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LETTERS TO THE EDITOR

Comments on simple climate models with periodic and stochastic forcing

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Two interesting papers on stochastic aspects of climate transitions, in particular the combined response to a periodic and a stochastic forcing, have recently been published in *Tellus* (Nicolis, 1982; Benzi et. al., 1982). The main conclusions from these investigations are that neither a periodic forcing nor a stochastic forcing alone can describe the temperature difference between an ice age and an interglacial period. It is, however, concluded that with both types of forcing acting at the same time, it is possible to get a much larger response than would be the case if one considers only the deterministic response to a suitable periodic forcing.

The models used in these investigations are of the Budyko–Sellers type. The single dependent variable is therefore the temperature averaged over the climatic system. The governing equation expresses the rate of change of this temperature as influenced by the net radiation, the periodic forcing and the stochastic forcing which is assumed to be white noise. In such a model, there are several important parameters such as the specification of the albedo as a function of temperature, the absorption due to outgoing long-wave radiation, the thermal inertia coefficient for the climatic system and the intensity of the stochastic forcing.

No numerical value is attached to the thermal inertia coefficient in either of the two papers. The reasons are, apparently, that the value of this constant is difficult to determine with accuracy and that it can be removed from the governing equation by changing the time scale. In the latter case, the uncertainty in the thermal inertia coefficient is transferred to an uncertainty in time. It may be useful to recall that the thermal inertia coefficient C, scaled by $C = \hat{C} \times 10^8$, may be calculated in principle from the following formula:

$$\hat{C} = 0.03h_w + 0.005h_l + 0.1, \tag{1}$$

where standard values have been used for the specific heats and the densities of water, land and air, and where it has been assumed that $\frac{3}{4}$ of the earth's surface is covered by oceans. The main difficulty in using eq. (1) is the uncertainties in h_w and h_l which are the depths of the water and land layers included in the climate system. It is, however, easy to see that the first term in eq. (1) dominates.

The intensity of the stochastic forcing should supposedly be estimated from climate data. This is not done in the investigation by Nicholis (1982). It is rather determined in such a way that the response to the combined periodic and stochastic forcing is close to its maximum value for a periodic forcing with a period of 10^5 years. In view of the treatment mentioned above, it becomes of interest to explore how sensitive the results of the investigations are to the numerical values of $\hat{C}$ and q^2, where q^2 measures the intensity of the white noise. To find the answer to this question, it was necessary to use Nicolis' theory leaving $\hat{C}$ and q^2 unspecified. Using this method it was possible to determine the relationship between the two parameters inherent in the theory. The numerical results are displayed in Table 1 which shows that q^2 decreases as $\hat{C}$ increases. We may first of all use the relationship displayed in Table 1 to determine the values of $\hat{C}$ and q^2 inherent in the theory by Nicolis. Since the thermal inertia coefficient enters

Table 1

$\hat{C}$	q^2	$\hat{C}$	q^2	$\hat{C}$	q^2
0.1	189.29	1.1	23.06	2.1	13.30
0.2	102.13	1.2	21.40	2.2	12.78
0.3	71.39	1.3	19.98	2.3	12.31
0.4	55.45	1.4	18.76	2.4	11.88
0.5	45.62	1.5	17.68	2.5	11.48
0.6	38.93	1.6	16.74	2.6	11.11
0.7	34.05	1.7	15.90	2.7	10.76
0.8	30.33	1.8	15.15	2.8	10.44
0.9	27.40	1.9	14.47	2.9	10.14
1.0	25.02	2.0	13.85	3.0	9.86

Table 2

$\hat{C}$	$(\hat{N}_-/n_-)_c$	$\hat{C}$	$(\hat{N}_-/n_-)_c$	$\hat{C}$	$(\hat{N}_-/n_-)_c$
0.1	0.1467	1.1	0.1095	2.1	0.0994
0.2	0.1357	1.2	0.1081	2.2	0.0988
0.3	0.1297	1.3	0.1069	2.3	0.0981
0.4	0.1252	1.4	0.1057	2.4	0.0974
0.5	0.1218	1.5	0.1047	2.5	0.0968
0.6	0.1189	1.6	0.1037	2.6	0.0961
0.7	0.1165	1.7	0.1027	2.7	0.0956
0.8	0.1145	1.8	0.1018	2.8	0.0950
0.9	0.1126	1.9	0.1010	2.9	0.0944
1.0	0.1110	2.0	0.1003	3.0	0.0939

in the potential it was necessary to write it in the form:

$$U = C^{-1}\,\Phi. \tag{2}$$

It was found that $\Delta\Phi$, corresponding to the so-called co-existence case was 283.74 J K m^{-2} s^{-1}, while ΔU for the same case according to Nicolis was 213 K^2 yr^{-1}. From these values, we find $C = 0.42 \times 10^8$ J K^{-1} m^{-2} and, from the formulas used for Table 1, $q^2 = 53.5$ $K^2\,yr^{-1}$. The following remarks may be made. Using eq. (1) with $h_1 \simeq 0$, we find that this value of C corresponds to $h_w \simeq 10.7$ m, which would appear to be too small a value for the ocean depth participating in the climate system. Correspondingly, the value of q^2 is much too large.

In view of these values it would appear that the results obtained from Nicolis' theory may be unrealistic. However, this question cannot be answered from the values given in Table 1. It is necessary to investigate the sensitivity of the final results to the values of C and q^2. The combined influence of the periodic and the stochastic forcing in Nicolis' theory is conveniently measured by the ratio $(\hat{N}_-/n_-)_c$ which may be written in the form:

$$\left(\frac{\hat{N}_-}{n_-}\right)_c = \frac{\sqrt{2}}{q^2 C}\,(\Phi_1(T_-) - \Phi_1(T_+))\,\varepsilon\,\frac{\Phi_{0_-}''^{\frac{1}{2}}}{\Phi_{0_+}''^{\frac{1}{2}} + \Phi_{0_-}''^{\frac{1}{2}}}. \tag{3}$$

Using the values of C and q^2 from Table 1, we may calculate the expression given in eq. (3). It is listed in Table 2.

It is seen from Table 2 that the ratio in eq. (3) varies relatively little as a function of C within the range included in the table. The main point, however, is that the parameters C and q^2 are not free parameters which may take any values. We may, for example, take the values of C used by Ghil (1976). The averaged value for the Northern Hemisphere is $C = 5264$ cal cm^{-2} $K^{-1} = 2.2\ 10^8$ J m^{-2} K^{-1}. This value corresponds to an ocean depth of about 70 m assuming $h_1 \simeq 0$. If Nicolis' theory were to apply with the result that $(\hat{N}_-/n_-)_c \simeq 0.1$, it would require that $q^2 \simeq 12.78$ K^2 yr^{-1}; but there is no evidence that the intensity of the stochastic forcing is so large. While the theory is thus mathematically correct, it does not seem to explain the natural phenomenon using the required parameter values.

A different approach to the same problem has been used by Benzi et al. (1982). They also start from a Budyko–Sellers, zero-dimensional model, but they take note of the observational evidence that "climatic changes appear confirmed to a temperature change of a few degrees, say 10 K". They therefore impose the main assumption that the steady state solutions to the model are two stable steady states T_1 and T_3 and an unstable state T_2 such that $T_1 < T_2 < T_3$. Their calculations are based on $T_1 = 280$ K and $T_3 = 290$ K. This requirement cannot be fulfilled without changing other aspects of the model in a radical way.

One may first of all say that the governing equation for the model is not based on any physical principle, but rather on the most simple mathematical expression which results in the required steady states. It cannot be expected that the model will behave in a physically correct manner. This may, for example, be seen by computing the albedo inherent in the model. It turns out that the function describing the albedo goes to infinity for a temperature of about 261 K, has a maximum at 277 K and a minimum at 294 K. There is of course no physical reason why the albedo should behave in this way. The model is therefore unsatisfying even as a simple climate model from a physical point of view. On the other hand, the authors demonstrate that as a mathematical model in which periodic and stochastic forcings are included, it is capable of showing transitions between the steady states T_1 and T_3 with a period of 10^5 years. To test the applicability of Nicolis' theory, it can furthermore be demonstrated that this theory will also reproduce the transitions with realistic values of C and q^2.

It would thus appear that with the present zero-dimensional models, it is difficult or impossible to reproduce the climatic transitions without either using unrealistic values of the parameters or stipulating mathematical forms which have no physical foundation.

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