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On the energetics of open systems

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ABSTRACT

The problems that result from attempts to define "energy conversions" in open systems are briefly reviewed and simple examples are used to illustrate the general inappropriateness of such concepts. Based on the kinetic theory of gases, a physical basis for boundary work is discussed and an unambiguous hierarchy of energy budgets for open systems is then developed. The resulting concepts of energy exchange within open systems, founded on first principles, remain consistent with those for closed systems. Within this framework thermodynamic, mechanical and boundary work remain separate and distinct physical processes. Two applications demonstrate how maintaining an explicit degree of freedom for boundary work influences the interpretation of diagnostic and theoretical results for open systems.

1. Introduction

Miller (p. 124, 1951) noted that various authors' points of view on the same energy phenomena may differ so much that they seem to be writing about quite different things. Despite the efforts of many to rationalize a physical basis for energy transformations, a problem still exists, particularly in regard to an open system, one which exchanges mass with an environment through motion relative to its boundary (Van Mieghem, 1951). In a closed system no boundary mass transport occurs and thus energy exchange by boundary transport and work are excluded. Much of the difficulty is identified with the combination of (i) energy transfer across the boundary of an open system and (ii) the concept of "conversion" between mechanical (kinetic and geopotential) and internal energies within an open system.

Reynolds (1894) credits Stokes (1846, 1847) with obtaining energy equations that contained a positive definite function for arbitrary stresses from a transformation of the equations of motion. This function represented the difference between the work done on the fluid by the environment through boundary stresses and the rate of increase of kinetic energy. He concluded that the positive definite function represented the conversion of kinetic energy into heat. Reynolds credits Lord Rayleigh as naming this positive definite quantity the dissipation function. In developing his criterion for the onset of turbulence, Reynolds emphasized (p. 132, 1894) that the difference between the rate of increase of kinetic energy for a fluid element and the rate at which work is done by boundary stresses equals the rate at which heat is converted into energy of motion. Thus the concept of a conversion between kinetic and internal energies within an open system that provided for the exchange of energy and momentum through the action of stresses was introduced early in the development of dynamical equations. A recent detailed treatment of energy transformations based upon the classical work of Stokes, Rayleigh and Reynolds for both open and closed systems is set forth by Van Mieghem (1973). This approach also formed the basis for Starr's (1948) claim for hydrostatic fluids that regions of horizontal velocity divergence are the primary source of kinetic energy, while regions of convergence are sinks of kinetic energy.

Another interpretation is that the conversion between kinetic and internal energy is represented by the scalar product of the velocity and pressure gradient force such as argued by Smith (1970). He emphasizes that boundary stresses serve to transfer internal energy across boundaries of open systems. Within this perspective, a gain of internal energy in one domain must be accompanied by a loss of internal energy in another domain at the same time that the internal energy is transferred across the common boundary.

A third viewpoint is summarized by Godske, Bergeron, Bjerknes and Bungaard (p. 249, 1957) who emphasize that one can "not say that one definite form of energy is transformed into another definite form". In his work with energy integrals, Pfeffer (1957) agreed with this conclusion regarding open systems, but accepted the concept of energy conversion integrals for closed systems.

The purposes of this paper are (1) to identify inconsistencies in the concepts of energy conversion for open systems, (2) to establish an inherent physical basis for boundary work stemming from the kinetic theory of gases, (3) to develop an alternative perspective for energy exchange within a hierarchy of energy models applicable in atmospheric science, and (4) to illustrate the physical insight gained from the application of this perspective. This perspective developed for either open or closed, hydrostatic or nonhydrostatic, rotating or nonrotating, systems defines roles for energy exchange through boundary work on a fluid element and establishes distinct source functions for internal and kinetic energies within an element, thereby maintaining consistency between integral and differential forms of the energy equations.

2. An examination of present concepts of energy conversion

The equations for mechanical energy and internal energy are expressed by

$$\frac{\partial}{\partial t}[\rho(k+\phi)] + \nabla\cdot[\rho(k+\phi)\mathbf{U}] = -\mathbf{U}\cdot\nabla p + \mathbf{U}\cdot\mathbf{F} \tag{1}$$

and

$$\frac{\partial}{\partial t}(\rho\imath) + \nabla\cdot(\rho\imath\mathbf{U}) = Q - p\nabla\cdot\mathbf{U}. \tag{2}$$

(see list of symbols in Appendix). The left-hand sides of these equations are the local change and the divergence of energy transport. The right-hand sides identify the sources and sinks. Within these equations, the source of mechanical energy (mechanical work) results from fluid displacement in the direction of the pressure and viscous forces acting on the fluid, as shown by $\mathbf{U}\cdot\nabla p$ and $\mathbf{U}\cdot\mathbf{F}$. The source of internal energy results from heat addition or by contraction of the fluid, $p\nabla\cdot\mathbf{U}$ (thermodynamic work).

The combination of kinetic and geopotential energy into mechanical energy permits simplification at this point, since geopotential energy change is solely linked with kinetic energy change through an exact differential associated with vertical displacement. Later the implications of hydrostatic balance are discussed.

The energy equations (1) and (2) as well as those which follow are valid for systems either inertially fixed, uniformly translating or uniformly rotating as long as the kinetic and geopotential energy are properly defined. In each of the systems the kinetic energy is defined to be the energy of motion relative to the coordinate system. In rotating coordinate systems, such as used in meteorology for earth relative motion, geopotential energy is defined to be the work of elevating mass "vertically" against the combined force of gravitation and centrifugal force of the rotating coordinate system (see definition in Table of Symbols). For systems undergoing linear or angular acceleration with respect to an inertial coordinate system, an additional source of kinetic energy would need to be added to reflect such changes. A discussion of the relation between energy equations expressed in inertial and rotating systems is presented by Van Mieghem (pp. 17–22 (1973)).

With the relation

$$\nabla\cdot p\mathbf{U} = p\nabla\cdot\mathbf{U} + \mathbf{U}\cdot\nabla p, \tag{3}$$

and the addition of (1) and (2), the total energy equation is

$$\frac{\partial}{\partial t}[\rho(k+\phi+\imath)] + \nabla\cdot[\rho(k+\phi+\imath)\mathbf{U}] = Q + \mathbf{U}\cdot\mathbf{F} - \nabla\cdot p\mathbf{U}. \tag{4}$$

The source of total energy results from heating, viscous mechanical work and boundary pressure work. For the purposes of identifying the overall problem, consider adiabatic inviscid conditions in two isolated systems with a common boundary.

The integrated total energy equation for each system is

$$\int_{V_i} \frac{\partial}{\partial t} [\rho(k + \phi + \imath)] \,\mathrm{d}V_i + \int_{\sigma i} \mathbf{n}_i \cdot [\rho(k + \phi + \imath) \mathbf{U}] \,\mathrm{d}\sigma_i = -\int_{\sigma i} \mathbf{n}_i \cdot (p\mathbf{U}) \,\mathrm{d}\sigma_i, \tag{5}$$

where $\mathbf{n}$ is the outer unit normal and i denotes system 1 and system 2. The exchange of energy from system 1 to system 2 occurs by either the transport of total energy across the boundary or by boundary work. A loss of energy by either process in one system requires a gain by the identical process in the other system. The presence of boundary pressure work in conjunction with the physical constraint expressed by (3) requires the existence of an imbalance between the source of kinetic energy by mechanical work and of internal energy by thermodynamic work within each system.

It should also be noted at this point that choice of the reference frame and a definition of velocity $\mathbf{U}$ relative to the reference framework serves to specify the boundary pressure work in (5). The choice of the reference frame does not spatially fix the open system since the boundaries of an open system may move with an arbitrary velocity $\mathbf{W}$ while the velocity relative to the boundary is $\mathbf{U} - \mathbf{W}$ (Johnson, 1970). A translating open system introduces differences in the degrees of freedom for boundary transport of thermodynamic energy $\mathbf{n} \cdot \rho(\mathbf{U} - \mathbf{W})\imath$ and boundary pressure work $(R/c_v)\mathbf{n} \cdot \rho\mathbf{U}\imath$. For simplicity in the analyses discussed herein, the boundaries of the open system are assumed fixed with respect to the reference framework. Thus the transport of all properties through the boundaries of the open system are defined through $\rho\mathbf{U}$ ($\mathbf{W} = 0$) and the ratio of boundary transport of thermodynamic energy and boundary pressure work are uniquely proportional to each other.

Now, the two contrasting views which stem from the definition of "conversion" between mechanical and thermodynamic energies within a system are examined. The classical approach summarized by Van Mieghem (1973) is to define the "conversion" of internal to kinetic and geopotential energies by

$$C_1(I, K + \Phi) = \int_V p\nabla \cdot \mathbf{U} \,\mathrm{d}V. \tag{6}$$

With the substitution of (3) the mechanical and thermodynamic energy equations are

$$\int_V \frac{\partial}{\partial t} [\rho(k + \phi)] \,\mathrm{d}V + \int_\sigma \mathbf{n} \cdot [\rho(k + \phi) \mathbf{U}] \,\mathrm{d}\sigma = C_1(I, K + \Phi) - \int_\sigma \mathbf{n} \cdot p\mathbf{U} \,\mathrm{d}\sigma \tag{7}$$

and

$$\int_V \frac{\partial}{\partial t} (\rho\imath) \,\mathrm{d}V + \int_\sigma \mathbf{n} \cdot (\rho\imath\mathbf{U}) \,\mathrm{d}\sigma = -C_1(I, K + \Phi). \tag{8}$$

Another approach summarized by Smith (1970) is to define the "conversion" by

$$C_2(I, K + \Phi) = -\int_V \mathbf{U} \cdot \nabla p \,\mathrm{d}V, \tag{9}$$

wherein the parallel equations to (7) and (8) are

$$\int_V \frac{\partial}{\partial t} [\rho(k + \phi) \,\mathrm{d}V + \int_\sigma \mathbf{n} \cdot [\rho(k + \phi) \mathbf{U}] \,\mathrm{d}\sigma = C_2(I, K + \Phi) \tag{10}$$

and

$$\int_V \frac{\partial}{\partial t} (\rho\imath) \,\mathrm{d}V + \int_\sigma \mathbf{n} \cdot (\rho\imath\mathbf{U}) \,\mathrm{d}\sigma = -C_2(I, K + \Phi) - \int_\sigma \mathbf{n} \cdot p\mathbf{U} \,\mathrm{d}\sigma \tag{11}$$

The contrast of (7) and (11) reveals the dichotomy noted by many, that boundary work may be viewed as either a source of thermodynamic energy or mechanical energy depending upon the arbitrary definition for the conversion term. This dichotomy is explored by considering two simple examples.

Example 1: Assume that the philosophy of conversion embraced by (6) and (7) is appropriate for a system in which a region of convergent flow is separated from a region of divergent flow by the open internal boundary b (Fig. 1). On the condition that the pressure is uniform throughout, the source of internal energy in region 1 is balanced by the sink of internal energy in region 2, since the net boundary work for the entire domain vanishes. With the magnitude of the boundary work at "a" equal to that at "c", both of which are greater than at "b", the role of the pressure work is to transfer energy from region 2 to region 1, without any production of kinetic energy. No conversion of internal to kinetic energy occurs. In this case the

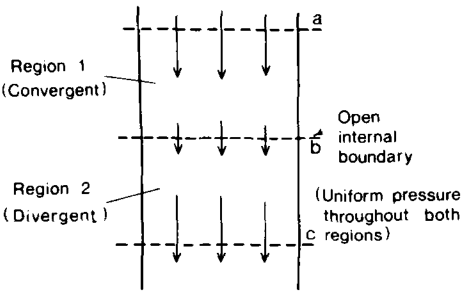

Fig. 1. Schematic of hypothetical system in which a region of convergent flow is separated from a region of divergent flow, by an open internal boundary. Pressure is uniform throughout. a, b and c are open internal boundaries.

transfer of internal energy in both domains occurs through thermodynamic work $p\nabla \cdot \mathbf{U}$ and boundary work $\nabla \cdot p\mathbf{U}$ in the absence of mechanical work $\mathbf{U} \cdot \nabla p$ and the production of kinetic energy. The logic for the conversion according to (6), (7) and (8) does not embrace the physical situation of this example. Thus, the physical concept of the conversion of thermodynamic and mechanical energy set forth by the definition (6) is illogical and inappropriate.

Example 2: Assume that the philosophy embraced by (9), (10) and (11) is appropriate for a system composed of two regions both of which have uniform non-divergent flow parallel to a uniform pressure gradient (Fig. 2). Each of the regions is characterized by a source of kinetic energy but changes of internal energy by thermodynamic work are precluded. Each of the boundaries "a", "b" and "c" (Fig. 2) are open. With the condition that the kinetic energy generations of each domain are equal, the following inequalities are obtained for the pressure work at the three boundaries

$$|p\mathbf{U}|_a > |p\mathbf{U}|_b > |p\mathbf{U}|_c, \tag{12}$$

and the magnitude of the pressure work at the internal boundary is given by

$$|p\mathbf{U}|_b = \tfrac{1}{2}\{|p\mathbf{U}|_a + |p\mathbf{U}|_c\}. \tag{13}$$

In this situation the kinetic energy change realized in region 2 through boundary work is not attended by any change in internal energy as a result of thermodynamic work. Again the logic for the conversion of energy expressed in (9), (10) and (11) does not embrace the physical situation of this example, wherein the transfer of energy by boundary work is uniquely related to kinetic energy production by mechanical work within the system. The interpretation by Smith (1970) following his equation (17) that boundary work is related to a "transformation of internal energy within σ_L (a system) to internal energy in the surrounding atmosphere (the environment of σ_L)" is also, in general, inappropriate. The relation between boundary work and changes of either mechanical or thermodynamical work within a system can only be determined by a direct evaluation of the sources/sinks of each form of energy within the system.

In both of these examples, the interpretation of energy exchange is compounded by the condition that the presence of boundary work (a mode of non-convective flux of energy) within an ideal gas occurs in conjunction with the boundary transport of internal energy (a mode of convective flux of energy). Note that through the equation of state, $\nabla \cdot (p\mathbf{U})$ is uniquely related to $\nabla \cdot (\rho\iota\mathbf{U})$ by the ratio R/c_v and that neither will vanish if an imbalance of thermodynamic and mechanical work exists within a system.

The analyses of these two hypothetical examples serve to reinforce the arguments of caution (Lettau, 1954; Pfeffer, 1957; and others) on the interpretation of conversion integrals between forms of energy. In the following section boundary work is established to be a basic physical process that is intrinsic to an energy conservation principle for open systems. Later, a hierarchy of energy budgets and concepts provide an unambiguous interpretation of the physical processes of energy transfer for open systems.

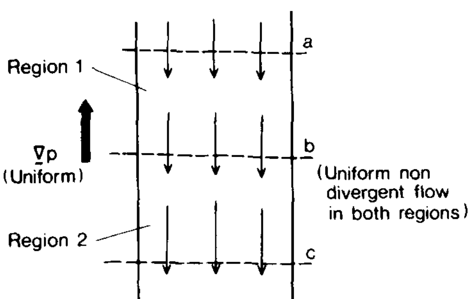

Fig. 2. Schematic of hypothetical system in which the pressure gradient is uniform and parallel to uniform non-divergent flow. a, b and c are open internal boundaries.

3. Boundary work and total energy budget for an open system

The general law of energy (e.g., as expressed by Miller, 1949, 1950) is:

Rate of increase of energy in the system	=	Rate at which energy is added through the boundary	−	Rate at which work is done by the system.	(14)

In the application to an open region of the atmosphere, the total energy is defined here to be the sum of mechanical and thermodynamic energies. The term thermodynamic energy includes the contributions of internal energy and the latent energy of phase changes. In nearly all meteorological applications, the heat released during phase changes is treated as an explicit proces of heat addition. In the present effort to differentiate clearly between boundary processes and sources or sinks of internal energy by adiabatic reversible processes within a fluid element, it is important to adopt a framework in which the total energy (E) comprises simply a mechanical ($K + \Phi$) plus a thermodynamic ($I + L$) component.

$$E = (K + \Phi) + (I + L) \tag{15}$$

The transport equation appropriate to Eq. (15) is

$$\frac{\partial E}{\partial t} = B(E) + H(E) + W(E), \tag{16}$$

where $B(E)$ denotes the convective transport, $H(E)$ denotes the energy flux by conduction, evaporation and radiation across the boundary, and $W(E)$ is the boundary work on the system by pressure and viscous stresses. A schematic for the total energy budget is shown in Fig. 3.

The interpretation of each of the physical mechanisms responsible for the changes in total energy is unambiguous at this stage. The convective transport of total energy for the open system results from bulk fluid motions through the boundary. Energy flux by the processes of conduction, radiation and evaporation is in each case identified with distinct physical mechanisms not related to bulk fluid motions. The boundary work on the system by pressure and viscous stresses in association with the bulk fluid motion is an independent mode of interaction of the system with its environment.

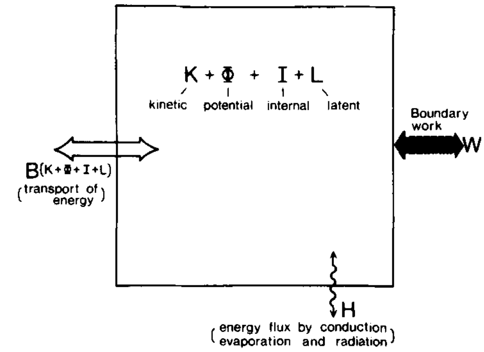

Fig. 3. Schematic showing the total energy budget of an open system.

Within the kinetic theory of gases the physical mechanism of boundary work is fundamental to the energy concept and does not arise from the *a priori* definition of the individual components of thermodynamic and mechanical energy and the requirement for a transformation function between these two forms of energy. This fact is evident from a derivation of the fundamental energy equations of fluid dynamics using Maxwell's generalized transport equations, when local changes of the sum of kinetic and internal energies stem from the divergence of a transport vector and external forces. (See Chapter V of Sommerfeld (1964) pp. 293–323.) The transport vector includes the transport of kinetic and internal energies by the mean motion, the transfer of internal energy by random molecular motion, and boundary pressure and viscous work. In its primitive form, the function does not distinguish between any of these boundary processes of energy exchange since the kinetic energy of the system has not been partitioned into the kinetic energy of mean and random molecular motion. Viewed in the context of statistical mechanics, the function describes the exchange of energy that occurs through the total motion of molecules.

Boundary work only becomes distinctly identified in the process of energy transport upon decomposition of the total motion of molecules into a mean and a random component. At this point combined exchange of mean and random molecular kinetic energy explicitly occurs by boundary pressure and viscous work and is uniquely identified with normal and tangential stresses on the surface of a volume element. However, sources of the random molecular kinetic energy (internal energy), thermodynamic work, and of mean molecular kinetic energy, mechanical work, have not been separated. Thus the boundary work has a unique origin independent of the source/sink functions of mean kinetic and internal energy. These sources become identified upon separation of the energy equation into separate mechanical and internal energy equations. If one wished to only address the question of changes of total energy for a system, the problem of energy conversion would be of no concern.

The key point is that boundary work, thermodynamic work and mechanical work each play a unique role in energy exchange even though the three physical processes are intimately related. The failure to recognize a role for each of these physical processes leaves an incomplete logical and physical structure to determine energy exchange and leads to questionable concepts of *in situ* conversions of one energy form to another. These considerations are now developed in the hierarchy of energy models summarized in the next section.

4. A heirarchy of energy models

4.1. *A thermodynamic and mechanical energy budget*

With the division of the total energy into its mechanical and thermodynamic components, additional degrees of freedom for sources and sinks of the two forms of energy within the system are introduced. The source of internal energy through thermodynamic work and the source of mechanical energy through mechanical work are coupled with each other and the boundary work of the open system (Fig. 4). Symbolically the energetics of the system can be written:

$$\frac{\partial}{\partial t}(K + \Phi) + B(K + \Phi) = S(K + \Phi), \tag{17}$$

$$\frac{\partial}{\partial t}(I + L) + B(I + L) = S(I) + H(I + L), \tag{18}$$

and

$$W(E) = S(K + \Phi) + S(I) = \int_V \nabla \cdot (\Psi \cdot \mathbf{U})\, dV$$

$$= \int_\sigma \mathbf{n} \cdot \Psi \cdot \mathbf{U}\, d\sigma, \tag{19}$$

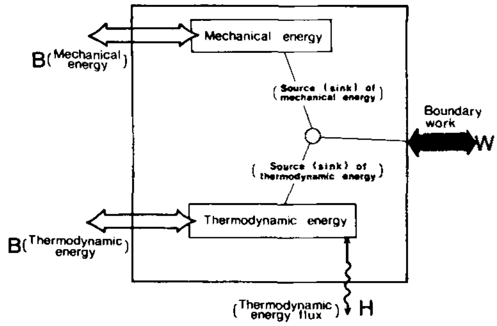

Fig. 4. Schematic showing the mechanical and thermodynamic component energy budget of an open system.

where $S(\ )$ denotes the source of the energy form denoted within the parentheses. The source of mechanical energy through mechanical work

$$S(K + \Phi) = \int_V \mathbf{U} \cdot (\nabla \cdot \Psi)\, dV \tag{20}$$

follows directly from Newton's second law and the definition of pressure and viscous forces (Ψ is the total stress tensor). The source of internal energy through thermodynamic work

$$S(I) = \Psi : \nabla U = \Psi : \Phi^* \tag{21}$$

follows from the thermodynamic equation. Eq. (19) establishes the relation between the boundary work on the system and the internal work by mechanical and thermodynamic processes. Note within the context of these energy equations emphasis is again placed on the constraint that boundary work on a system requires an imbalance between the thermodynamic and mechanical work within the system, a situation that prevails in most open atmospheric domains. Furthermore, this physical mechanism of boundary work is distinct from the mechanism of internal energy transport across the boundary once mechanical and thermodynamic energies have been explicitly separated and defined.

In endeavoring to pose a consistent physical basis for energy transformation functions, the work by surface stresses is now partitioned into components associated with the pressure and viscous structure of the system. To this point the definition of work and its mechanical and thermodynamic components have been independent of any assumptions regarding the physical origin of the stresses.

The total stress tensor for the classical Navier-Stokes equation (Batchelor, 1970) is given by

$$\Psi = -pI + \Upsilon, \tag{22}$$

where

$$\Upsilon = 2\mu[(\Phi^* - \tfrac{1}{3}\nabla\cdot\mathbf{U})I], \tag{23}$$

and

$$\Phi^* = \tfrac{1}{2}[\nabla\mathbf{U} + (\nabla\mathbf{U})^T] \tag{24}$$

are the respective definitions for the deviatoric and strain tensors. Strain in the fluid has been assumed to be proportional to the deformation of the velocity structure; however, the coefficient of proportionality need not be uniform in space or time. With this definition, the boundary work is divided into two parts,

$$W(E) = W_1(E) + W_2(E), \tag{25}$$

where the components associated with normal pressure and tangential viscous stresses are

$$W_1(E) = -\int_V \nabla\cdot p\mathbf{U}\,\mathrm{d}V, \tag{26}$$

and

$$W_2(E) = \int_V \nabla\cdot\Upsilon\cdot\mathbf{U}\,\mathrm{d}V. \tag{27}$$

An expansion of (26) and (27) yields

$$W_1(E) = S_1(K + \Phi) + S_1(I + L), \tag{28}$$

and

$$W_2(E) = S_2(K + \Phi) + S_2(I + L), \tag{29}$$

where

$$S_1(K + \Phi) = -\int_V \mathbf{U}\cdot(\nabla\cdot pI)\,\mathrm{d}V$$
$$= -\int_V \mathbf{U}\cdot\nabla p\,\mathrm{d}V \tag{30}$$

$$S_2(K + \Phi) = \int_V \mathbf{U}\cdot(\nabla\cdot\Upsilon)\,\mathrm{d}V \tag{31}$$

$$S_1(I + L) = -\int_V pI:\nabla\mathbf{U}\,\mathrm{d}V = -\int_V p\nabla\cdot\mathbf{U}\,\mathrm{d}V \tag{32}$$

$$S_2(I + L) = \int_V \Upsilon:\nabla\mathbf{U}\,\mathrm{d}V = \int_V \Upsilon:\phi^*\,\mathrm{d}V. \tag{33}$$

The energy transformation functions associated with this partitioning are presented in Fig. 5, where the boundary work is explicitly divided into components associated with the pressure and the viscous stresses. The results emphasize the intrinsic coupling of boundary work with the internal structure of mechanical and thermodynamic work associated with each of the stress components. Sources of mechanical energy within the system cannot be uniquely related to sinks of internal energy for either form of stress. The existence of boundary work in the system through either form of stress again requires an imbalance between mechanical and thermodynamic work within the system by a corresponding component of stress on the system.

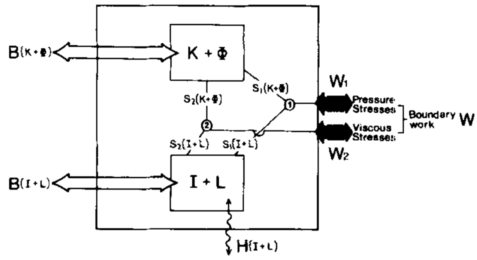

Fig. 5. Schematic showing the mechanical and thermodynamic component energy budget with explicit separation of the boundary work into pressure and viscous stress components.

In atmospheric diagnostic studies of open systems, the source of kinetic energy through the component of mechanical work identified with pressure stresses is normally the result of the difference of the two large and opposing effects of boundary work and internal energy changes by thermodynamic work within the system. A complete solution and understanding of the energetics of the system require that all of the energy transformation functions be accurately determined. Conclusions regarding the transfer of energy based on the determination of only one transformation function are incomplete and can be quite misleading.

The same problem exists in the interpretation of sinks of kinetic energy and sources of internal energy by viscous stresses. Only the sign of the $S_2(I + L)$ term in (33) is uniquely positive since the integrand is composed of a linear combination of quadratic quantities. At any given point in the system a viscous source of internal energy does not require a corresponding sink of kinetic energy by viscous mechanical work since a sink/source of momentum by tangential stresses within a fluid requires a transfer of momentum across the surface and the kinetic energy changes may be positive. Accordingly the arrangement of the energy transformation functions in Fig. 5 summarizes the necessary logical and physical consistency between integral and differential forms of the energy equations and the basic concept of work.

4.2. *A component energy budget*

With the explicit formulation of the boundary work and its relation to the components of mechanical and thermodynamic energy by pressure and viscous stresses the total energy budget is now divided into kinetic, geopotential, internal and latent energy components. The component equations for the system are:

$$\dot{K} = S_1(K) + S_2(K) + C(\Phi, K) + B(K) \tag{34}$$

$$\dot{\Phi} = -C(\Phi, K) + B(\Phi) \tag{35}$$

$$\dot{I} = S_1(I) + S_2(I) + H(L, I) + H(I) + B(I) \tag{36}$$

$$\dot{L} = -H(L, I) + H(L) + B(L) \tag{37}$$

where

$$C(\Phi, K) = -\int_V \rho g w \, \mathrm{d}V \tag{38}$$

$$H(L, I) = \int_V -\rho L \frac{\mathrm{d}q}{\mathrm{d}t} \mathrm{d}V. \tag{39}$$

The term $C(\Phi, K)$ is defined to be the conversion between geopotential and kinetic energy (the justification of which is established in a later paragraph). $H(L, I)$ is the transformation between internal and latent energies during phase changes. $H(I)$ and $H(L)$ represent energy addition by boundary molecular flux while $B(I)$ and $B(L)$ are boundary transport by mean motion. The budget is shown schematically in Fig. 6. The form of closure for the energy exchange between geopotential, kinetic and internal energies corresponds with Wiin-Nielsen's (1968) form for non-hydrostatic closed systems.

The $S_1(\ )$ and $S_2(\ )$ functions for $\dot{K}$ and $\dot{I}$ in (34) and (36) are identical in form to the same functions defined for the mechanical and thermodynamic partitioning in (20) and (21). However, with the explicit separation of mechanical and thermodynamic energies the source functions $S_i(K + \Phi)$ and $S_i(I + L)$ in (28) through (33) respectively become the source functions $S_i(K)$ and $S_i(I)$ in (34) and (36). Thus, the physical process of boundary work is now identified with changes in K and I that do not directly involve changes in Φ or L. However, in systems that tend toward hydrostatic balance, the implicit redistribution between geopotential and internal energies is closely coupled with the vertical components of pressure stress.

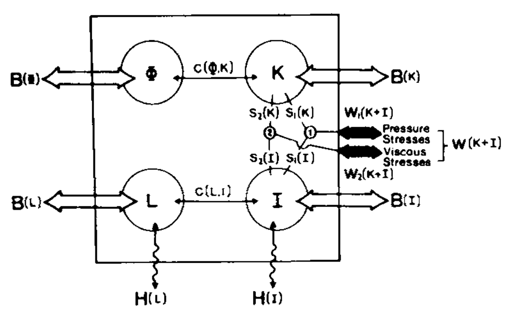

Fig. 6. Schematic showing geopotential, kinetic, internal, and latent heat components of the energy budget, again with explicit separation of the boundary work term.

In this closure we have defined a conversion term between geopotential and kinetic energy based on (1), the necessary condition that expressions of opposite sign appear in both energy equations; and (2), a sufficient condition that the source/sink function has a physical basis founded on first principles that is unique to each of the two equations and whereby an *in situ* sink of one form of energy is an *in situ* simultaneous source of the other form of energy. In this case, the mechanical work involved with an increase of geopotential energy is equal to the mechanical work involved with the decrease of vertical kinetic energy of the parcel. Even though vertical displacements involve pressure work that couples kinetic and internal energy changes, the physical mechanism of this process is clear and distinct. The coupling of mechanical and thermodynamic energies in hydrostatic systems will now be addressed.

4.3. *Mechanical and thermodynamic energies of hydrostatic systems*

To reveal the essence of the arguments, consider a system in which the only component of the thermodynamic energy is the internal energy of dry air and assume isentropic motion, a constraint that precludes heating and internal viscous dissipation of kinetic energy. These assumptions do not invalidate the considerations involved in contrasting the energy balance of closed hydrostatic and non-hydrostatic systems previously considered by Wiin-Nielsen (1968).

With the assumption of hydrostatic balance, horizontal kinetic energy (K_H) replaces the three-dimensional kinetic energy in the mechanical energy equation. While kinetic energy of vertical motion is present, its magnitude is small and in effect assumed negligible. The mechanical and thermodynamic energy equations become

$$\dot{K}_H + \dot{\Phi} = S_1(K_H + \Phi) + B(K_H + \Phi). \tag{40}$$

$$\dot{I} = S_1(I) + B(I) \tag{41}$$

$$W_1(E) = S_1(K_H + \Phi) + S_1(I) \tag{42}$$

In this partitioning, the form and physical meaning of the boundary work and sources/sinks of the energy forms for open hydrostatic systems shown in Fig. 7a are identical to those portrayed previously for non-hydrostatic systems in Fig. 5.

4.4. *Kinetic, internal and geopotential energies of hydrostatic systems*

In this section the strategy is to seek a physical basis for closure between kinetic, geopotential and internal energies in open hydrostatic systems that is consistent with the closure between mechanical and thermodynamic energies established in the previous section.

The component equations are

$$\dot{K}_H = S_{1h}(K_H) + B(K_H) \tag{43}$$

$$\dot{\Phi} = S_{1h}(\Phi) \tag{44}$$

$$\dot{I} = S_1(I) + B(I) \tag{45}$$

$$W_1(E) = S_{1h}(K_h) + S_{1h}(\Phi) + S_1(I) \tag{46}$$

where

$$S_{1h}(K_H) = \int_V - \mathbf{U} \cdot \nabla_z p \, \mathrm{d}V \tag{47}$$

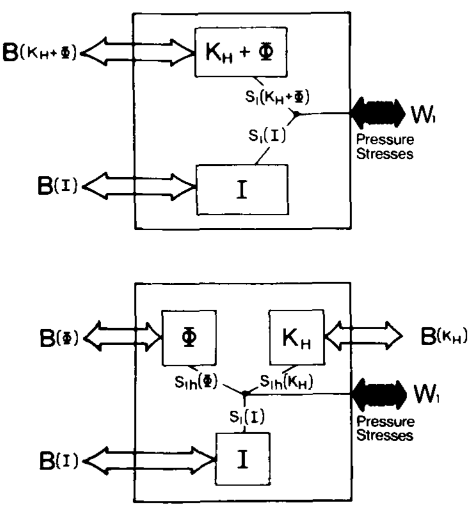

Fig. 7. Schematics of the energy budget for a hydrostatic isentropic open system. (a) Mechanical and thermodynamic partitioning. (b) Kinetic, geopotential, and internal energy partitioning.

$$S_{1h}(\Phi) = \int_V \rho g w \, \mathrm{d}V = \int_V - w \frac{\partial p}{\partial z} \mathrm{d}V \tag{48}$$

(subscript h denotes hydrostatic).

The partitioning of the mechanical energy into its two components for the hydrostatic system requires that the source functions for the geopotential and kinetic energies be modified from the non-hydrostatic case, while the boundary work on, and the thermodynamic work within the system, remain unique and unaltered. The physical basis for the uniqueness of the latter two processes is associated with the condition that three dimensional velocity divergence within the system, as well as boundary work associated with vertical motion and vertical components of pressure stress, are well defined. However, through the hydrostatic assumption the exchange of the geopotential and kinetic energy is constrained. As a result, the combination of vertical motion and vertical pressure stress through the boundary work component, $\partial(pw)/\partial z$, becomes the link between the sources of geopotential energy by mechanical work $S_{1h}(\Phi)$, and sources of internal energy owing to expansion or contraction of the fluid along the vertical axis, a component of the thermodynamic work $S(I)$, Fig. 7b. The source of horizontal kinetic energy by mechanical work is solely owing to motion in the presence of horizontal pressure gradients that remain coupled to lateral boundary work and sources of internal energy due to expansion or contraction of the fluid along horizontal axes within the system (Fig. 7b). With the hydrostatic constraint that the boundary work directly couples sources of geopotential and kinetic energy with the source of internal energy, the closure for such open systems is portrayed in Fig. 7b by a branch point representing boundary work which links the three source functions of energy.

This closure for energy transformation in open hydrostatic systems (Fig. 7b) departs from the method outlined by Wiin-Nielsen (1968) for closed hydrostatic systems; as noted previously the closure argued for open hydrostatic systems is consistent with the closure established by Wiin-Nielsen for closed hydrostatic systems. He pointed out that the closures of energy transformations within non-hydrostatic and hydrostatic systems are dissimilar in the sense that the "conversion" between geopotential and kinetic energy is direct in a non-hydrostatic system while within a hydrostatic system the energy conversion occurs between geopotential and internal energy, but not with kinetic energy. However, this argument is only applicable for closed systems. In an open system a degree of freedom must be provided for increase of geopotential and horizontal kinetic energies through boundary work under the plausible kinematic condition of no thermodynamic work in association with non-divergent flow ($\nabla \cdot \mathbf{U} = 0$), an energy exchange explicitly permitted in Fig. 7b. It is important to recognize that the boundary work in hydrostatic systems is uniquely coupled to the imbalance between sources of internal, geopotential and horizontal kinetic energies within the system. The closure developed herein provides explicit degrees of freedom from these boundary processes and stems logically from physical principles. If in effect the system is closed due to the absence of boundary processes, the linking paths for the transformation functions in Fig. 7b remain unaltered and unambiguous.

5. On the necessary and sufficient conditions for the definition of conversion integrals

It is generally agreed that a necessary condition for the definition of a conversion integral or function is that identical expressions of opposite sign appear in separate energy equations and that a fundamental physical basis be provided for its interpretation (Lettau, 1954; Pfeffer, 1967). Interestingly enough with rearrangement of energy equations expressions of opposite sign can always be ascertained and the challenge to provide logical interpretations is usually attempted. The integrands of the energy equations will be some function of the physical variables and represent some form of temporal or spatial energy exchange. Are such conditions sufficient?

Sufficiency in our application, which maintains a consistent mathematical and physical basis between differential and integral equations, requires that the conversion function must involve *in situ* a sink of one form of energy and a simultaneous source of a second form of energy. Furthermore, the source/sink function must have a physical basis founded in the first principles. The function cannot involve boundary terms (a similar requirement was argued by Pfeffer (1957)), a third form of energy, or be determined as a result of integration by parts. At first these necessary and sufficient conditions might seem extreme because nearly all expressions used to represent energy conversions are eliminated. However, due consideration reveals that application of the sufficient conditions maintains a clear, comprehensive perspective that requires identification of all the physical processes involved in energy exchange and precludes ambiguities.

In the definition of the energy conversion $C(\Phi, K)$ used earlier in this paper, the above criteria for necessary and sufficient conditions were employed since the sink of kinetic energy by mechanical work against gravity ($-\rho U \cdot \nabla\phi$) and the corresponding source of geopotential energy ($\rho\, \mathrm{d}\phi/\mathrm{d}t$) are unique for a fluid element. Since $(\partial\phi/\partial t_{\phi}) = 0$, the two expressions are identical, of opposite sign and can be interpreted from first principles. The basis for the uniqueness in this case stems from the condition that the curl of the geopotential force vanishes, thus ensuring a geopotential energy function that is uniquely related to mechanical work. In the case of the link between mechanical and thermodynamic work, boundary work is an intrinsic process. With the sufficiency set forth here it is not permissible to define conversion between these two forms of energy for either closed or open systems.

6. Illustrations of boundary work processes

6.1. *Interpretation of a diagnostic result*

An advantage of avoiding the concept of conversion between mechanical and thermodynamic energies and maintaining an explicit degree of freedom for boundary work is now illustrated by examining some of Kung's (1966, 1967) budget results. The vertical profiles of the time averaged dissipation calculated in these studies show two large positive dissipation regions, one in the boundary layer, the other near the tropopause. A region of small negative dissipation occurs in the vicinity of 400–500 mb just below the maximum positive values near the tropopause. This region of negative dissipation appears spurious to some investigators if (as many do) one assumes that the physical mechanism of the sink of kinetic energy by viscous mechanical work is identical with the ever present positive source of internal energy by viscous thermodynamic work. The retention of a distinct definition for viscous boundary work suggests the following interpretation.

In the absence of momentum transfer to the earth's surface, the existence of a viscous mechanical force in a layer of the atmosphere requires from momentum conservation principles an equal and opposite viscous mechanical force in a juxtaposed layer. Through this mechanism, momentum must be transferred between the two layers. Within the jet stream regions of the middle latitudes, the westerly momentum near the maximum wind region of the tropopause is in all likelihood transferred vertically to layers adjacent to the maximum wind level, thereby creating westerly momentum in other layers. In conjunction with the momentum transfer by viscous stresses, a viscous sink of kinetic energy by mechanical work must be coupled with viscous thermodynamic work within the layer or transfer of energy by viscous boundary work to the adjacent layers. It is reasonable to expect that the source layer of momentum by viscous stresses may also be coincident with the layer to which some energy is transferred by viscous boundary work and within which a small amount of kinetic energy of the westerlies may be created by positive viscous mechanical work in layers adjacent to the maximum wind level. Thus viscous sources of momentum and kinetic energy of the westerlies located adjacent to the region of upper level sinks of momentum and kinetic energy dissipation, seem feasible and may well have been isolated in Kung's results.

6.2. *An application of Bernoulli's theorem*

In the previous sections the fundamental importance of boundary work for open systems has been emphasized by maintaining an interpretation of energy integrals and their transformation that parallels the basic differential equations for the total energy and its components. The application of these results to an idealized steady solution for isentropic flow (e.g., a stationary Rossby wave or steady flow over mountains) will serve to illustrate the importance of explicitly separating the concepts of mechanical, thermodynamic and boundary work.

For steady isentropic flow, Bernoulli's theorem

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(k + \phi + \frac{c_p}{c_v}\,\iota\right) = 0 \tag{49}$$

expresses the conservation of kinetic and geopotential energies and enthalpy. Alternative forms of (49) for steady isentropic flow are

$$\frac{\mathrm{d}}{\mathrm{d}t}(k + \phi + \iota) = -\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{R}{c_v}\,\iota\right) = -\mathbf{U}\cdot\nabla\left(\frac{R}{c_v}\,\iota\right) \tag{50}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}(k + \phi + \iota) = -\rho^{-1}\,\nabla\cdot(p\mathbf{U}) \tag{51}$$

$$\nabla\cdot\left[\rho\left(k + \phi + \frac{c_p}{c_v}\,\iota\right)\mathbf{U}\right] = 0 \tag{52}$$

$$\nabla\cdot[\rho(k + \phi + \iota)\mathbf{U}] = -\nabla\cdot\left(\rho\frac{R}{c_v}\,\iota\mathbf{U}\right) \tag{53}$$

Eqs. (50) and (51) relate the change of the specific total energy to temperature advection and to boundary work. Eq. (52) expresses Bernoulli's conservation relation in flux form. Eq. (53) shows the relation between the divergence of the energy transport to boundary work. If one now assumes that in some region of the steady flow kinetic energy increases result from boundary work and a sink of internal energy in the absence of geopotential energy change, then from (50)

$$\frac{\mathrm{d}}{\mathrm{d}t}(k) = -\frac{\mathrm{d}}{\mathrm{d}t}(c_v + R)\,T = -\frac{\mathrm{d}}{\mathrm{d}t}(c_p\,T). \tag{54}$$

It is clear that a fraction (c_v/c_p) of the source of kinetic energy by mechanical work comes from internal energy by thermodynamic work within the system while a fraction (R/c_p) comes from boundary work by the environment. It is also clear that boundary work cannot contribute to an increase in both kinetic and internal energy of the system (in (54) $\mathrm{d}T/\mathrm{d}t > 0$ is incompatible with $\mathrm{d}k/\mathrm{d}t > 0$). Thus the source of kinetic energy through mechanical work in the steady flow requires the presence of both boundary work and a sink of internal energy by thermodynamic work in the aforementioned proportions. The concepts of conversion summarized by either Smith (1970) or Van Mieghem (1973) for this apparently simple flow lead to inconsistencies, since the kinetic energy production cannot be uniquely related to a conversion function from existing energy forms within the system. In contrast the concepts developed herein do not lead to any inconsistencies.

7. Summary and conclusions

In this paper an unambiguous hierarchy of energy budgets for open systems is presented in which the concepts of energy exchange within closed systems remain consistent with those for open systems. The hierarchy is established through an emphasis that thermodynamic, mechanical and boundary work must be considered separate and distinct physical processes in assessing exchange and transformation of energies. Through an examination of conversion functions frequently employed in atmospheric studies, logical and physical inconsistencies are revealed. Beginning with fundamental principles of energy conservation from kinetic theory, the distinct role that boundary work enjoys in total energy exchange is emphasized. With a subsequent partitioning of total energy into its mechanical and thermodynamic components, the separate roles of mechanical and thermodynamic work are stressed and schematized within traditional energy diagrams through the use of branch points. The branch points denote the link between a boundary process and related sources/sinks of individual partitioned components of the total energy of a system.

In two applications of these concepts it is emphasized that the presence of boundary work requires an imbalance of mechanical and thermodynamic work within an open system and an interaction with an environment which must also have an imbalance between mechanical and thermodynamic work. As a result, total energy is transferred by boundary work between the open system and its environment. In nearly all atmospheric phenomena, transfer of energy by boundary work is an important mode of energy exchange, a result that is in accord with concepts of strong scale interaction. These more general concepts properly place in perspective the energy transfer between different regions of the atmosphere and the reversible adiabatic energy transformations within a region.

8. Appendix

Symbols

k, K	kinetic energy
L	latent heat energy
ϕ, Φ	geopotential energy ϕ is equal to $\phi_a - \Omega^2 R^2/2$ where ϕ_a is gravitational energy, Ω is angular velocity of reference frame and R is perpendicular distance to axis of rotation
ι, I	internal energy (also unit tensor)
$k+\phi, K+\Phi$	mechanical energy
$I+L$	thermodynamic energy
$H(\ )$	boundary flux of energy by radiation and the molecular transfer of latent and sensible heat
$B(\ )$	boundary transport of the energy
$W(\ )$	transfer of mechanical and thermodynamic energy by boundary work
$S(\ )$	source (sink) of energy
$C(\Phi, K)$	conversion of potential to kinetic energy
Ψ	stress tensor
Υ	deviatoric stress tensor
Φ^*	strain tensor
$(\ )^T$	transpose of quantity
$\dot{(\ )}$	time rate of change
$\mathbf{U}$	velocity vector
p	pressure
∇	del operator
$:$	double dot product used for multiplication of tensors
T	temperature
c_v, c_p	specific heats
$\mathbf{F}$	friction vector
ρ	density
Q	heating rate
σ	surface area
V	volume
C_1, C_2	conversion integrals according to different authors
w	vertical component of velocity
g	acceleration due to gravity defined by $\mathbf{g}$ equal to $-\nabla\phi$
q	specific humidity

Subscripts

H	horizontal
h	hydrostatic
i	system identification

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