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Analysis of an 89-year-long sea level record from the Kattegat with special reference to the barotropically driven water exchange between the Baltic and the sea

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ABSTRACT

A long record (1887–1975) of daily mean values of the sea level from the Kattegat is analysed. The variance of the sea level is 340 cm^2. 60% of this is caused by fluctuations with periods of 2 months and shorter. This high frequency variance changes throughout the year by a factor of 4, with a maximum during early winter.

From the dynamics of the water exchange through the Belts and the Sound a non-linear response function for the Baltic is constructed. Applying this, it is found that the effective barotropically driven water exchange between the Baltic and the Sea is, to about 42%, executed by high-frequency fluctuations (2 months and shorter). It is found that no single 4-year mean of the effective barotropically driven water exchange has deviated by more than about 10% from the mean of the whole period under consideration and that the variations seem to be random.

The amplitudes of some long-period, phase-locked components are determined. Among other things it is found that the 1-year period has the largest amplitude, about 10 cm.

1. Introduction

The data set from Varberg, Sweden (57° 10′ N, 12° 15′ E) 70 km south of Gothenburg, which is analysed here, consists of daily mean values of the sea level. Most of the influence of shorter-period oscillations upon the resulting data set is in this way eliminated. However, the influence of monochromatic, phase-locked disturbances, like the tidal components whose frequencies are not multiples of (1 day)$^{-1}$, is not completely removed by the averaging process. A fraction of the original variability caused by these components will be spread to lower frequencies. However, the tides in the Kattegat are extremely weak. For instance, the largest tidal component, the M_2, has an amplitude of only about 4 cm at Varberg. Therefore the spreading of the sea level variability in the frequency domain caused by the chosen averaging process is particularly small in this area.

The present record permits the study of sea level fluctuations on all scales ranging from the Nyquist period (2 days) to the length of the record (89 years). Also influences from fluctuations with still larger periods may be seen as trends in the series. The uplift of land at Varberg is fairly small, about 0.05 cm yr^{-1} (Svansson, 1972).

The sea level fluctuations in the range 2 days–100 years are mainly caused by, directly and/or indirectly, the interaction of the sea with the atmosphere together with the radiation exchanged with space. In addition there are some long-period tidal components. There is one very pronounced and phase-locked period for the atmospheric influence upon the sea, namely the 1-year period. The weather varies with the 1-year period (exactly). This periodicity in the weather has a very firm and well-known mechanistic explanation. Superposed upon this yearly cycle there are more-or-less random (not phase-locked) fluctuations.

In this paper, we chiefly concentrate upon the sea level variations in the Kattegat as the main cause of the water exchange between the Baltic and the Sea. One of the questions we will try to answer is whether or not there has been a change of this forcing during the period which our sea level record covers. During this period the Baltic has become saltier, see e.g., Kullenberg (1981). One obvious explanation for this is the observed decrease of the river runoff to the Baltic. The prediction of the effect of a decreasing freshwater supply upon the salinity of the Baltic surface layer put forward by the present author (Stigebrandt, 1983) quantitatively explains the observed amplitude of the salinity change. However, the observed change of state of the Baltic could also, at least partly, be an effect of a changed barotropic water exchange between the Baltic and the Sea (Welander, 1974).

Before we start to study the barotropic exchange of water between the Baltic and the Sea, we will present some analyses of the sea level record in order to give a general overview of the fluctuations.

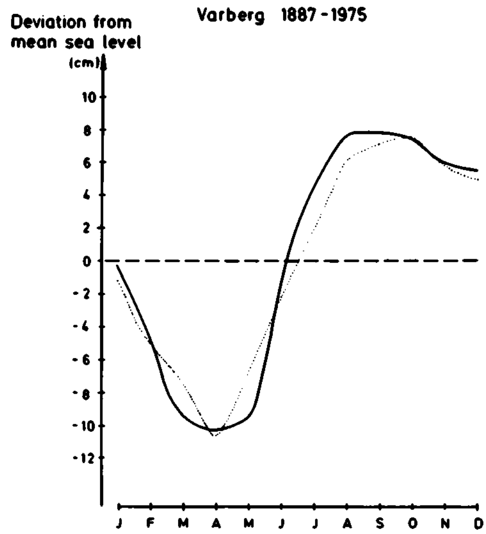

Fig. 1. The yearly cycle of the monthly mean sea level before (——) and after (······) correction for the static effect of atmospheric pressure.

2. Analysis of the time series

A time series made up of daily mean values of the sea level at Varberg for the period 1 January 1887–31 December 1975 was transferred to a magnetic disk. The notation used in the following is such that h_{nij} means the mean sea level during the nth day of the ith month of the jth year. A few short periods were missing (in total a few % of the data points). Analyses giving an overview of some of the properties of the record are described below.

2.1. *The yearly cycle of the sea level*

The mean sea level, h_{ij}, during the ith month of the jth year is calculated from

$$h_{ij} = (1/N_{ij}) \sum_n h_{nij}, \quad (1)$$

where N_{ij} is the number of days of the ith month of the jth year. The mean monthly mean sea level, h_i, is calculated from

$$h_i = (12/M) \sum_j h_{ij}, \quad (2)$$

where M is the number of months in the record (M is a multiple of 12). The average yearly cycle that appears from the calculation is shown by the solid line in Fig. 1. The yearly cycle is certainly caused by a combination of several effects. One contribution comes from the varying air pressure. This effect can be removed following the method described by Lisitzin (1974). If this is done, and using the monthly means of air pressure in Gothenburg (1931–1960), the dotted line in Fig. 1 is obtained. The remainder of the yearly cycle is caused by local and/or regional variations of the sea water density (the freshwater content in the Kattegat surface layer varies during the year), systematic variations of the regional and local wind patterns and other effects such as the annual and semiannual tides.

The effect of a varying freshwater content in the Kattegat surface layer upon the sea level can be roughly estimated. The vertical stratification in the Kattegat can be described as approximately two-layered (see Stigebrandt, 1983). The lower layer is composed of Skagerrak water of salinity S_{2K}. The upper layer, of depth H_{1K}, is less salty because of the outflow from the Baltic, and the salinity here is S_{1K}. The sea level difference between the open Skagerrak (where the salinity is S_{2K} from the surface and downwards) and the Kattegat is Δh. Assuming no horizontal pressure gradient between the lower layer of the Kattegat and the Skagerrak, one obtains

$$\Delta h = H_{1K}\, \beta \cdot (S_{2K} - S_{1K}), \quad (3)$$

where the linear equation of state for brackish water

$$\rho = \rho_f \cdot (1 + \beta \cdot S) \quad (4)$$

has been used. In eq. (4) ρ_f is the density of freshwater and $\beta = 8 \cdot 10^{-4}$ ‰$^{-1}$.

The present author (Stigebrandt, 1983) gives the time mean values for H_{1K} and S_{1K} to 15 m and 23‰, respectively. Using these values and S_{2K} = 33‰, one obtains $\Delta h \approx 12$ cm as a mean value for the Kattegat. Experience from the Kattegat indicates that instantaneous values of Δh should be expected in the range 0–25 cm. An amplitude of about 4 cm for the yearly cycle caused by this effect seems quite reasonable. However, there is as yet no investigation undertaken of the mean yearly cycle of the freshwater height in the Kattegat (based on long time series of salinity from several depths). The investigation of a large number of sea level records by Pattullo et al. (1955) shows a large annual cycle in all records from the northeastern Atlantic Ocean. Their investigation also shows regional modulations of the annual cycle. The varying freshwater content in the surface layer is probably the largest single modulator in the Kattegat.

2.2. *The yearly cycle of high frequency sea level variations*

The variance of the sea level during the ith month of the jth year is defined by

$$\text{Var}_{ij} = (1/N_{ij}) \sum_n (h_{nij} - h_{ij})^2. \quad (5)$$

Var_{ij} contains the variance of the sea level fluctuations with periods from 2 days to 2 months. The monthly mean of the variance is obtained from

$$\text{Var}_i = (12/M) \sum_j \text{Var}_{ij} \quad (6)$$

The mean yearly cycle of the high frequency variance (2 days–2 months) is shown in Fig. 2. As can be seen, the yearly cycle is very pronounced and the variance is largest in January and smallest in July. This yearly cycle is caused by the yearly weather cycle. The winds are strongest and the air pressure fluctuates the most during late autumn and early winter. In connection with the barotropic forcing of the water exchange between the Baltic and the Skagerrak, it is of great importance to know how this high frequency variance is distributed among the different period bands. Therefore the Fourier components were determined for 32 days long (and slightly overlapping) periods starting days 1, 31, ..., 331 for each of the 89 years. These periods approximately coincide with the months of the year. The variance was grouped into 5 spectral bands (2 months–3.5 weeks, 3.5–2 weeks, 2–1 week, 7–4 days and 4–2 days). The result is shown in Fig. 3 where it can be seen that the variance contribution from fluctuations with periods 2 months–3.5 weeks is the largest one.

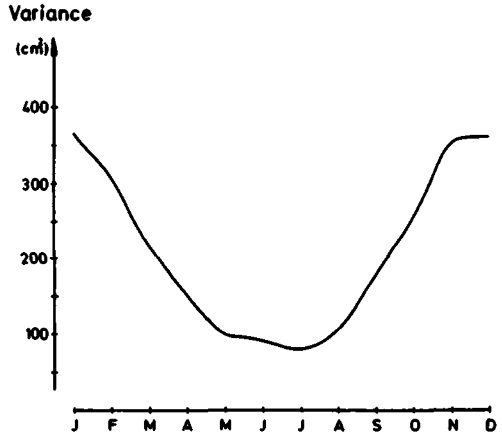

Fig. 2. Monthly means of the variance for periods of 2 days–2 months.

2.3. *The distribution of the total variance among different period bands*

We studied, above, the variance in the record contributed by fluctuations in the range 2 days–2 months. This variance is calculated relative to the "local" mean sea level, h_{ij}. The total variance, Var, in the series also contains the contributions from periods longer than 2 months. It should be calculated relative to the mean sea level for the whole period, z_0. Thus

$$\text{Var} = (1/M) \sum_{ij} (1/N_{ij}) \sum_n (h_{nij} - z_0)^2. \quad (7)$$

We manipulate eq. (7) in the following way

$$\text{Var} = (1/M) \sum_{ij} (1/N_{ij}) \sum_n ((h_{nij} - h_{ij})^2 + 2h_{nij}h_{ij} + z_0^2 - 2h_{nij}z_0)$$

$$= (1/M) \sum_{ij} (1/N_{ij}) \sum_n ((h_{nij} - h_{ij})^2 + (z_0 - h_{ij})^2 + 2(h_{ij} - h_{nij})(z_0 - h_{ij}))$$

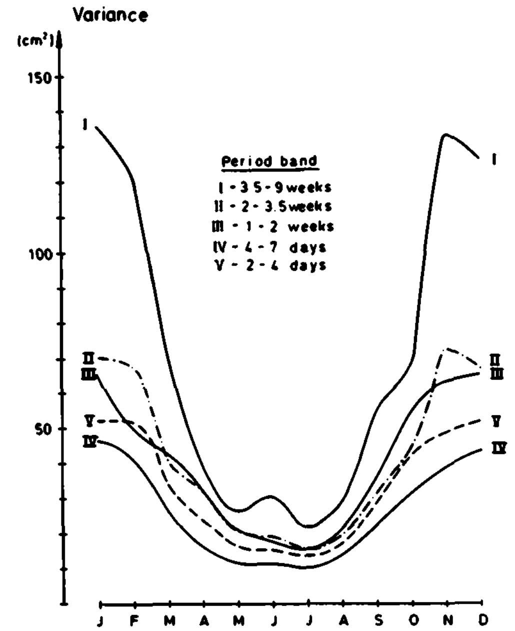

Fig. 3. The distribution of variance shown in Fig. 2 among different period bands.

or, using eq. (5),

$$\mathrm{Var} = 1(M) \sum_{ij} (\mathrm{Var}_{ij} + (z_0 - h_{ij})^2 + \mathrm{Rest}_{ij}), \quad (8)$$

where

$$\mathrm{Rest}_{ij} = (2/N_{ij})(z_0 - h_{ij}) \sum_n (h_{ij} - h_{nij}). \quad (9)$$

The three components of the variance in eq. (8) were determined and the following results were obtained:

$$(1/M) \sum_{ij} \mathrm{Var}_{ij} = 205.30\ \mathrm{cm}^2, \quad (60\%)$$

$$(1/M) \sum_{ij} (z_0 - h_{ij})^2 = 135.24\ \mathrm{cm}^2, \quad (40\%)$$

$$(1/M) \sum_{ij} \mathrm{Rest}_{ij} = O(10^{-3})\ \mathrm{cm}^2.$$

From this it is evident that the approximation

$$\mathrm{Var} = (1/M) \sum_{ij} (\mathrm{Var}_{ij} + (z_0 - h_{ij})^2) \quad (10)$$

is a very good one for the present time series.

The figures given above show that, with this partition of the time series, 60% of the variance is due to fluctuations with periods equal to or shorter than 2 months.

In order to obtain a better resolution with regard to the distribution of the variance among different period bands, the Fourier components of the series h_{nij} (32,500 values—the whole series minus 6 values at the end) were determined. This Fourier line spectrum (the amplitude of the components squared and divided by 2) is shown in Fig. 4. Note the large peaks for the periods 1 and ½ years, respectively. The distribution of the variance among different period bands is shown in Table 1. For comparison we also determined the Fourier components of the series h_{ij} (1068 values). This spectrum and that shown in Fig. 4 are nearly identical for periods of about 1 year and longer.

The yearly cycle of the monthly mean sea level in Fig. 1 is asymmetrical. Therefore the higher harmonics of the 1-year period appear in Fig. 4. The weather and the processes in the sea induced by the weather are not symmetrical. Note that the solar semi-annual tidal component, Ssa, also has the ½-year period, and from the record we cannot determine the partition of the amplitude between this component and the weather-induced component. From the spectral analysis based upon the h_{ij} series, we find that the oscillation with the 1-year period has the amplitude 9.2 cm and the maximum occurs on the 4th of October.

All phase-locked, continuous processes of significant amplitude should show up in the spectrum in Fig. 4. From this spectrum we can gain some information regarding some tidal constituents, see Table 2. Here also the amplitude of the tidal constituent with the 18.61-year period is given.

Table 1. *The distribution of the variance among different period bands*

Period band	Variance (h_{ij}) (cm²)	Variance (h_{nij}) (cm²)
1.03–89 years	36.0	36.1
0.98–1.03 years	43.8	45.2
0.166–0.98 years	55.5	60.1
1–2 months		42.6
0.5–1 year		53.4
8–15 days		47.8
4–8 days		31.4
2–4 days		24.2

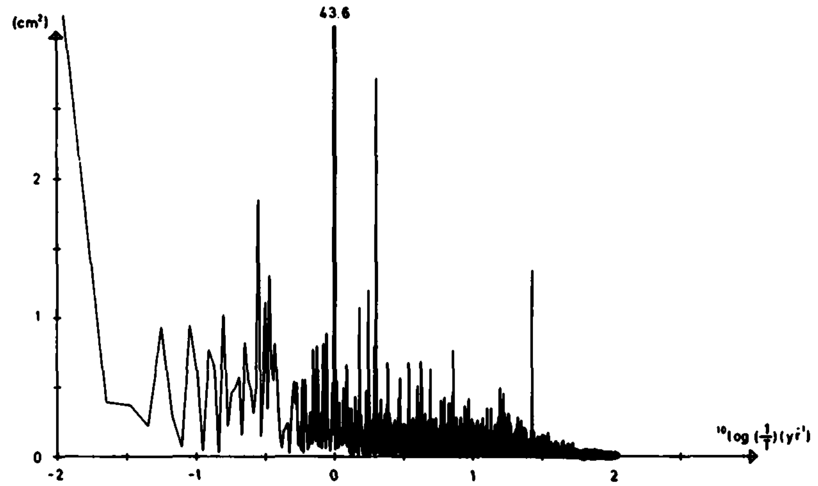

Fig. 4. Fourier spectrum for the 89 year long series (32,500 days).

Table 2. *The amplitude of some long-period tidal constituents*

Constituent	Period	Amplitude (cm)
Mf_N	13.63 days	0.45
Mf	13.66 days	1.89
M_M	27.55 days	0.18
M_N	18.61 years	1.78

This latter was obtained from a reduced record with 27,170 data points (the record length equal to 4 × 18.61 years). We also looked for the so-called Chandler period (see Lisitzin, 1974). A few spectral analyses were performed with different lengths of the series (32,506, 32,500 and 27,170 data points). They all show a significant concentration of variance in an interval around the 440-day period. The amplitude of this component is about 1.7 cm. With our high resolution in period space, we can state that the Chandler period is longer than the about 434 days earlier suggested (cf. Lisitzin, 1974).

3. The barotropically-driven water exchange between the Baltic and the sea

The exchange of water and salt between the Baltic and the Skagerrak has recently been modelled by the present author (Stigebrandt, 1983). One of the important forcing functions in that model, and in the prototype as well, is the barotropic flow through the Kattegat–Belt Sea area. This flow is mainly driven by the sea level difference between the Kattegat and the southwestern Baltic (just inside the Belt Sea–Öresund). This was shown in Stigebrandt (1980b) where a quasi-steady model for the barotropic exchange was developed and tested. In that non-linear model, the longitudinal barotropic pressure gradient (the slope of the sea surface) is balanced by bottom friction (which is proportional to the velocity squared).

With the help of Table 1 in Stigebrandt (1980a) and the information given in Stigebrandt (1980b), the amplitude response (the gain) of the Baltic to external (Kattegat) sea level fluctuations can be constructed. Because of non-linearities in the system, the gain varies with the amplitude of the forcing fluctuation. The calculated gain functions for two different forcing amplitudes are shown in Fig. 5. As can be seen, full response is achieved for forcing time scales of the order of 1 year. 50% response occurs for forcing periods in the range 1–2 months, depending on the forcing amplitude. For forcing periods of the order of 1 week, the response is only about 10%.

In order to investigate how the barotropic exchange between the Baltic and the Sea has varied during the period 1887–1975, we may use the barotropic model described in Stigebrandt (1980b) or the condensed information from that model

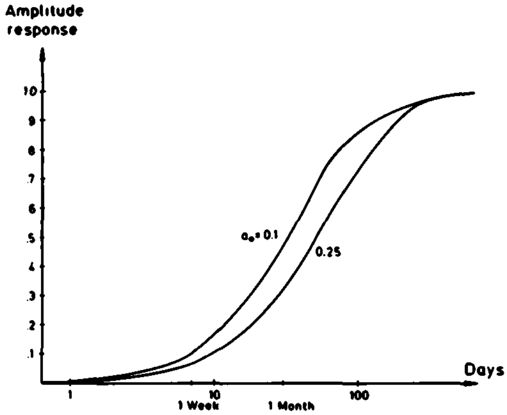

Fig. 5. The amplitude response of the Baltic to sea level fluctuations of the amplitude a_0 in the Kattegat. The freshwater supply is here taken to be 10^4 m^3 s^{-1}.

Table 3. *The means of relative contributions from each of the period bands to the barotropic water exchange*

	Contribution to the transport	
Period band	9 components (%)	6 components (%)
1.1–4 years	1.9	3.3
0.9–1.1 years	5.3	9.4
0.5–0.9 years	4.1	7.2
0.25–0.5 years	7.9	13.9
1.5–3 months	14.0	24.7
0.5–1.5 months	23.5	41.5
7–14 days	20.7	—
3.5–7 days	12.8	—
2–3.5 days	9.8	—

represented by gain functions of the type shown in Fig. 5. These functions take care of the dynamics of the barotropic exchange and give the response for single harmonic forcing components. There are many forcing components of small amplitude operating simultaneously, and non-linear effects will decrease the total exchange caused by these. This is accounted for in the following way: the variance of the whole time series is about 340 cm^2. This is nearly equivalent to a single harmonic variation of amplitude $a_0 = 0.25$ m. Therefore, it seems reasonable to use the gain function valid for $a_0 = 0.25$ m for the computations that follow below.

In order to calculate the distribution of the variance of the Kattegat sea level among different periods (the spectrum) for 4-year long records (1461 data points), we invert the time series to the frequency space by means of Fourier analysis. The overlap of neighbouring series was chosen to be 2 years. In this way, we obtain 43 sea level spectra (covering the periods 1887–1891, 1889–1893, ...). For each of the spectra, we gather the variance contributions into 9 different period bands, see Table 3. We substitute the fluctuations in each period band with one single fictive fluctuation whose period is equal to the central period of the band and with such an amplitude, a_{mn} ($m = 1, \ldots, 43$, $n = 1, \ldots, 9$), that the variance contribution from the band is conserved. Thus the variance of the time series number m is

$$\mathrm{Var}_m = \tfrac{1}{2} \sum_{n=1}^{9} a_{mn}^2.$$

The contribution to the barotropic exchange for each 4-year period, m, from each of the 9 fictive components a_{mn} is

$$T_{mn} = 2 \cdot a_{mn} \cdot F_n \cdot \omega_n \cdot Y \quad \begin{matrix} m = 1, \ldots, 43 \\ n = 1, \ldots, 9 \end{matrix}, \tag{11}$$

where F_n is the gain factor and ω_n is the frequency of the fictive component number n and Y is the area of the Baltic Sea ($Y = 0.37 \cdot 10^{12}$ m^2). The barotropic exchange, T'_m, for the mth 4-year period then is

$$T'_m = \sum_{n=1}^{9} T_{mn}. \tag{12}$$

The mean value of the barotropic exchange for the whole period,

$$T = \tfrac{1}{43} \sum_{m=1}^{43} T'_m$$

is 1311 km^3 yr^{-1}. The mean values of the contributions from the 9 different period bands (T_n) are given in Table 3, the second column. However, the barotropically-driven exchange of water must not necessarily lead to an *effective exchange* of water. The same water may possibly just oscillate back and forth in the mouth of the Baltic. Other processes, causing mixing and/or withdrawal from the transition area, must act together with the barotropic pumping in order to give an efficient water exchange. Such processes, especially efficient for longer-period oscillations, operate in this area, see Stigebrandt (1983). The highest frequencies, however, are not believed to execute any significant water exchange. Therefore we calculate

$$T_m = \sum_{n=1}^{6} T_{mn} \tag{13}$$

which is assumed to approximately represent the effective barotropic exchange. The mean value of T_m is 743 km^3/year. The mean values of the contributions from the 6 different period bands are given in Table 3, the third column. In Fig. 6 we have plotted T_m for all periods. It can be seen that the effective barotropic exchange has apparently been fairly steady during the period investigated. The maximum deviation from the mean is about 10%. Thus no systematic change of the statistical properties of the sea level variation in the Kattegat and the barotropically driven water exchange between the Baltic and the Sea seems to have occurred, although a 15–20-year-long cycle can be imagined from Fig. 6.

Because of the net freshwater supply, Q_f (about 450 $km^3\ yr^{-1}$) to the Baltic, the difference between the long-term means of the outflow and the inflow is Q_f. Very roughly, we may estimate the effective outflow to be equal to $T_m + Q_f/2$ (about 970 $km^3\ yr^{-1}$) and the effective inflow to be equal to $T_m - Q_f/2$ (about 520 $km^3\ yr^{-1}$). These figures are not far from the classical figures for the Baltic water exchange (see e.g., Kullenberg (1981)). In fact this figure further justifies the omission of components with periods shorter than 2 weeks in the expression for the effective exchange (eq. (13)). The barotropically-driven effective volume exchange, T_m, was found earlier to have been rather stable during the period investigated (less than 10% deviation from the long-term mean for the different 4-year means). However, the effective inflow, $T_m - Q_f/2$, may also change because of variations of Q_f. During the years 1975–76, for instance, the freshwater supply was only about 300 $km^3\ yr^{-1}$, see e.g., Stigebrandt (1983). Such a low freshwater supply would give an increase to the effective volume inflow amounting to about 15%. The effect of a changing inflow upon the salinity of the Baltic also depends on the salinity of the exchanged water. However, this topic is beyond the scope of the present paper. The reader is referred to Stigebrandt (1983) for a discussion of this subject.

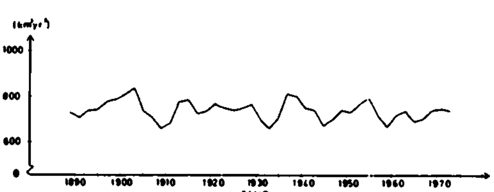

Fig. 6. The barotropically-driven mean water exchange between the Baltic and the Sea for 4-year periods (T_m).

From Table 3, it can be seen that about 42% of the barotropically driven exchange is executed by oscillations with periods of 1.5 months and shorter. These high-frequency fluctuations have been shown, in Fig. 3, to vary strongly throughout the year with a maximum during late autumn and early winter. From this it follows that the outflow from the Baltic should be maximal during this period.

Svansson (1975) shows a graph suggesting that the yearly cycle of surface salinity in the Kattegat, with the lowest salinity during summer, is caused by the yearly variation of the sea level in the Baltic. However, the suggested correlation may be fortuitous. In Table 3 we show that the variation of the sea level with the 1-year period contributes only about 9% of the total effective volume exchange. Such a small component cannot possibly play a dominant rôle for the surface salinity in the Kattegat. The low surface salinity in the Kattegat in summer is most probably an effect of the weak winds during this season which make the vertical entrainment of saltier water from below particularly small.

4. Summary

The effective barotropically-driven volume exchange between the Baltic and the Sea seems to have been fairly stable during the period 1887–1975. The maximum deviation from the long-term mean is, for 4-year periods, about 10%. The variations seem to be random. The changes of the hydrographic state of the Baltic, observed to occur during the period of the record, seem to be caused by factors other than changes of the barotropic forcing.

5. Acknowledgements

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