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SHORT CONTRIBUTION

On entrainment observed in laboratory and field experiments

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ABSTRACT

The consistency between results from mixing experiments in two-layered fjords and the results in the laboratory is investigated. The field data are interpreted by using a model in which the interfacial shear stress is regarded as the source of turbulent energy. Using the definition of the interfacial shear stress $\tau_i = \rho_0 u_*^2 = \rho_0 c_i U^2$, where the drag coefficient c_i is experimentally determined, the field results are found to agree quite well with the results of the laboratory experiments characterized by values of the overall Richardson number Ri_* in the interval $90 < \mathrm{Ri}_* < 400$.

1. Introduction

In the last ten to fifteen years much effort has been devoted to explaining the phenomenon of shear induced entrainment. Most of the work done is based on laboratory experiments, for instance Kato and Phillips (1969) (called the KP experiment) and Kantha, Phillips and Azad (1977) (called the KPA experiment). These two sets of experiments were carried out using the same laboratory equipment.

The KP experiment started with a lineary stratified fluid and the following relation was obtained

$$\frac{u_e}{u_*} = 2.5\mathrm{Ri}_*^{-1}, \qquad (1)$$

where u_e is the entrainment velocity, $u_* = (\tau_0/\rho_0)^{1/2}$ is the friction velocity due to the surface stress τ_0, $\mathrm{Ri}_* = g' D/u_*^2$ is the overall Richardson number, g' being the reduced gravity force and D the depth of the upper layer.

The KPA experiment started with a fluid consisting of two homogeneous layers separated by a jump in density. For the same value of Ri_* a definite 100* higher entrainment rate was found compared to the KP experiment. This discrepancy has been subject to a number of studies, e.g. Phillips (1976), Kantha (1978), Price (1979) and Kitaigorodskii (1981).

Since laboratory experiments and theoretical work are usually made to explain things happening in nature, it is of great interest to compare the results obtained in the laboratory with results from field experiments.

2. Field experiments

The circulation in two-layered fjords is very much dependent on the vertical mixing across the pycnocline. In situ observations of the mixing across the pycnocline in two-layered fjords have been carried out by means of the dye diffusion technique developed by Kullenberg (1969, 1974).

The fjords investigated all belong to type 1 according to the classification given by Pickard (1961). These types of fjords consist of two nearly homogeneous layers separated by an abrupt change in density. The velocity is almost uniform in the upper layer with direction out of the fjord, while a weak compensation flow occurs in the lower layer.

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Buch (1981) assumed that the vertical mixing was generated solely by the interfacial shear stress

$$\tau_i = \rho_0 c_i U^2, \tag{2}$$

where c_i is the interfacial drag coefficient, U is the mean velocity of the upper layer, and a simple model for the interfacial shear stress induced mixing was developed by studying the turbulent transport in a thin layer of thickness dz at the pycnocline in a Cartesian coordinate system with the z-axis positive upward. The local value of the flux Richardson number at the interface is

$$\mathrm{Rf} = \frac{(g/\rho_0)\rho' w'}{-u' w' \dfrac{\mathrm{d}u}{\mathrm{d}z}} = \frac{-(g/\rho_0)K\dfrac{\mathrm{d}\rho}{\mathrm{d}z}}{\dfrac{\tau_i}{\rho_0}\dfrac{\mathrm{d}u}{\mathrm{d}z}} = \frac{KN^2}{\dfrac{\tau_i}{\rho_0}\dfrac{\mathrm{d}u}{\mathrm{d}z}}, \tag{3}$$

where it is assumed that the Reynolds stress equals the interfacial shear stress, and where the K-approximation is applied for expressing the buoyancy flux, K being the vertical turbulent diffusion coefficient and N the Brunt-Väisälä frequency.

Combining eqs. (2) and (3) yields:

$$K = \mathrm{Rf}\, c_i \left(\frac{U}{N}\right)^2 \frac{\mathrm{d}u}{\mathrm{d}z}. \tag{4}$$

This relation is consistent with previously found theoretical relations, e.g. Welander (1968) which can be seen by the following reasoning. The vertical momentum transfer coefficient K_m can be expressed

$$K_m = \frac{\mathrm{Ri}}{\mathrm{Rf}} K \tag{5}$$

where $\mathrm{Ri} = N^2(\mathrm{d}u/\mathrm{d}z)^{-2}$ is the gradient Richardson number. Inserting eq. (4) into eq. (5) reveals:

$$K_m = \frac{c_i U^2}{\mathrm{d}u/\mathrm{d}z} \Rightarrow \tag{6}$$

$$K_m \frac{\mathrm{d}u}{\mathrm{d}z} = \frac{\tau_i}{\rho_0} = c_i U^2. \tag{7}$$

It is also noted that eq. (4) is consistent with the relation

$$K = \beta \varepsilon N^{-2}, \tag{8}$$

where β is a constant and ε is the rate of dissipation. The latter is given by the energy flux from mean motion to turbulent motion, which is equal to

$$\frac{\tau_i}{\rho_0}\frac{\mathrm{d}u}{\mathrm{d}z}.$$

Buch (1981) found the factor $\mathrm{Rf}c_i$ to be $2.0 \cdot 10^{-4}$ by plotting observed values of K and $(U/N)^2\, \mathrm{d}u/\mathrm{d}z$ versus each other.

Various investigators have suggested that the value of Rf integrated over the upper layer in a subcritical fjord flow is 0.05, Stigebrandt (1976) and Pedersen (1980). In eq. (3) the local flux Richardson number is defined at the interface, i.e. in the most stable part of the water column. This implies that Rf here will approach its critical value, which is higher than the integrated one. The critical value of Rf has been found to vary in the range 0.10–0.20, e.g. Turner (1973), Kullenberg (1974) and here the value $\mathrm{Rf}_c = 0.10$ will be used, leading to:

$$c_i = 2.0 \cdot 10^{-3}. \tag{9}$$

The production of turbulent energy per unit time and area at the interface in a flow wide enough to neglect side-wall friction due to flow in the upper layer, may be given in the form:

$$E_{\mathrm{prod}} = \tau_i(U - u_i), \tag{10}$$

u_i being the velocity at the interface.

The rate of increase of potential energy due to the entrained mass in a water column of unit area can be expressed as, e.g. Turner (1973),

$$\frac{\mathrm{d}E_{\mathrm{pot}}}{\mathrm{d}t} = \tfrac{1}{2}\rho_0 g' u_e D. \tag{11}$$

Using the definition of the flux Richardson number together with eqs. (10) and (11):

$$\mathrm{Rf} \equiv \frac{\dfrac{\mathrm{d}E_{\mathrm{pot}}}{\mathrm{d}t}}{E_{\mathrm{prod}}} = \frac{\tfrac{1}{2}\rho_0 D g' u_e}{\tau_i(U - u_i)} \tag{12}$$

from which an entrainment function can be found:

$$\frac{u_e}{U} = 2\mathrm{Rf}c_i(1 - u_i/U)F_\Delta^2 = \lambda F_\Delta^2 \tag{13}$$

where

$$F_\Delta^2 = \frac{U^2}{g' D} \tag{14}$$

is the densimetric Froude number.

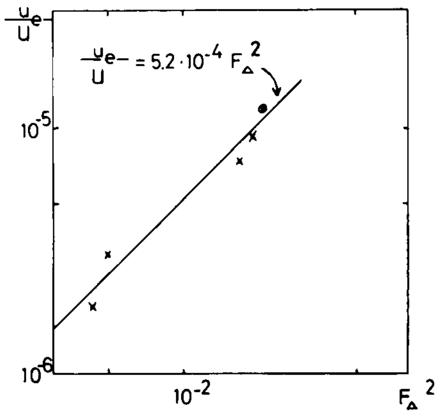

Fig. 1. The entrainment as a function of the densimetric Froude number, determined from field experiments.

In order to use eq. (13) for the interpretation of results from field experiments, it is necessary to express u_e in terms of parameters observed during the dye experiments. This has been done by Kullenberg (1977)

$$u_e = \frac{KN^2}{g'}. \tag{15}$$

Plotting u_e/U versus F_Δ^2 (Fig. 1) it is found that:

$$\frac{u_e}{U} = 5.2 \cdot 10^{-4} F_\Delta^2. \tag{16}$$

Although many dye experiments are carried out in several different areas, only a few fulfil the conditions assumed in the present study. Hence it is of importance to compare the results with the results given by appropriately intercomparable laboratory experiments.

3. Comparison of the KPA experiment and the field experiments

The KPA experiments revealed no distinct power law dependence on Ri_* and only in the range $90 < Ri_* < 400$, was u_e/u_* found to be proportional to Ri_*^{-1}, giving:

$$E_{KPA} = \frac{u_e}{u_*} = 6 Ri_*^{-1} \tag{17}$$

in qualitative agreement with the KP experiment.

In order to make a comparison between eq. (16) and eq. (17) the former equation is rewritten into a form equivalent to eq. (17):

$$\frac{u_e}{u_*} = \frac{u_e}{U}\frac{U}{u_*} = 5.2 \cdot 10^{-4} \frac{U^2}{g'D}\frac{U}{u_*} \Rightarrow \tag{18}$$

$$\frac{u_e}{u_*} = 5.2 \cdot 10^{-4} Ri_*^{-1} \left(\frac{U}{u_*}\right)^3. \tag{19}$$

To proceed further it is necessary to find an expression for U/u_* in terms of known parameters. In the laboratory experiments the turbulence was generated at the surface and the energy was partly dissipated and partly diffused down to the interface where it was used for entraining water from the lower layer. In the field experiments the turbulence was assumed to be generated at the interface and some of this energy was consumed for entrainment across the interface.

The interfacial shear stress may be written

$$\tau_i = \rho_0 u_*^2 = \rho_0 c_i U^2, \tag{20}$$

giving

$$\frac{U}{u_*} = c_i^{-1/2}, \tag{21}$$

which is equivalent to what Kitaigorodskii (1981) found in his theoretical analysis of the KP and KPA experiments.

Inserting eq. (21) into eq. (19) and using the experimentally obtained value of $c_i = 2.0 \cdot 10^{-3}$

$$\frac{u_e}{u_*} = 5.8 \cdot Ri_*^{-1}, \tag{22}$$

a result very near that obtained by KPA in the interval $90 < Ri_* < 400$, see Fig. 2.

Equation (13) was derived assuming that the turbulence was shear-generated and as seen in Fig. 1 the field experiments tend to corroborate this assumption.

The close resemblance between eqs. (22) and (17) consequently suggests that in the laboratory the turbulence was shear-generated in the experiments characterized by values of Ri_* in the interval $90 < Ri_* < 400$. The experiments with Ri_* values outside this interval are seen to diverge from the relation given in eq. (22).

For $Ri_* < 90$ the divergence is probably a real tendency since it is expected that the exponent of Ri_* will approach zero for $Ri_* \to 0$, also showed by the KP experiments.

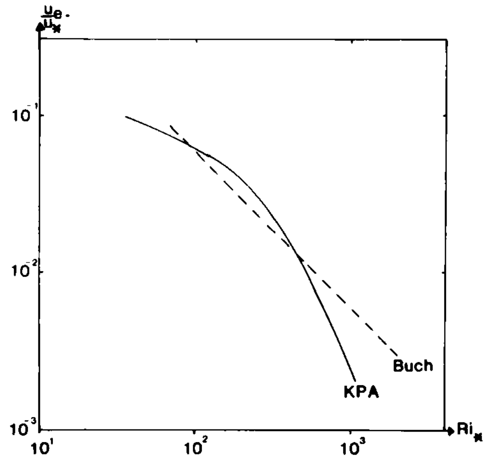

Fig. 2. The entrainment rate as a function of the overall Richardson number.

Pedersen (1980) argued that with the experimental set-up used in the KP and KPA experiments, molecular effects would highly influence those experiments with values of $Ri_* > 400$, a point also discussed by KPA themselves.

With no extraneous effects present in the laboratory experiments the entrainment velocity $u_e = dD/dt$ is constant, i.e. the depth of the mixed layer D is a linear function of time, and departure from linearity indicates the onset of unwanted sidewall, and other effects. As seen in Fig. 4 in the KPA paper, the sidewall effect becomes increasingly noticeable as Ri_* increases, and the methods used by KPA for reduction of the sidewall effects become critical at high values of Ri_* (see Figs. 7 and 8 in the KPA paper). Therefore insufficient compensation of the friction at the walls may be another possible explanation of the divergence from the Ri_*^{-1} power law for $Ri_* > 400$ in the KPA experiments.

4. Summary and conclusions

The entrainment rate in some Nordic fjords was estimated by means of *in situ* dye diffusion experiments, interpreted by means of an earlier described model (Buch, 1981).

The purpose of this note is to investigate the relation between the results of the field experiments and the laboratory investigations of Kantha, et al. (1977). Such a relation was found by applying the definition of the interfacial shear stress $\tau_i = \rho_0 u_*^2 = \rho_0 c_i U^2$ where the drag coefficient c_i is experimentally determined. For Ri_* outside the interval $90 < Ri_* < 400$, the results of the laboratory experiments diverge from the Ri_*^{-1} power law, which may be due to molecular effects, insufficient reduction of sidewall friction for $Ri_* > 400$ and the fact that the exponent of Ri_* approaches zero for $Ri_* \to 0$.

The relation between the laboratory and field experiments found in this paper is rather encouraging and may be of importance to future work on mixing and circulation in fjords, where appropriate mixing experiments are often difficult and expensive to carry out.

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