when stratification is included. His main model includes a top frictional layer, an upper stratified layer, and a deep, homogeneous bottom layer, in which the current generally is assumed to be small. As usual, the analysis is carried out meticulously, including all terms which cannot be rigorously eliminated. In Section III, Ekman introduces the formal components of the

¹ Stommel, H. 1948. The westward intensification of wind driven ocean currents. *Trans. Amer. Geophys. Un.*, 29, no. 2

² Bibliography, in reprint of the paper “On the influence of the earth's rotation on ocean currents”, published by R. Swedish Academy of Sciences, Almqvist and Wiksells Printers, Uppsala 1963.

Elementary Current, and defines the transport when the stratification is included (identical with the modern "mass transport"). He proceeds to consider the effects of precipitation and evaporation, and estimates the magnitude of the currents which these can provide, which he correctly finds to be negligibly small. In Section IV, he gives the complete set of field equations for the problem, in a form analogous to the ones found in his 1923 paper, but now with stratification included. Firstly, he deals with the problem in an f -plane, but later the β -effect is introduced. One can track down expressions that essentially reproduce the Sverdrup relation (see, for example, the formula just before eq. (33) in the following translation) for a case with no net divergence, stating that the meridional transport is proportional to curl T .

Ekman finishes with a comment on the path of the Gulf Stream, noting that it ought to turn into deep water as it pushes north. He states that he intends to return to this problem to complete his theory, but, as we know, this was not to happen.

The paper seems to me to be of enough interest historically as well as scientifically to warrant publication, in a translated English version. The translation, which follows, is intentionally made with preservation of the somewhat rigid, Swedish style which Ekman uses. His notations are kept unchanged: the reader should note that some variables (V , γ , S , G , T , etc.) are vectors, their components are written γ_x , γ_y , etc. Derivatives are, on the other hand, written out in full.

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Outline of a Unified Ocean Current Theory

By V. WALFRID EKMAN,

(From an invited paper for the doctoral promotion in Lund, Sweden 31 May 1924)

I

The theoretical studies of the dynamics of ocean currents, to date, have mainly been confined to one or the other of two main types of currents: wind currents and convection currents¹. The former are currents which are generated by the wind in an ocean basin with homogeneous water, while the latter run without direct influence of the wind, through the disturbance in hydrostatic balance between water masses having different densities.

As the theories for these two types of currents have followed widely different lines, these furthermore appear isolated, and attempts to unify them to a general theory for ocean currents have un-

fortunately been unsuccessful. This is regrettable since some of the most spectacular and important motions manifested in the sea obviously represent effects of both the wind and the disturbed hydrostatic balance, which accordingly have not been subjected to a logical theoretical study. For the same reason, one has had little success in the discussion of the question whether the wind or the internal equilibrium changes represent the most important cause of the ocean currents, and this old controversy has not yet been removed from the agenda.

The attempt which will be made here to build a foundation on which a unified theory for ocean currents of different origins can be built, is most closely connected to the theory for wind currents, which I have developed in some earlier publications. Among these publications, one finds in particular

¹ One exception is the peculiar theory which was forwarded by Mohn in 1887, but this exhibits such fundamental shortcomings that it nowadays is only of historic interest.