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On the relation between sea-surface heat flow and thermal circulation in the ocean

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ABSTRACT

A theoretical framework for the description of the thermal state and circulation in the ocean is presented. Relations between differential heating, diffusive heat flux and surface drift are derived and discussed in the light of the basic mechanical property of the system, i.e. the tendency for light water to spread on top of heavier water. We find that for low and medium temperatures, the poleward surface drift can be determined directly from a knowledge of the heat flux through the sea surface. Preliminary quantitative estimates indicate that the "Hadley circulation" for the entire ocean involves a volume flux of order 70 $\pm$ 30 Sverdrup. For the North Atlantic we find a poleward drift of order 10 Sverdrup which compares well with previous estimates of the southward deep flow. Mean values for the diffusive flux in the ocean are found to be of order 20 W m^{-2} at 15 °C and 60 W m^{-2} at 25 °C.

1. Introduction

The thermal circulation in the ocean is created by differential heating through the sea surface causing density differences which are acted upon by the gravitational field. Due to the thermal circulation, water is moved between high and low latitudes in a so-called "Hadley cell". An obvious feature of the circulation is that a water parcel has to change its temperature throughout the range of temperatures present in the ocean, i.e. water parcels experience a motion in "temperature space", which has to be coordinated with motions in physical space. This means that knowledge of the motion in "temperature space" can be translated into knowledge about the circulation in physical space. In the present paper these possibilities are explored.

The circulation has different "arms" which together form a closed circuit. Water will thus experience:

(i) Poleward motion in the surface layer accompanied by cooling.
(ii) Sinking in the poleward region and equatorward motion accompanied by some mixing of water masses with slightly different properties.
(iii) Motion towards the surface through the thermal stratification of the ocean, accompanied by heating through diffusion from above.

Any one of these arms may be studied, giving independent information about the strength and character of the thermal circulation. Existing studies are primarily concerned with the last part of arm (i), i.e. the final cooling before sinking, and (ii), i.e. the motion of cold water masses towards regions where the water will rise to the surface. For a review of our present knowledge based on such studies see Gordon (1975).

The present effort is primarily concerned with arm (i) above, i.e. the motion of water along the surface through the whole temperature range. We are thus studying that part of the circuit where the forcing of the whole circulation occurs. We try to link the strength and character of the circulation as directly as possible to the primary forcing caused by heat transfer through the ocean surface.

In this study it is just as interesting to trace how water is being transformed from say 9 to 8 °C in the surface layer as to examine the final transformation occurring before sinking. This means that we are concerned with the whole

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"Hadley" circulation, not only the part which traverses the very deepest parts of the ocean.

In Section 2 the theoretical framework is outlined. In Section 3 we discuss how diffusive fluxes and the thermally forced circulation are related to the heat supply at the surface of the ocean. In Section 4 we discuss the mechanism which keeps the real ocean in a steady state under the influence of differential heating, and the omnipresent tendency for light water to spread on top of heavier water. We also give some quantitative estimates of the thermal "Hadley" circulation in the world ocean and in the North Atlantic.

It is obvious that information regarding the thermal circulation of the type discussed in this paper can be used for the estimation of various geochemical fluxes e.g. the flux of carbon between the surface layer and the deep water. It is also believed that the theoretical framework outlined in Section 2 is a most suitable tool in the modelling of long-term fluctuations in the state of the atmosphere and ocean. Such possibilities have not yet been explored.

As outlined above we discuss in this paper the relation between thermal forcing at the sea surface and the thermal "Hadley" circulation. We thus claim that a relation exists which does not explicitly involve the strength and character of the mechanical forcing, e.g. the wind stress at the sea surface. This result, however, does not imply that mechanical forcing is unimportant for the thermal circulation. Changing wind conditions would create new conditions at the sea surface (e.g. a different temperature field) which would change the thermal forcing and thereby the thermal circulation. The new thermal circulation, however, would be related to the new thermal forcing in the same way as before.

2. Definitions and basic relations

We consider the thermal properties of a certain oceanic region. For this purpose we will define a number of functions which in condensed but precise form describe:

(i) the thermal state of the region;
(ii) thermal processes inside the region;
(iii) thermal interaction with the surroundings.

We will also derive relations between these functions which follow from heat and volume conservation.

2.1. *Definitions*

We consider a region R bounded by the ocean surface A, a control surface B facing adjacent oceanic regions, and the bottom. Furthermore we define:

$T_0(\mathbf{r}, t)$: Temperature at the point $\mathbf{r}$ at time t, $\mathbf{r}$ being inside R;
R_T: subregion to R in which $T_0(\mathbf{r}, t) < T$, T being a parameter which we will use as an independent variable;
A_T: subregion of the surface A on which $T_0(\mathbf{r}, t) < T$;
S_T: surface inside R on which $T_0(\mathbf{r}, t) = T$;
B_T: subregion of the control surface B on which $T_0(\mathbf{r}, t) < T$. B and B_T may consist of several disconnected parts B^ν and B_T^ν;
$V(T, t)$: volume of R_T;
$A(T, t)$: area of A_T;
$S(T, t)$: area of S_T;
$G(T, t)$: volume flux through S_T;
$H(T, t)$: heat flux through S_T;
$M(T, t)$: volume flux through B_T;
$Q(T, t)$: total heat flux through A_T including sensible, latent, and radiative heat transfer;
$E(T, t)$: volume flux through A_T.

All fluxes $G(T, t)$, $H(T, t)$ etc. are counted positive when directed into R_T.

Among these functions of (T, t) the first three $V(T, t)$, $A(T, t)$, $S(T, t)$ describe the state of R, while $G(T, t)$, $H(T, t)$ describe processes inside R, and $M(T, t)$, $E(T, t)$, $Q(T, t)$ describe the interaction of the region R with the surroundings.

The heat flux $H(T, t)$ may be considered as the result of an advective heat flux $c \cdot GT$ and a diffusive flux $F(T, t)$, c being the heat capacity per unit volume. We thus define $F(T, t)$ through the relation:

$$F(T, t) = H(T, t) - cGT. \qquad (2.1)$$

In the following we let $F(T, t)$ replace $H(T, t)$ as one of our main dependent variables. The definitions given above are illustrated in Fig. 1.

2.2. *Continuity of volume and heat*

In the ocean mass, continuity implies volume continuity to a very good approximation. Adopting

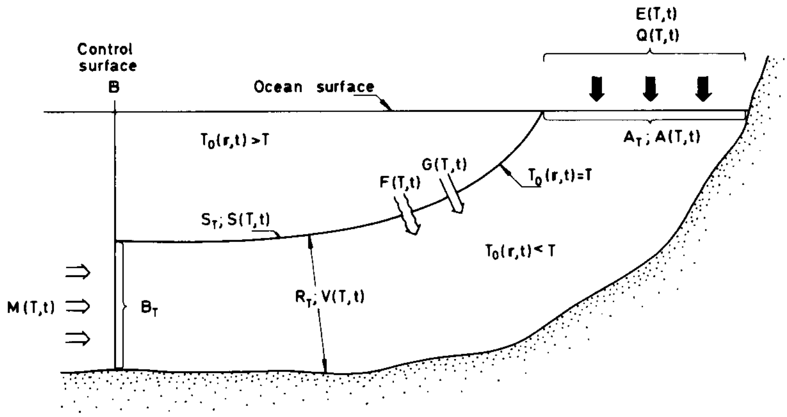

Fig. 1. Illustration of the definitions made in Section 2. R_T denotes the region with volume $V(T,t)$ in which the temperature $T_0(\mathbf{r},t)$ is lower than T etc., see text.

conservation of volume we thus obtain

$$\frac{\partial}{\partial t} V(T,t) = M(T,t) + E(T,t) + G(T,t). \tag{2.2}$$

From conservation of heat we obtain

$$c\frac{\partial}{\partial t}\int_{-\infty}^{T} vT\,\mathrm{d}T = c\int_{-\infty}^{T} (m+e)T\,\mathrm{d}T + cGT + F + Q, \tag{2.3}$$

where

$$v = \frac{\partial V}{\partial T}, \quad m = \frac{\partial M}{\partial T}, \quad e = \frac{\partial E}{\partial T}. \tag{2.4a,b,c}$$

The heat balance as given by (2.3) requires that the heat flux through B_T is given by

$$c\int_{-\infty}^{T} mT\,\mathrm{d}T,$$

i.e. that the diffusive flux through B vanishes. B should thus be chosen essentially perpendicular to the isotherms. Since we consider B as fixed in space, while the isotherm slope may be variable, this condition is not always strictly fulfilled.

The error we make in ignoring that the diffusive flux might have a component through the control surface B is entirely equivalent to an uncertainty in $F(T,t)$. We simply do not know precisely how much of the diffusive flux should be considered to be inside B, i.e. included in $F(T,t)$. Obviously this uncertainty becomes more important if mixing is particularly strong in the vicinity of the control surface B.

Differentiating (2.2) and (2.3) with respect to T we obtain

$$\frac{\partial v}{\partial t} = m + e + \frac{\partial G}{\partial T}, \tag{2.5}$$

$$cT\frac{\partial v}{\partial t} = c(m+e)T + cT\frac{\partial G}{\partial T} + cG + \frac{\partial F}{\partial T} + \frac{\partial Q}{\partial T}. \tag{2.6}$$

Multiplying (2.5) with cT and subtracting from (2.6) we obtain

$$cG = -\frac{\partial F}{\partial T} - \frac{\partial Q}{\partial T}. \tag{2.7}$$

Note that the time derivatives cancel in the derivation of (2.7). Eqs. (2.5), (2.6) and (2.7) express conservation of volume and heat for the infinitesimal volume $v(T,t)\,\mathrm{d}T$ in the neighbourhood of the isothermal surface S_T.

We consider next the volume $v'(T,t)\,\mathrm{d}T$ in the neighbourhood of S'_T, S'_T being an arbitrary part of S_T (see Fig. 2). We then obtain a corresponding set of equations. In particular we find the counterpart of (2.7).

$$cG' = -\frac{\partial F'}{\partial T} - \frac{\partial Q'}{\partial T}, \tag{2.8}$$

where G', $\partial F'/\partial T$ and $\partial Q'/\partial T$ are those parts of G, $\partial F/\partial T$ and $\partial Q/\partial T$ which can be assigned to S'_T.

We note that for the derivation of (2.8) to hold the control surfaces b_1 and b_2 should be chosen in such a way that the diffusive heat flux through

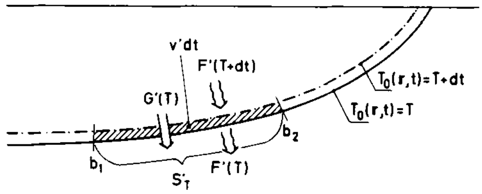

Fig. 2. Illustration of the subregions S'_T and $v'\,\mathrm{d}T$ used to prove that a local heat supply produces a local cross-isothermal volume flux. The heat supply may be from external sources $(\partial Q'/\partial T)\,\mathrm{d}T$ or from convergences in the field of diffusion $(\partial F'/\partial T)\mathrm{d}T$. The volume $v'\mathrm{d}T$ is represented by the shaded area. In the illustrated case $\partial Q'/\partial T = 0$ since Q' is localized to the ocean surface.

b_1 and b_2 vanishes, i.e. b_1 and b_2 should be essentially perpendicular to S_T.

(It is not, however, necessary that b_1 and b_2 are fixed in space. Eq. (2.8) is valid at all times and can be derived on the basis of the instantaneous location of S_T. This means that the diffusive flux through b_1 and b_2 can be assumed to vanish by definition. The interpretation of G' as obtained from (2.8), however, becomes increasingly arbitrary if the isotherm pattern changes quickly in time.)

Equation (2.8) tells us that the cross-isothermal volume flow $G(T,t)$ is located at that part of S_T where the diffusive accumulation of heat $\partial F/\partial T$ and the external heat supply $\partial Q/\partial T$ actually occur.

In the presentation given above we have excluded certain processes which may be of importance in specific processes, e.g. heat flux through the ocean bottom and heat and volume flux resulting from interaction with sea or land ice. It is possible, without difficulty, to include such processes in the formalism.

We have included the volume flux $E(T,t)$ through the ocean surface. The importance of this term is measured by the parameter $c\Delta T/l$, where l is the heat of evaporation per unit volume. In the discussion in Section 3 we will ignore the influence of the flux $E(T,t)$ because of the smallness of $c\Delta T/l$.

3. Application

The bulk of oceanographic observations concerns the state of the ocean, i.e. primarily the temperature and salinity fields. From ship observations there also exists an enormous amount of information on the conditions at the sea surface and lower atmosphere. This information can bc used to compute estimates of the functions $V(T,t)$, $A(T,t)$ and $Q(T,t)$.

On the other hand we have very poor observations, if any at all, which can be used for direct computation of the functions $G(T,t)$ and $F(T,t)$ (or $H(T,t)$) describing the flow of heat and volume in the ocean. Perhaps such observations will never be available.

Interaction as described by $M(T,t)$ takes on an intermediate position. From certain connexions between ocean basins we have rough approximations of the flow involved, but more often our knowledge is only qualitative.

The most direct use of the formalism of Section 2 concerns deducing information about what is grossly unknown, i.e. G, F and/or M from a knowledge of the state of the ocean, $V(T,t)$ and the forcing at the sea surface given by $Q(T,t)$, or its mean value in time $\bar{Q}(T)$.

3.1. *Computations of* $\bar{Q}(T)$

At the department of Oceanography in Gothenburg some efforts have been made to compute the function $Q(T,t)$. Part of the results are presented in Andersson et al. (1982). Computations for the North Atlantic have been presented previously (Andersson and Rudels, 1979).

The reliability of our computations for the North Atlantic is probably quite good being based on a recent heat flux computation by Bunker (1976). Our computations for the world ocean are based on data taken from the atlas prepared by Budyko (1963). The results in Andersson et al. (1982) show a troublesome discrepancy between $\bar{Q}(T)$ for the North Atlantic based on the Bunker data and Budyko's atlas. For estimates of $\bar{Q}(T)$ for the world ocean the best we can do at present is to use our results based on Budyko and correct them fully or partially in proportion to the difference obtained for the North Atlantic.

3.2. *Diffusive heat transport in world ocean*

Let us apply eqs. (2.2, 2.3) to the long term mean state of the world ocean. Noting that $\bar{M}(T)$ obviously vanishes (since there is no boundary B) and ignoring $\bar{E}(T)$ we obtain

$$\bar{G}(T) = 0, \tag{3.1}$$

$$\bar{F}(T) + \bar{Q}(T) = 0. \tag{3.2}$$

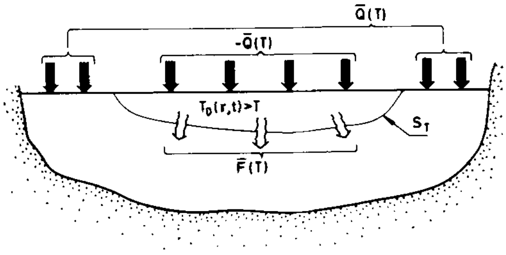

Fig. 3. Heat balance for the entire ocean. The mean volume flux $\bar{G}$ through S_T must vanish (ignoring net rainfall). The heat balance simplifies to $\bar{Q}(T) + \bar{F}(T) = 0$.

These equations are intuitively obvious (see Fig. 3). Nevertheless the information about $\bar{F}(T)$ that can be deduced from (3.2), if $\bar{Q}(T)$ were known, is far from trivial.

The strength in (3.2) stems from its directness. There is e.g. no arbitrary assumption regarding advective processes involved (which is typically the case when diffusive processes are estimated with conventional diagnostic procedures).

Our results regarding $\bar{Q}$ and $\bar{F}$ for the world ocean (see Fig. 9 and Andersson et al., 1982) can be summarized as follows. The heat loss from the ocean surface for temperatures below T, i.e. the diffusive flux through S_T, increases monotonically with temperature at a roughly constant rate up to a maximum value of order $(5\text{–}10)\cdot 10^{15}$ W at a temperature well above 20 °C. Our results for higher temperatures are probably less accurate. $\bar{F}(T)$ and $\bar{Q}(T)$, however, have to drop to zero at some temperature level close to 30 °C.

Since the area of S_T is well known, being for all practical purposes equal to $A(\infty) - A(T)$ (where $A(\infty)$ is the area of the whole ocean) we can use our result for $\bar{F}(T)$ to compute the flux $f(T)$ per unit area of $S(T)$. This function is of considerable interest in itself since it will expose e.g. the influence of the stability in the ocean on the diffusivity.

Let us illustrate with some numerical examples. We define κ and $f(T)$ from

$$\bar{F}(T) = \bar{S}(T) f(T) \simeq \bar{S}(T)\kappa \overline{\frac{\partial T_0}{\partial z}} c, \qquad (3.3)$$

where $\overline{\partial T_0/\partial z}$ is a typical mean value of the vertical temperature gradient. Our computations indicate

$$\bar{F}(15\,°\mathrm{C}) \simeq 5\cdot 10^{15}\ \mathrm{W}, \quad \bar{S}(15\,°\mathrm{C}) \simeq 2.5\cdot 10^{14}\ \mathrm{m}^2,$$

$$\bar{F}(25\,°\mathrm{C}) \simeq 8\cdot 10^{15}\ \mathrm{W}, \quad \bar{S}(25\,°\mathrm{C}) \simeq 1.4\cdot 10^{14}\ \mathrm{m}^2,$$

i.e. from (3.3) we have

$$f(15\,°\mathrm{C}) \simeq 20\ \mathrm{W\,m^{-2}}, \qquad f(25\,°\mathrm{C}) \simeq 60\ \mathrm{W\,m^{-2}}.$$

Furthermore let us take

$$c \simeq 4.2\cdot 10^6\ \mathrm{W\,s\,m^{-3}\,°C^{-1}},$$

$$\left(\frac{\partial T_0}{\partial z}\right)_{15\,°\mathrm{C}} \simeq 0.25\cdot 10^{-1}\ °\mathrm{C\,m^{-1}}.$$

We then obtain

$$\kappa \simeq 2\cdot 10^{-4}\ \mathrm{m^2\,s^{-1}}.$$

3.3. *Thermal conditions on surface drift*

Let us now consider a system like the North Atlantic as illustrated in Fig. 4. The control surface B is essentially vertical and traverses the ocean surface where the temperature is high. Water with lower temperature meets the surface inside the region and communicates through B at depth only. The combined action of gravity and thermal forcing at the surface will keep our system in circulation. Our concern now is to find out how this circulation may be related to the thermal forcing.

Let us assume that diffusive fluxes in the system may be ignored, i.e. that

$$F(T,t) = 0. \qquad (3.8)$$

We then have from eq. (2.7)

$$cG(T,t) = -\frac{\partial}{\partial T} Q(T,t). \qquad (3.9)$$

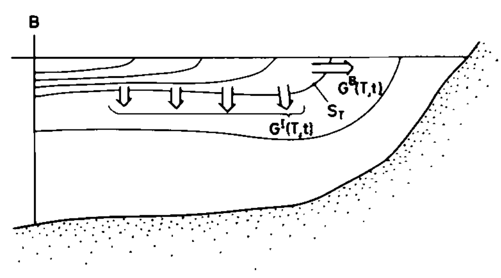

Fig. 4. Illustration of the separation of $G(T,t)$ into $G^I(T,t)$ and $G^B(T,t)$.

Furthermore from (2.2) and (3.9) we have

$$c\bar{M}(T) = \frac{\partial}{\partial T}\bar{Q}(T). \tag{3.10}$$

Note that (3.9) is valid at all times while (3.10) is only valid at long mean time values. We conclude that the forcing as given by Q creates a cross-isothermal volume flux given by (3.9).

Our next concern is where this volume flux takes place. We refer to our general rule expressed by (2.8) that the cross-isothermal flux is located at the same place as the heating which creates it. The physics is exceedingly simple. Water in the temperature range $(T, T + \mathrm{d}T)$ is given a supply of heat amounting to $(\partial Q/\partial T)\mathrm{d}T$. Water in this temperature interval thus becomes warmer (if $\partial Q/\partial T > 0$) and "moves" across the isotherms. Quite naturally this "motion" will occur at the surface where the heating takes place. With the notation G^{B} and G^{I} for flow at the surface and elsewhere respectively we thus have

$$cG^{\mathrm{B}}(T,t) = -\frac{\partial}{\partial T}Q(T,t). \tag{3.11}$$

Equations (3.10) and (3.11) thus tell us that the thermal forcing Q will create a general thermal circulation consisting of a cross-isothermal surface drift controlled by (3.11) and an outflow through B, the mean strength of which is given by (3.10) (see Fig. 5).

Next we will consider how this simple picture is modified by diffusive fluxes. Assuming for the moment $Q = 0$, we obtain

$$cG = -\frac{\partial}{\partial T}F. \tag{3.12}$$

It is important to notice that the cross-isothermal flow depends on the variation of F with T. Physically this is because the flow G requires a net heat supply to the temperature interval $(T, T + \mathrm{d}T)$ which in turn requires that $F(T) \neq F(T + \mathrm{d}T)$, i.e. that $\partial F/\partial T \neq 0$. Our task is thus to find out how $\partial F/\partial T$ is distributed over S_T.

Let us assume to begin with that F is distributed evenly over the surface S_T with intensity $f(T,t)$ per unit area, i.e. that F may be written

$$F(T,t) = S(T,t)f(T,t). \tag{3.13}$$

We then find

$$\frac{\partial}{\partial T}F = S\frac{\partial}{\partial T}f + f\frac{\partial}{\partial T}S. \tag{3.14}$$

We now claim that the volume flux associated with the first term is distributed evenly over S_T while the latter term should be interpreted as a surface flux, i.e. that

$$cG^{\mathrm{I}}(T,t) \simeq -S(T,t)\frac{\partial}{\partial T}f(T,t), \tag{3.15a}$$

$$cG^{\mathrm{B}}(T,t) \simeq -f(T,t)\frac{\partial}{\partial T}S(T,t). \tag{3.15b}$$

To see this we refer to Fig. 6. If we apply eq. (2.8) to the subrange S'_T in Fig. 6 it is clear that the second term in (3.14) will essentially drop out since neighbouring isotherms inside S'_T have essentially the same area. We thus have

$$cG'(T,t) \simeq -S(T,t)\frac{\partial}{\partial T}f(T,t). \tag{3.16a}$$

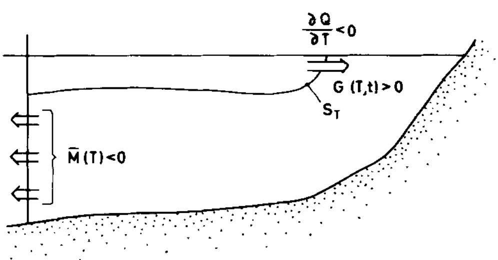

Fig. 5. The flow generated by surface heat flux assuming $F(T,t) = 0$ and $\partial Q/\partial T < 0$, which corresponds to heat loss at temperatures around $T_0(\mathbf{r},t) = T$. Note that $G^{\mathrm{B}}(T,t)$ is controlled instantaneously by $\partial Q(T,t)/\partial T$ while the control on $M(T,t)$ is only on the mean flow $\bar{M}(T)$. The cross-isothermal flux $G^{\mathrm{I}}(T,t)$ vanishes since $F(T) = 0$.

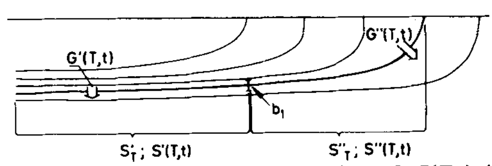

Fig. 6. Illustration of the separation of $G(T,t)$ into $G'(T,t)$ and $G''(T,t)$. The control surface b_1, is chosen in such a way that for temperatures around T we have $\partial S/\partial T \simeq \partial S''/\partial T$, and $S \simeq S'$. The main part of the area of S_T is within S'_T while the main part of the variation with T of the area of S_T is within S''_T.

Considering instead the subregion S_T'', we find that the first term in (3.14) is small since the area of S_T'' is much smaller than that of S_T', i.e. we have

$$cG''(T,t) \simeq -f(T,t)\frac{\partial}{\partial T}S(T,t). \tag{3.16b}$$

Identifying G' with G^{I} and G'' with G^{B} we find the postulated equations (3.15a,b).

It may be pointed out that the surface flux $-f\,\partial S/\partial T$ occurring in this case is entirely analogous to the boundary layer flux which occurs in a stratified fluid along a sloping insulated boundary (see Walin, 1971). The physics behind the surface flux $-f\,\partial S/\partial T$ is illustrated in Fig. 7.

Next we consider the possibility that the local intensity of $F(T,t)$ is much larger in the vicinity of the sea surface. as illustrated in Fig. 8. It is easily recognized that this will give a contribution $-c^{-1}\,\partial F^{\mathrm{s}}/\partial T$ to the surface drift, where F^{s} is the excess diffusive heat flux close to the ocean surface. This flux is of the same order as $-c^{-1}S\,\partial f/\partial T$ or $-c^{-1}f\,\partial S/\partial T$ only if $F^{\mathrm{s}} \sim F$. Note that a locally high or even very high mixing intensity near the surface does not imply that F^{s} is comparable with the total diffusive flux traversing the whole isothermal surface S_T. We assume here that $F^{\mathrm{s}} \ll F$, i.e. that $-c^{-1}\,\partial F^{\mathrm{s}}/\partial T$ may be ignored compared with $-c^{-1}f\,\partial S/\partial T$.

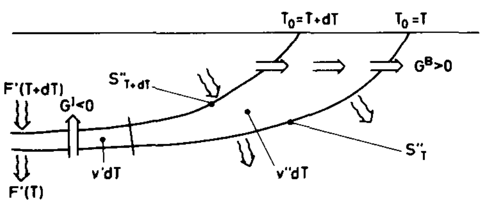

Fig. 7. The generation of surface drift by diffusion. Cross-isothermal flow requires net supply of heat to the infinitesimal volumes $v'\,\mathrm{d}T$ and $v''\,\mathrm{d}T$. Heat supply to the volume $v'\,\mathrm{d}T$ depends on $F'(T + \mathrm{d}T) - F'(T)$, which if $\partial F'/\partial T > 0$ generates "upwards" flow $G^{\mathrm{I}} < 0$. The volume $v''\,\mathrm{d}T$ will experience a heat loss since $S''(T) > S''(T + \mathrm{d}T)$. This heat loss generates a volume flux towards lower temperatures i.e. $G^{\mathrm{B}} > 0$.

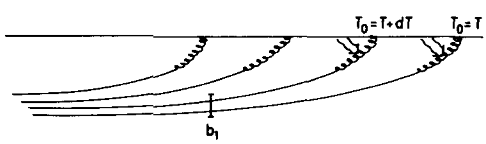

Fig. 8. Illustration of a near surface concentration of diffusive flux $F^{\mathrm{s}}(T,t)$. Generation of cross-isothermal flow depends on $\partial F^{\mathrm{s}}/\partial T$ which may be positive or negative. This contribution can compete with the term $-f\,\partial S/\partial T$ (see Fig. 7) only if $F^{\mathrm{s}}(T,t)$ is comparable with the total diffusive flux $F(T,t)$

It should also be recognized that in the evaluation of the term $-f\,\partial S/\partial T$ in (3.14), f should be interpreted as the mean value from the whole area of S_T, not as the local intensity of mixing near the ocean surface. (Intuitively one might expect that since $-f\,\partial S/\partial T$ represents a surface flux, f should be interpreted as the intensity at that part of S_T where the flux occurs, which is not correct.)

Summarizing the contributions to the cross-isothermal flux from surface heat flux and diffusive activity in the ocean, we have

$$cG^{\mathrm{I}}(T,t) \simeq -S(T,t)\frac{\partial}{\partial T}f(T,t), \tag{3.17a}$$

$$cG^{\mathrm{B}}(T,t) \simeq -f(T,t)\frac{\partial}{\partial T}S(T,t) - \frac{\partial}{\partial T}Q(T,t), \tag{3.17b}$$

where $f(T,t)$ is defined by

$$F(T,t) = S(T,t)f(T,t). \tag{3.17c}$$

4. On the thermal ocean circulation

In this section we will discuss some qualitative aspects of the thermal circulation which follow directly from Section 3. We will also present some quantitative estimates of the volume and heat fluxes involved which are based on our computations presented in Andersson et al. (1982).

In Fig. 9 the principal features of $Q(T)$ and $S(T)$ for the world ocean are illustrated. Let us now discuss the connexion between the surface drift $\bar{G}^{\mathrm{B}}$ and the properties of $\bar{Q}(T)$ and $\bar{S}(T)$.

Letting T_{m} be the temperature level at which $\partial\bar{Q}/\partial T$ changes sign (see Fig. 9), we find from the definition of $f(T)$ and the properties of $\bar{Q}(T)$ that

$$\frac{\partial}{\partial T}\bar{Q} \lesseqgtr 0 \quad \text{for } T \lesseqgtr T_{\mathrm{m}}, \tag{4.1a}$$

$$f\frac{\partial}{\partial T}\bar{S} < 0 \quad \text{for all } T, \tag{4.1b}$$

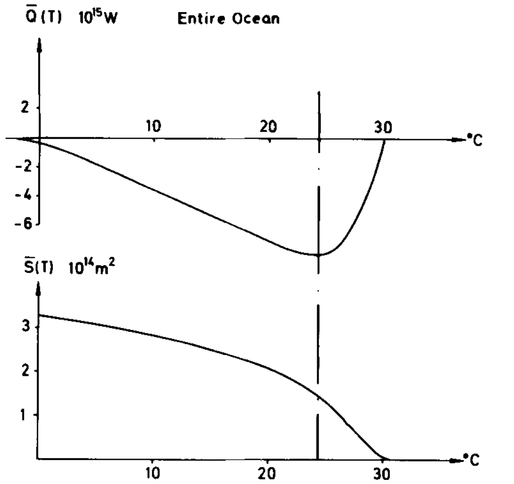

Fig. 9. The qualitative behaviour of $\bar{Q}(T)$ and $\bar{S}(T)$ for the world ocean, $\bar{Q}(T)$ being the heat supply to the ocean at temperatures below T, and $S(T)$ the area of the isothermal surface S_T. $|\bar{Q}|$ has been reduced by roughly 30% relative to our results based on Budyko's atlas. The reason is that our calculations for the North Atlantic indicate that our results based on Budyko are systematically too high.

i.e. from (3.17b)

$$G^B > 0 \quad \text{for } T < T_m \tag{4.2a}$$

$$G^B > 0 \quad \text{for } T > T_m \quad \text{only if} \left| f\frac{\partial}{\partial T}\bar{S} \right| > \left| \frac{\partial}{\partial T}\bar{Q} \right|. \tag{4.2b}$$

The surface drift $\bar{G}^B$ has to be compatible with mechanical laws, which essentially tells us that light water tends to spread on top of heavier water, i.e. we expect

$$\bar{G}^B > 0, \quad \text{for all } T. \tag{4.3}$$

From the very rough estimates of $\bar{Q}(T)$ and $\bar{S}(T)$ given in Fig. 9 we obtain for the entire ocean

$$\frac{\partial}{\partial T}\bar{Q} \simeq 1.4 \cdot 10^{15}\ \mathrm{W\ °C^{-1}} \quad \text{for } T \simeq 27\ °\mathrm{C}, \tag{4.4a}$$

$$\frac{\partial}{\partial T}\bar{Q} \simeq -0.35 \cdot 10^{15}\ \mathrm{W\ °C^{-1}} \quad \text{for } T \simeq 15\ °\mathrm{C}, \tag{4.4b}$$

$$f\frac{\partial}{\partial T}\bar{S} \simeq -2.1 \cdot 10^{15}\ \mathrm{W\ °C^{-1}} \quad \text{for } T \simeq 27\ °\mathrm{C}, \tag{4.4c}$$

$$f\frac{\partial}{\partial T}\bar{S} \simeq -0.13 \cdot 10^{15}\ \mathrm{W\ °C^{-1}} \quad \text{for T} \simeq 15\ °\mathrm{C}. \tag{4.4d}$$

Furthermore $\partial\bar{Q}/\partial T$ is essentially constant in the range $0\ °\mathrm{C} < T < 20\ °\mathrm{C}$ while the importance of $f\ \partial\bar{S}/\partial T$ lessens with decreasing temperature.

From eqs. (4.1–4.4) we make the following interpretation (see Fig. 10). From mechanical laws it follows that surface water will drift polewards on the surface in the whole temperature range. For medium and low temperatures the physical picture is very simple. The water is simply cooled through the ocean surface while moving polewards. The mixing term $-f\ \partial S/\partial T$ is of no vital importance for the system in the temperature range $T < T_m$.

In the high temperature range $T > T_m$, the situation is completely different. Both terms in (3.17b) are much larger than for $T < T_m$ and tend to cancel each other. Furthermore the term $-f\ \partial\bar{S}/\partial T$ is absolutely vital for the system, i.e. to give the surface drift the right sign.

Suppose e.g. that for some reason the term $-f\ \partial\bar{S}/\partial T$ in (3.17b) is too small to overcome the term $-\partial\bar{Q}/\partial T$ in the range where $\partial\bar{Q}/\partial T > 0$. The mechanically forced surface drift will then spread the warm water out to larger areas. This will increase the spacing between isotherms, i.e. $\partial S/\partial T$, until $f\ \partial S/\partial T$ has been raised to an appropriate level for a steady state to exist.

The surface drift $\bar{G}^B$ is of course closely linked to the so-called Hadley circulation in the ocean. We have to recognize that all of this circulation does not penetrate to the deepest parts of the ocean. It is generally hard to evaluate from $\bar{G}^B(T)$ whether water is leaving the surface at say $+1\ °\mathrm{C}$ or $-1\ °\mathrm{C}$, since this depends on the details of the shape of $\bar{Q}(T)$ close to $T \sim 0\ °\mathrm{C}$.

Our result that $-f\ \partial\bar{S}/\partial T$ is less important or even negligible for temperatures well below T_m implies that the volume flux involved in the Hadley circulation can be estimated directly from $\partial\bar{Q}/\partial T$ at say $T \lesssim 15\ °\mathrm{C}$.

From the rough estimate of $\bar{Q}(T)$ in Fig. 9 we obtain for the entire ocean

$$\bar{G}^B \simeq -\frac{1}{c}\frac{\partial}{\partial T}\bar{Q} \simeq (70 \pm 30) \cdot 10^6\ \mathrm{m^3\ s^{-1}}$$

$$\text{at } T \lesssim 15\ °\mathrm{C}. \tag{4.5}$$

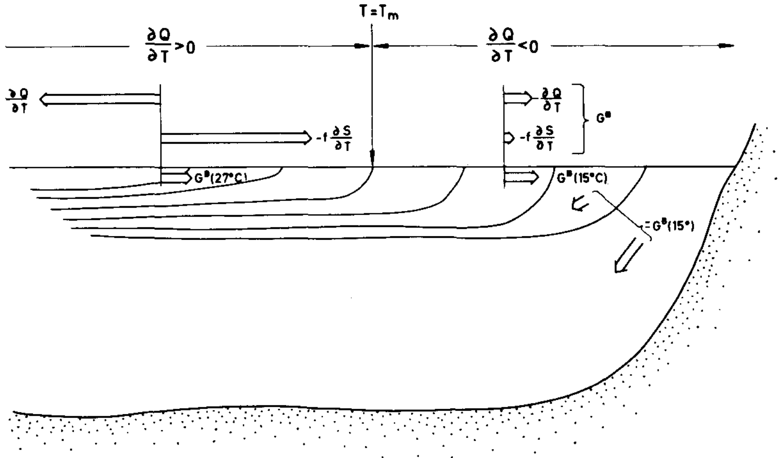

Fig. 10. Illustration of the thermal control of the surface drift. Mechanical forces induce a "poleward" drift at every temperature level which implies that $c\bar{G}^B > 0$ for all T. Note the importance of the term $-f\ \partial\bar{S}/\partial T$ in the region of heat supply ($\partial\bar{Q}/\partial T > 0$).

Our estimate for the North Atlantic based on results in Andersson et al. (1982) becomes

$$\bar{G}^B \simeq 10 \cdot 10^6\,\mathrm{m^3\,s^{-1}} \quad \text{at } T \lesssim 15\,°\mathrm{C}. \tag{4.6}$$

This result is well in accordance with estimates of the southward deep flow in the North Atlantic reviewed by Gordon (1975). In the North Atlantic it is thus appealing to identify the surface drift as given by (3.17b) with the deep water formation occurring in the Greenland area.

We note (see Andersson et al., 1982) that $\bar{Q}(T)$ has roughly the same shape and amplitude for the North Pacific as for the North Atlantic. We thus have a surface drift $\bar{G}^B$ of the same order of magnitude in the two basins despite the lack of deep water formation in the Pacific. To understand this, we recall that the surface drift $\bar{G}^B$ is a mechanical necessity forced by gravity acting on the meridional temperature gradient. Since the temperature field is qualitatively similar in the Pacific and the Atlantic, we should expect $\bar{G}^B$ to be of the same order of magnitude in the two basins. In the North Pacific, a small part of this cool water formation is leaving through the Bering straits: $1.5 \cdot 10^6$ m^3 s^{-1} according to Stigebrandt (1981). The major part of the flow will create a Hadley circulation which does not penetrate to the bottom layers in the Pacific. In order to close the Hadley cell we thus expect a southward flow in the North Pacific at an intermediate depth with approximate magnitude $10 \cdot 10^6$ m^3 s^{-1}. Whether such a flow is compatible with observations is presently not known.

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