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Storm track representation and verification

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ABSTRACT

A vortex representation of highs and lows is presented which mathematically describes individual local features by a small number of parameters representing magnitude, location, size, and shape. Parameter values for a representation of a particular data set are determined by minimizing the integral of the squared difference between the mathematical representation and the data set. Various objective verification measures can be based on these parameters by fitting vortex representations to both forecasts and corresponding verifying analyses. The differences in values of the parameters provide measures of errors in magnitude, location, size, and shape of individual features. An example applying this representation to a 10-day forecast by ECMWF and to the corresponding NMC verifying analysis is discussed. Representations of storm tracks in the forecast and analysis are defined by mapping the envelope of individual features with time, i.e. by determining the volume swept out by the storm. Such a two-dimensional horizontal field or map indicates the path of each storm as well as its strength and size along its track during the period included in the mapping. The usefulness of such mapping as a forecast product as well as a means for verification is illustrated.

1. Introduction

Subjective examination of numerical forecasts often indicates that the paths of disturbances are better forecast than the actual instantaneous positions. Because of truncation errors, forecast cyclones can lag behind their observed positions to the point where they are 180° out of phase. The instantaneous forecast at this time is useless, giving exactly the opposite conditions to reality. Yet when one follows these cyclones with time they often seem to move along the correct path in the forecast.

A more objective indication that storm tracks might be more predictable than instantaneous positions is provided by Druyan et al. (1975) who found some skill in time averages of medium-range forecasts (from day 7–14) when individual cyclones were no longer predictable. Such a time-averaging procedure removes the instantaneous shape aspects of the faster-moving, more local features, leaving some indication of their paths. However, in doing so, it obscures important details about the cyclones such as depth at various points along the path. In addition, in the vicinity of storm tracks the time average is highly dependent on the translation speed of the systems as well as the period of the average. Thus, by itself, it does not provide useful information about the extremes that could be expected in the period covered.

As suggested above, the paths of cyclones might be more predictable than their instantaneous positions. Similarly, changes in some characteristics of the cyclones such as size or intensity might be predicted at the proper geographical location along the path even if the exact timing is unpredictable. For example, a model may deepen lows when they move off the east coast of the U.S. properly but again at the wrong time because the forecast low moved to that position at the wrong time. A procedure to isolate certain spatial characteristics of the forecast and to eliminate certain aspects of its temporal variation would allow easier examination of these local geographical aspects. With such a procedure we might be able to establish whether some of these geographical aspects are predictable for longer periods.

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Conventional global verification measures such as r.m.s. error or S_1 skill score do not isolate the local nature of cyclones. Such scores include all scales with an implicitly assumed scale-dependent weighting. Some indication of the contribution from various scales to the total error is provided by computing these skill scores over the fraction of the field included in specified spectral wavenumber ranges (Miyakoda et al., 1972) or defined with band-pass filters (Bettge and Baumhefner, 1980). However, when the errors become relatively large, the phase-amplitude coupling can confuse the meaning of these scores. For examinple, if a wave is out of phase by 180°, an improvement in amplitude with no corresponding change in phase can produce a larger r.m.s. error even though one aspect of the forecast is improved. This phase-amplitude coupling problem can be circumvented by computing the phase and amplitude contributions separately (Williamson, 1978). Errors in individual cyclones are difficult to relate to such wavenumber separations. In fact, an individual cyclone has a Fourier decomposition with amplitudes in all scales including the largest global waves. Therefore, an error in an individual low will be expressed as an error in all Fourier components.

A representation of individual synoptic systems such as cyclones and troughs is needed to verify these individual systems. Such a representation separates local features from a remaining global scale background field and can be used as a basis of verification of these features at selected times. In addition, once important properties of individual features are identified and quantified for a sequence of times, the temporal aspects of these properties can be isolated and examined in different ways.

A few attempts have been made in the past to define a separation into local and global scales. These attempts involved applying spatial filters to the original complete field to define a global scale field. The local perturbation field is obtained by subtracting this global scale field from the original complete field. Wolff (1955) specified a global component by averaging the 500 mb height field over the edge of 20° latitude–longitude boxes. This average value defines the global component at the grid point at the center of the box. Holl (1963) developed an inherent-scale technique which defines a global scale field to be that which would essentially result if the original complete field were subject to diffusion at a specified rate for a specified length of time. The degree of smoothing is determined by arbitrary parameters related to the rate and time. Holl does point out one problem with these two approaches where intense vortices appear flattened out in the global scale component and the complementary or remaining portion appears in the disturbance field. Thus, many features are not assigned completely to one or the other component but rather are divided unsatisfactorily between them so that troughs in the global scale field coincide with lows in the disturbance field, and similarly for ridges and highs. In addition, these approaches do not identify and isolate important properties of individual storms. The opposite approach of isolating and representing individual features directly avoids many of these problems. Such a vortex representation is described below. Its application to storm track representation and verification is then discussed.

2. Vortex representation

A vortex representation for atmospheric cyclones isolates individual features at a given time so that each one may be examined individually and followed in time separate from the remainder of the flow. In the terminology of pattern recognition, this representation is a method of feature extraction so that individual features are characterized by a small number of parameters. Since these parameters may serve as a basis of verification, it is desirable that they represent aspects which have physical meaning to meteorologists such as magnitude, location, size, and shape of highs and lows.

In order to quantify these parameters, individual features are represented mathematically. An individual high or low is characterized as an amplitude A times some radial function $g(r)$

$$\text{Feature} \simeq Ag(r)$$

where the radial function g is one at the center of the feature ($r = 0$) and zero when the radius, measured from the center of the feature, is greater than some size parameter R.

The center of the feature is denoted by (x_0, y_0) where x and y are the horizontal coordinates. Since highs and lows are not perfectly circular, an elliptical shape is assumed for the features with two additional parameters to represent the eccentricity and orientation of the ellipse. The radial measure is then given by

$$r = \{[(x - x_0)\cos\alpha + (y - y_0)\sin\alpha]^2 + \varepsilon[-(x - x_0)\sin\alpha + (y - y_0)\cos\alpha]^2\}^{1/2}.$$

The angle between the x axis and the major axis of the ellipse is α measured counterclockwise from the positive x axis and the ratio of the length of the semimajor and semiminor axes is $\sqrt{\varepsilon}$. In order to have a unique specification, we require $\varepsilon \geq 1$ and $0 \leq \alpha < \pi$. The size or radius parameter R is the distance along the major axis of the ellipse at which the feature becomes zero. The radial function is given the following form

$$g(r) = \begin{cases} \frac{1}{2}(1 + \cos(\pi r/R)) & \text{for } r \leq R \\ 0 & \text{for } r \geq R. \end{cases}$$

which is continuous and has a continuous first derivative. The choice of the functional form is arbitrary; however, it is convenient for it to be a smooth function of x and y.

With the functional forms above, each feature is determined by six parameters representing amplitude, size, center, eccentricity, and orientation (A, R, x_0, y_0, ε, α) and is also a function of the spatial coordinates x and y. The ith feature is denoted by

$$f_i(x, y; A_i, R_i, x_{0i}, y_{0i}, \varepsilon_i, \alpha_i)$$

and the problem is to find the parameter values (A_i, R_i, x_{0i}, y_{0i}, ε_i, α_i) which give the best fit to a particular map. These parameters are determined by minimizing the squared difference between the observed field and the field as specified by the features

$$\min \sum_x \sum_y \left[\sum_i f_i(x, y; A_i, R_i, x_{0i}, y_{0i}, \varepsilon_i, \alpha_i) - h(x, y)\right]^2$$

where $h(x, y)$ is the observed field. More than one feature is included in the expression because the functions defining the features are not orthogonal when summed over x, y space and hence the minimization must be done over all parameters of all features at once. Highs and lows can overlap and partially cancel each other. In fact, if the parameters defining a high and a low are identical except with opposite sign for the amplitude they cancel exactly. To prevent such cancellation, we fit all lows at once to the total field without including any highs in the representation and if highs are also desired we then separately fit all highs at once to the total field without including any lows rather than fitting both highs and lows simultaneously. A complete representation of both highs and lows is then given by the combination of the separate fits of the lows and of the highs.

In the examples described in this report, the discrete grid is taken to be the National Meteorological Center (NMC) octagon on a stereographic projection with Cartesian coordinates x, y. The sum over x, y space for the minimization is taken over the uniform grid on the projection and is weighted by the area of the sphere represented by each grid point.

Except for the amplitude, the parameters defining the features all enter the expression to be minimized nonlinearly. Thus we have a non-linear least-squares problem to find $6n$ parameters where n is the number of features included. The "Adaptive nonlinear least-squares algorithm" of Dennis, Gay, and Welsch (1977) is used to solve the minimization problem. Their subroutine NL2SOL based on this algorithm is distributed by International Mathematical and Statistical Libraries, Inc. This procedure requires as input a first guess of the parameters (A_i, R_i, x_{0i}, y_{0i}, ε_i, α_i) for each feature included in the representation. If an interactive graphics terminal is available the first guess can easily be determined by indicating a few points with a light pen or cursor on a plot of the field to be represented. First the center of a feature is indicated, providing x_0 and y_0. The amplitude A is taken as the value of the field interpolated to this point. Next, the point along the semimajor axis of the ellipse where the feature becomes 0 is indicated. The radius R is determined as the distance along a line from the center to this point and the angle α can be computed from the angle this semimajor axis line makes with the horizontal axis. The graphics terminal can then determine and plot the line through the center perpendicular to the semimajor axis. The point on this line where the feature becomes zero is indicated by the operator and the length of the semiminor axis is calculated as the distance from this point to the center giving ε. If a vortex representation is being determined for a time sequence of fields an adequate first guess is provided by the fit of the field 12 hours earlier. Occasionally a feature must be added in the manner described above or deleted.

All general methods to solve the nonlinear optimization problem described here find a local rather than a global minimum. Thus a solution is not necessarily unique. In addition, as pointed out above, the features are not orthogonal, so that the parameters found for any one feature will be influenced by those of other features overlapping it and if additional features are included, the parameters can change. The choice of the functional form in the representation is arbitrary and has not been based on physical principles. Nevertheless, this type of representation could be extremely useful for many applications and can only be evaluated by actual application to atmospheric data and examination of the properties of the parameters defining a series of fitted states.

An example applying this representation to the NMC 500 mb height analysis of 00Z November 29, 1970 is discussed in Williamson (1979a). A second example applying this representation to a single 24 hr NMC 500 mb height forecast valid 00Z December 3, 1970 and its corresponding verifying analysis is given in Williamson (1979b). The latter example illustrates the nature of the parameters used to define the features and shows how the parameters themselves can be used as a basis of forecast verification. A string of three lows across the Atlantic is singled out for examination. The parameters indicate that in the forecast the amplitudes of two of the lows are 50% too small while the third one is 33% too small. The latitudes of the centers are nearly correct, but the longitudes in the forecast for all three are 5° too far west indicating a translation speed error. Thus amplitude and phase errors of individual systems are easily determined from the parameters and can be combined to give average amplitude and phase errors of the forecast.

3. Representation of a medium-range forecast

In the following, we are primarily interested in the time evolution of the parameters and in the use of the representation as a basis for storm track representation. We consider a 10-day forecast made by the European Centre for Medium Range Weather Forecasts (ECMWF) starting 00Z January 16, 1979 and the corresponding verifying NMC analysis and concentrate on only the 500 mb height field. This is the forecast period discussed by Bengtsson (1981) in his investigation of the predictability of atmospheric blocking, concentrating on the eastern Atlantic region. In the following we concentrate on the paths of cyclones in the eastern Pacific, across the U.S. and out into the western Atlantic.

We first illustrate the vortex representation for the initial data. The upper left panel of Fig. 1 shows the 500 mb NMC height analysis for 00Z January 16, 1979 and the upper right this 500 mb height analysis minus its zonal average. This deviation field is seen to be a collection of highs and lows and as such serves as a basis for the vortex representation. The lower right panel shows the vortex representation of the deviation field and the lower left the difference between the vortex representation and the original field. The contour interval of 50 meters is the same in all figures presented in this report. The difference field consists mainly of the zero contour meandering through the domain with a few nonzero contours indicated by a plus or minus enclosing very small areas. The zero contour indicates the dominant scale of the difference field. This representation consists of 15 lows and 13 highs and the variance of the difference field is 1.5% of the variance of the original NMC analysis.

Fig. 2 shows the vortex representation of the initial data for the ECMWF forecast in the upper right and the difference between the vortex representation and the original ECMWF data in the upper left. Again, the difference field is seen to consist primarily of a zero contour line with a few 50 m contour lines indicated by plus or minus. The lower right panel shows the individual features of the representation superimposed (not summed) over each other and the lower left just the lows for clarity. Each feature is arbitrarily assigned an index for identification. The index is not necessarily the same for a low in the ECMWF forecast and a corresponding low in the NMC analysis.

Fig. 3 shows a longitudinal spectral decomposition of these fields at 40° and 60° latitude. The vortex representation given by the combination of highs and lows provides a complete description of

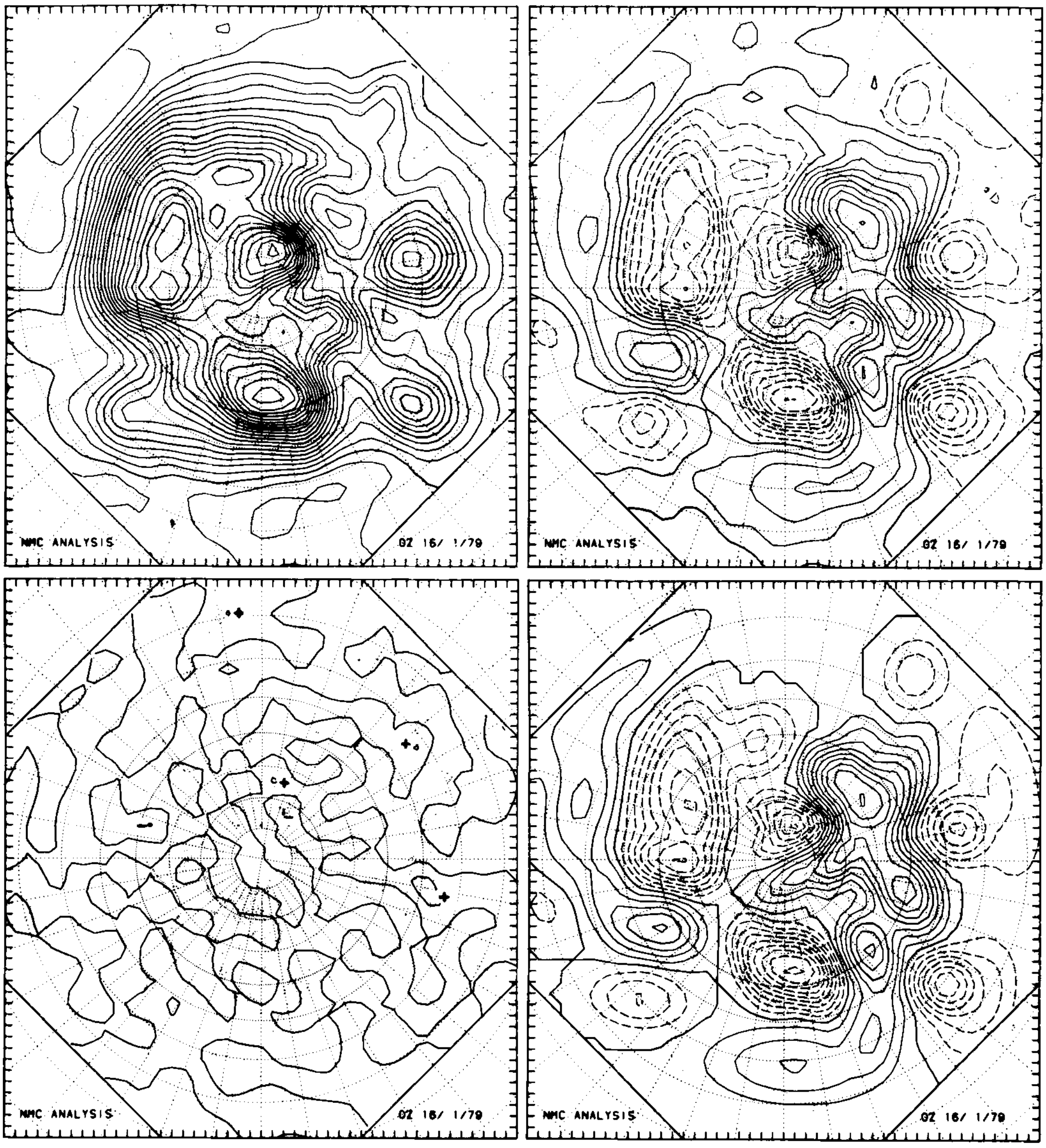


Fig. 1. Upper left: 500 mb NMC height analysis. 00Z January 16, 1979. Upper right: deviation of 500 mb height field from its zonal average. Lower right: vortex representation of deviation of height field. Lower left: difference of vortex representation and original field. Contour interval is 50 m, positive values have solid lines, dashed negative. In the difference map nonzero contours are indicated with a + or —.

the height field for the large and medium scales. The difference is smaller than either the original field or the vortex representation for wavenumbers less than 15 at 40° and 10 at 60°.

In order to examine the storm tracks, we have determined a vortex representation every 12 hours for the ECMWF forecast 500 mb height field and for the corresponding NMC verifying analysis for 00Z January 16, 1979 to 00Z January 26, 1979. Fig. 4 shows the percent variance of the difference between the complete fields and the vortex representation compared to the variance of the complete

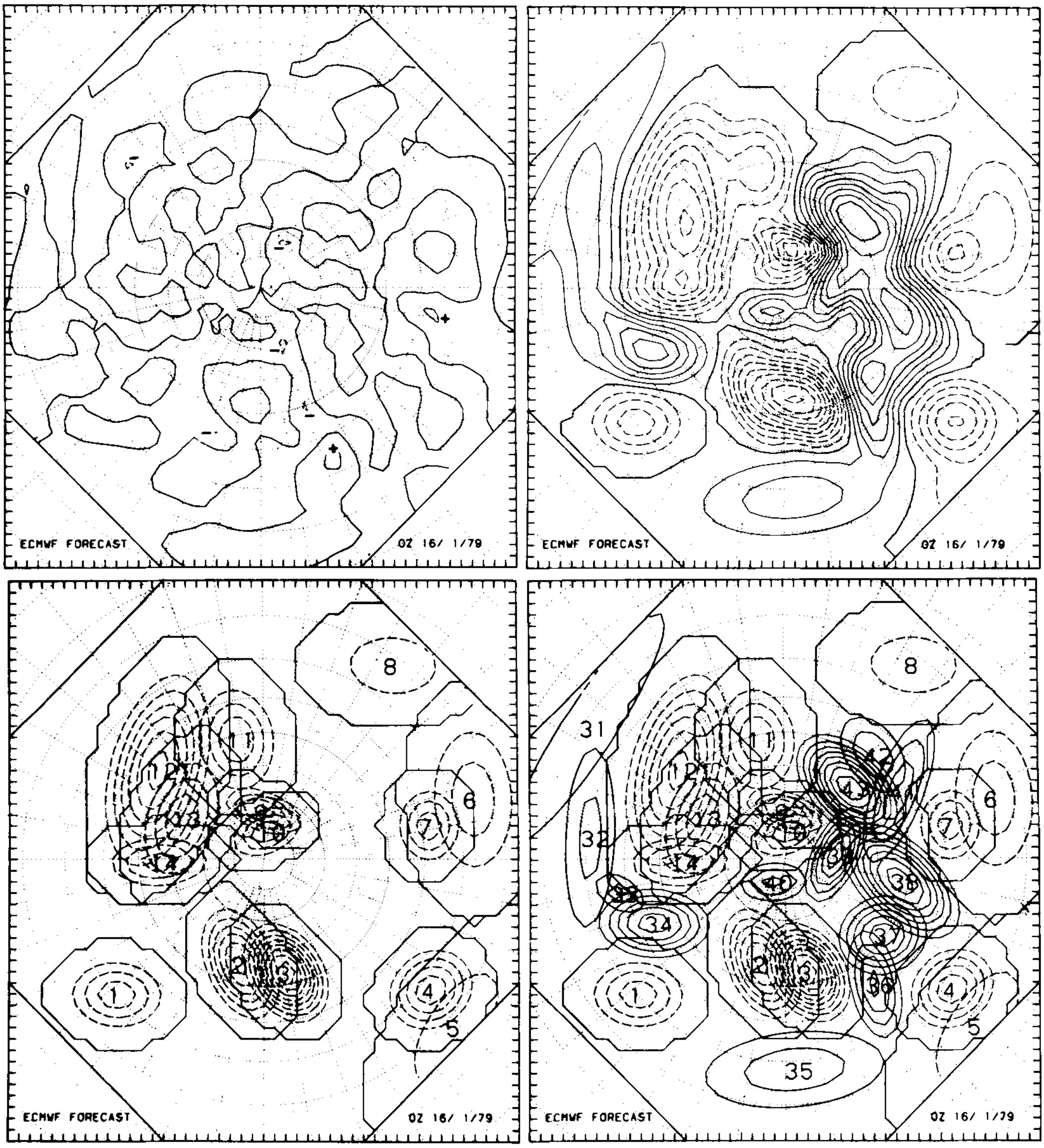


Fig. 2. Upper right: vortex representation of deviation from zonal average of 500 mb ECMWF height for 00Z January 16, 1979. Upper left: difference between vortex representation and original data. Lower right: individual features superimposed. Lower left: lows only superimposed. Contour values same as Fig. 1.

field for this period. The variance of the difference varies from 1 to 3% for the ECMWF forecast and from $1\frac{1}{2}$ to 4% for the NMC analysis. This difference between the two cases is probably due to additional smoothing of the smallest scales of the ECMWF forecast accompanying the interpolation from the model grid to the NMC octagonal grid. Fig. 4 also shows the number of highs and lows used for each fit. In the remainder of this paper we consider only the representation of the lows.

In order to examine the storm tracks, the evolution of the individual lows must be considered. Therefore Fig. 5 presents the 500 mb height field of the ECMWF forecast and NMC verifying analysis

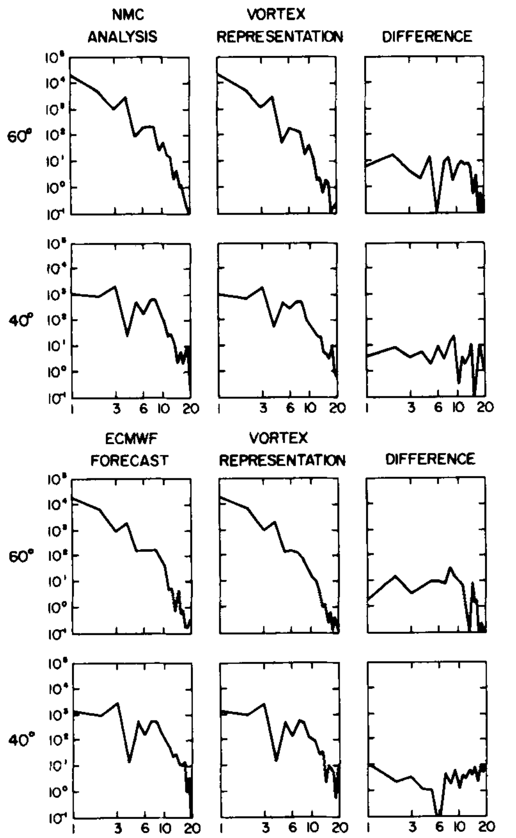


Fig. 3. Variance of height field as a function of longitudinal wavenumber at 40° and 60° latitude of complete data, vortex representations, and differences (m²).

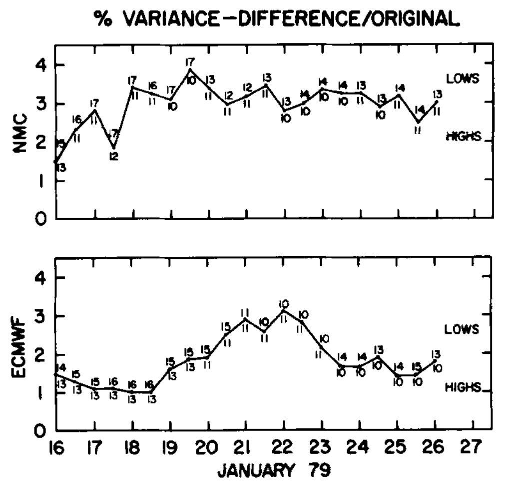


Fig. 4. Percent variance of height of the difference field compared to the complete fields.

for the lower half of the NMC octagon. These are the fields meteorologists customarily examine, however not the ones used to determine the parameters of the vortex representation. To determine the parameters, the deviation of the 500 mb height field from its zonal average is used so that troughs appear as closed lows amenable to the vortex representation. These fields are shown in Fig. 6 for the forecast and verifying analysis. The individual lows of the vortex representations are shown superimposed over each other in Fig. 7. Each low is arbitrarily assigned an index for reference. These indices, included in Fig. 7, are not necessarily the same for similar lows in the ECMWF forecast and NMC analysis. Continual reference to Figs, 5, 6, and 7 is useful to understand the properties of the vortex representation and of the storm track representation to be discussed in the next section. The discussion of the movement of the individual lows illustrated by Figs. 5, 6, and 7 will be postponed until they can be described in conjunction with the storm track representation.

Except for Figs. 3 and 4, all figures in this paper were directly machine produced on 35 mm microfilm by the DICOMED D48 Graphic COM System of the NCAR Computing Division and are therefore operationally feasible.

4. Storm track representation

We now consider how the evolution of individual lows can be mapped for verification purposes. Fig. 8 shows several possible geographical mappings for two examples of particular lows, index 1 from the NMC analysis and index 17 from the ECMWF forecast. The top row is a plot of just the central position every 12 hours connected by straight lines. The plotted number represents 00Z on that day of January 1979. This plot provides very limited information with no information about other characteristics of the storm such as size or intensity and could be obtained much easier than with the vortex representation.

The second row has the zero contour added at 12 hr intervals and indicates the areal size of the storm. This display has several shortcomings since comparison with Figs. 5, 6, and 7 shows it imparts a feeling of a larger size for the storms than the original maps do and provides no indication of the intensity. In order to indicate both size and

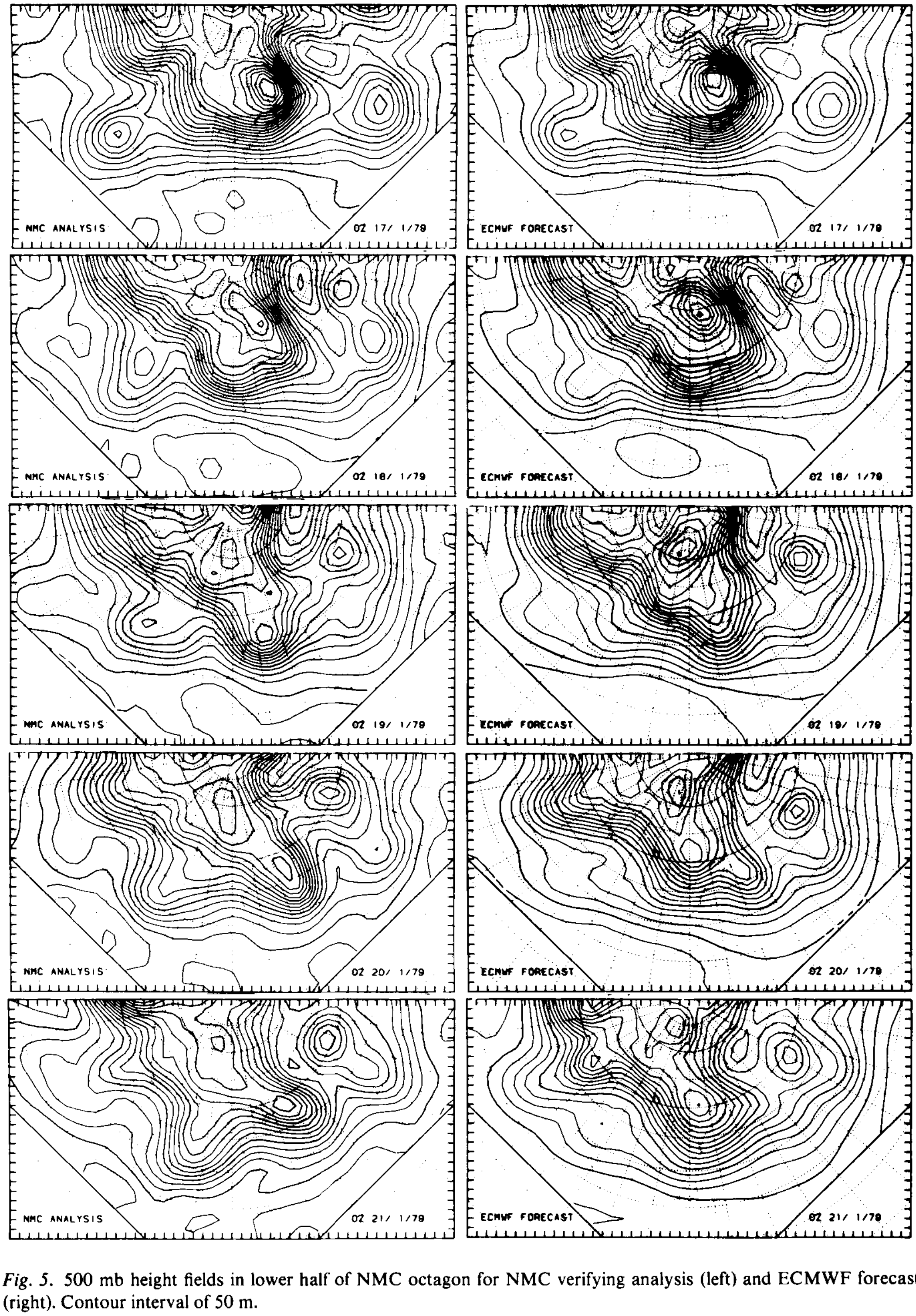
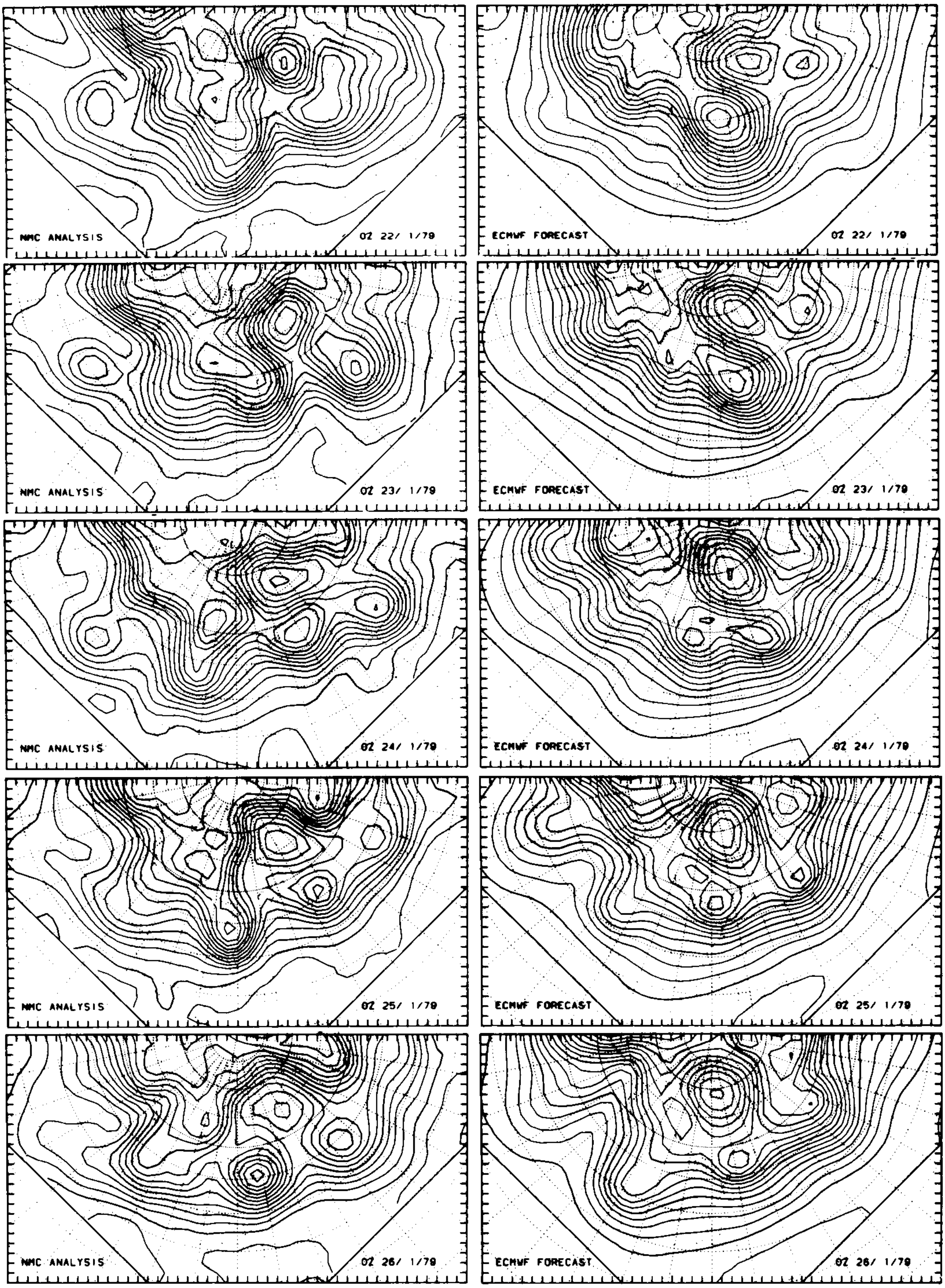


Fig. 5. 500 mb height fields in lower half of NMC octagon for NMC verifying analysis (left) and ECMWF forecast (right). Contour interval of 50 m.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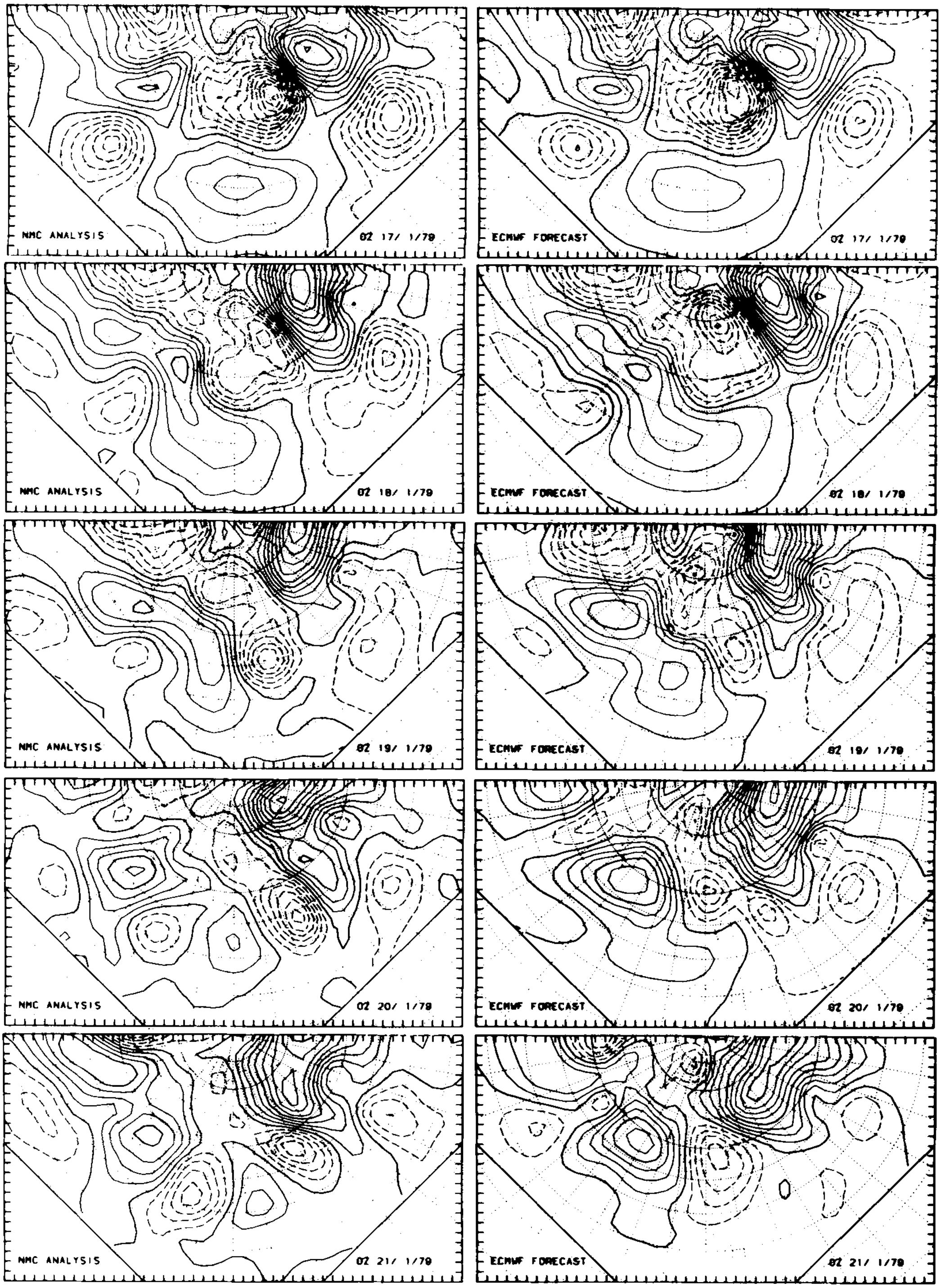
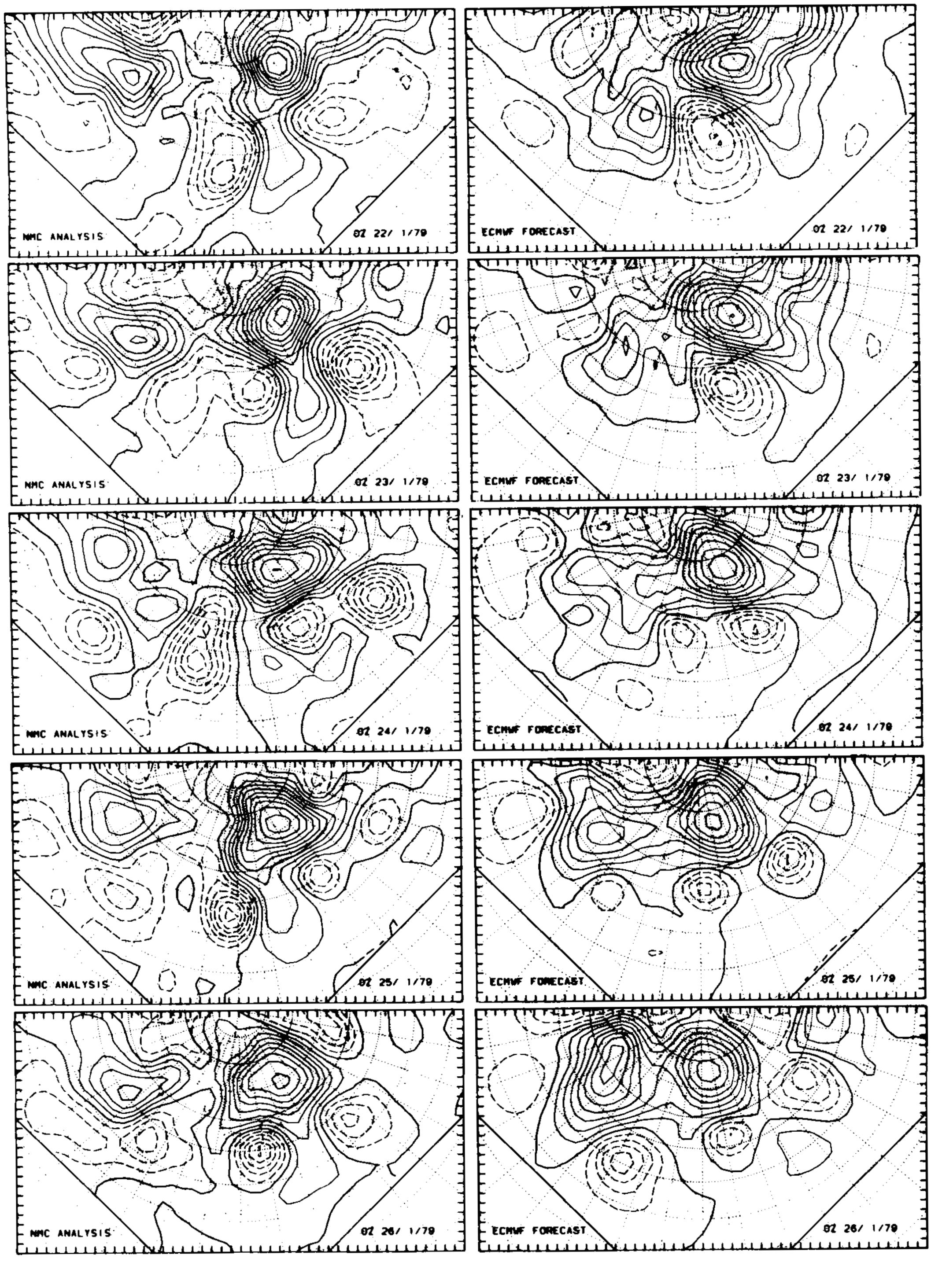


Fig. 6. Deviation of 500 mb height field from its zonal average in lower half of NMC octagon for NMC verifying analysis (left) and ECMWF forecast (right). Contour interval of 50 m. Positive contours are solid, negative dashed.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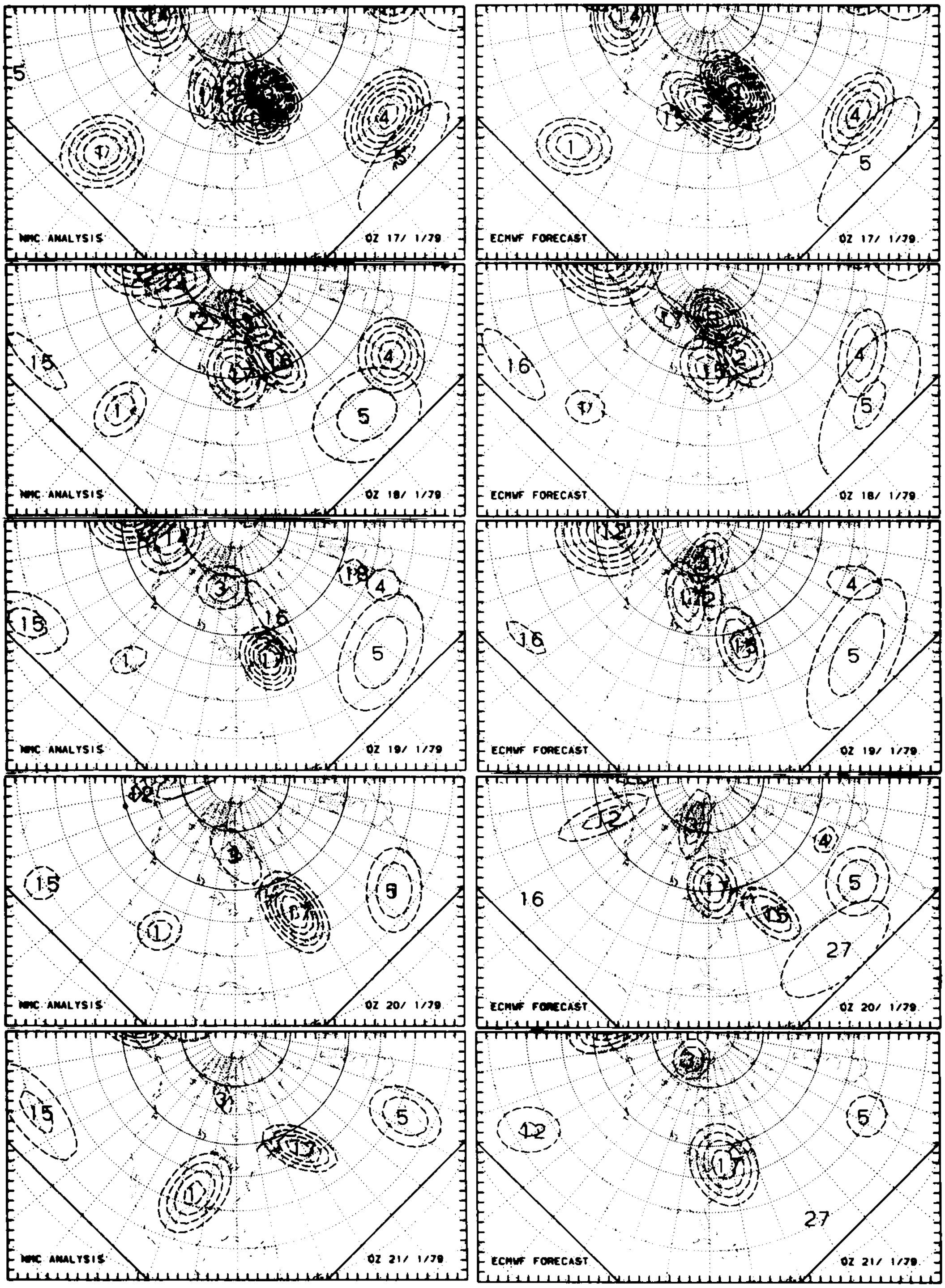
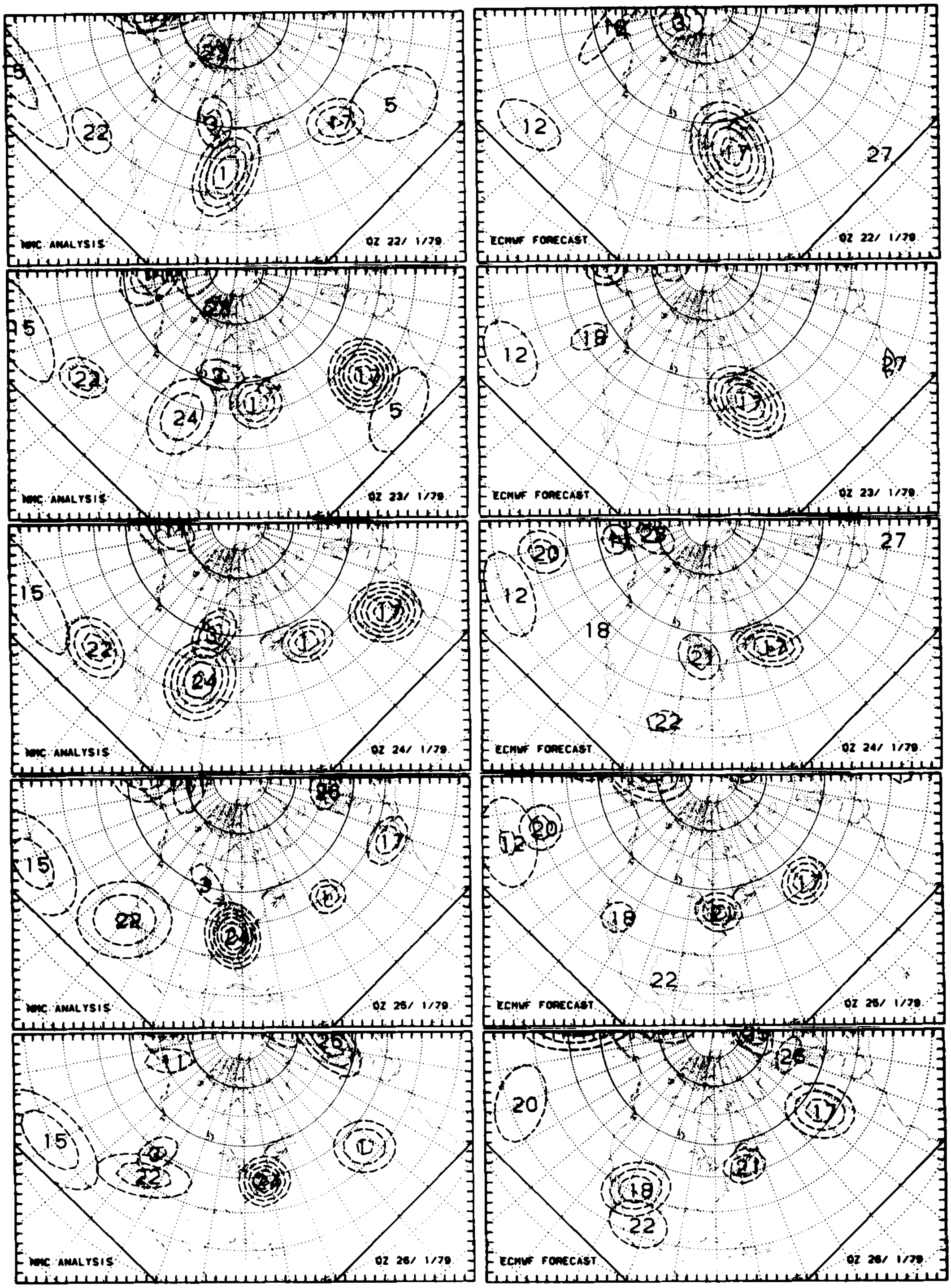


Fig. 7. Individual lows of vortex representation of fields in Fig. 6. Contour interval same as Fig. 6. Zero contour suppressed.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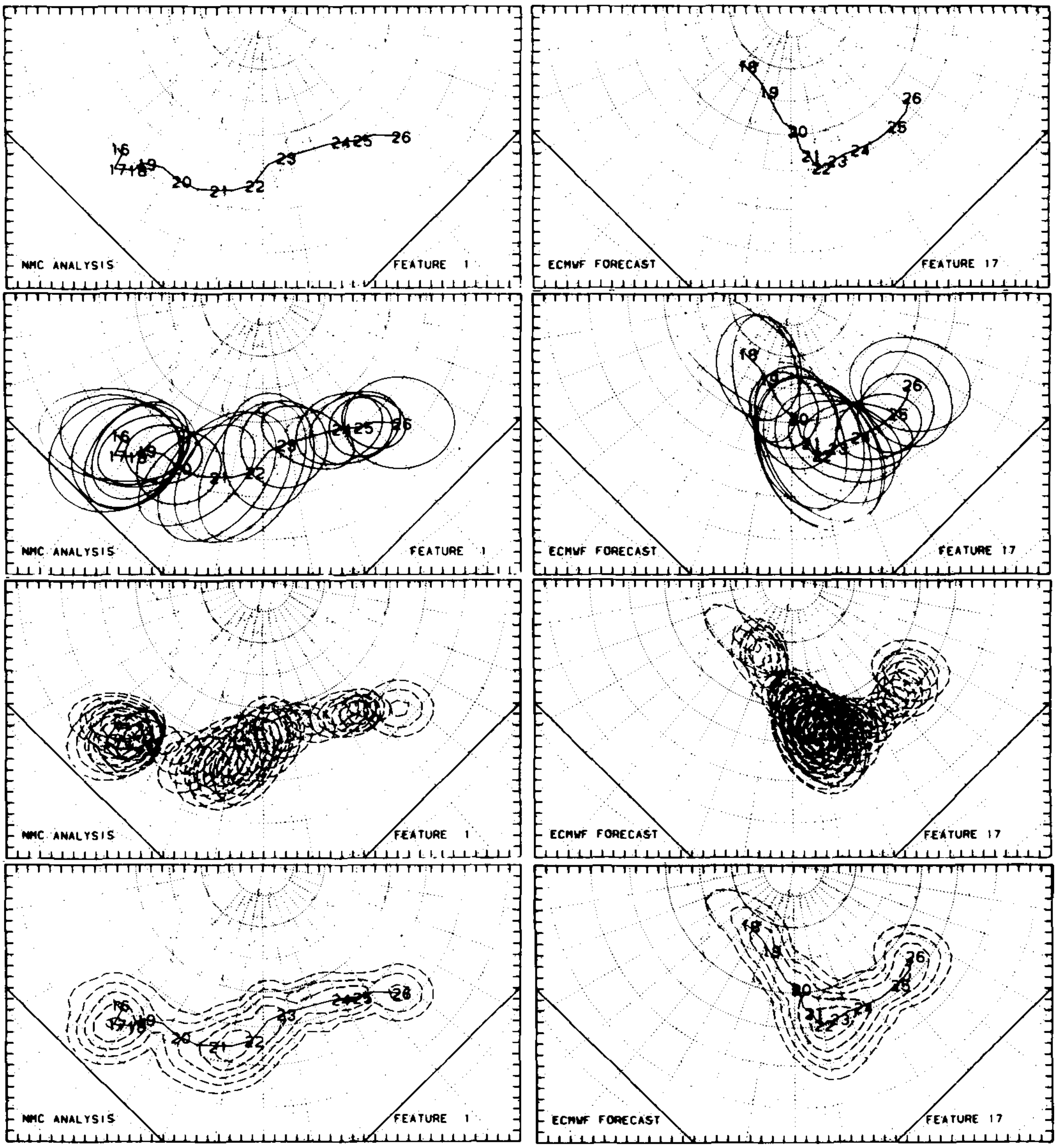

Fig. 8. Possible storm track representations for two different lows. Contour interval of 50 m, zero contour suppressed in bottom two rows.

intensity, the third row includes all nonzero contours at 12 hr intervals. Such a plot often has too much overlap at which point details become obscure. Less frequent plotting could be used, but with some information loss.

A clearer display is a plot of the envelope of each contour during the period of interest as illustrated in the bottom row of Fig. 8. These plots give a clear indication of how the systems move and grow or intensify in various geographical locations.

With this last approach, a two-dimensional field is defined which can be used as a basis of computing skill scores such as r.m.s. to express the skill of forecasting the storm tracks in terms of a single number. Of course, the one case presented here isn't enough to warrant such calculations, since in order for such scores to be useful, it is necessary to compare several cases against each other and also to establish reference scores which provide certain standards. Nevertheless, such a skill score is feasible and would be a meaningful, useful measure.

The tracks of several cyclones from the forecast and from the verifying analysis are shown in Fig. 9 to illustrate further the potential of this representation. The features are numbered arbitrarily in each data set so there is not necessarily a correspondence between features with the same number in the forecast and in the verifying analysis. Reference back to Figs. 5, 6, and 7 in conjunction with the following discussion will help establish the meaning of these plots.

Consider first Feature 1 in the NMC analysis. This low is initially off the southwest coast of the U.S. It decays and moves inland during the first few days of the period, then amplifies and moves more rapidly across southern U.S., up along the Appalachians, and out into the Atlantic. The same low in the ECMWF forecast (Feature 1) decays and moves toward the coast but then damps as it moves through the long wave ridge. This low in the ECMWF forecast illustrates a problem in the method used here to define the field which is to be represented by vortices, i.e. the removal of the zonal average. The complete 500 mb height field in Fig. 5 shows a trough corresponding to Feature 1 on 00Z January 19; however, the deviation from the zonal average in Fig. 6 shows only positive values in the region of this trough. The trough is indicated by a decrease of intensity of the high, but since the field has positive values there it cannot be represented by a vortex with negative amplitude.

Feature 17 of the analysis first appears in the representation on January 17 in northwestern Saskatchewan. Previous to this date, it was a short wave trough visible in the 500 mb analysis but too small to appear in the vortex representation. It amplifies while moving southeastward above the Great Lakes and out into the Atlantic off northern New England. It reaches maximum intensity January 20, then moves eastward across the Atlantic and dies out at the end of the 10-day period. A similar short wave trough in the ECMWF forecast becomes Feature 15 just before January 17. It is centered a little farther south than Feature 17 of the analysis when first recognizable by the representation, but moves eastward the first day, then southeastward following the path of Analysis Feature 17 very closely. However, once over the Atlantic, Forecast Feature 15 dies out after January 20, rather than continuing across the Atlantic. In the forecast, a second short wave trough deepens (Feature 17) and moves southeast following Forecast Feature 15. Once over the Atlantic, it moves much as Analysis Feature 17 except about two days behind. In fact, from January 22 on, Forecast Feature 17 looks more like Analysis Feature 1. It appears that the model forecasts the storm track through Canada and across the Atlantic properly, although the instantaneous storm locations are not correct, while missing the track across southern U.S.

The clusters of features 2, 3, 16, 17 in the NMC analysis and 2, 3, 15 in the ECMWF forecast on 00Z January 17, indicate how the parameters defining the features can depend on the number of features included in a more complex structure. Fig. 7 indicates that the parameters are quite different for the representations at this time with 4 features to represent the NMC analysis and 3 the ECMWF forecast, yet the structures of the total field in this area appear fairly similar (Figs. 5 and 6). A fourth feature, number 17, was added to the ECMWF forecast before 00Z January 18, at which time the parameters for the representations of the NMC analysis and ECMWF forecast are closer to each other. A fourth feature probably should have been included in this cluster for the representation of the ECMWF forecast on January 17.

We also point out two other features of the forecast, namely Features 18 and 21. Forecast Feature 18 moves across the Pacific in the latter half of the forecast, decreasing in intensity to less than 50 m on January 24, then moves over southwestern U.S. and intensifies. There is a similar low in the analysis in the early part of this period, Feature 13. However, it decreases in intensity to the point where it cannot be followed with this representation. A feature does amplify in the Pacific (Analysis Feature 22) and comes into the U.S. at about the same time and location as Forecast Feature 18.

Analysis Feature 24 is a short wave trough which amplifies east of the Rockies then moves across the U.S. during the last three days of the forecast period. A corresponding low in the forecast (Feature 21) develops one day later, too far northeast, appearing less like lee cyclogenesis

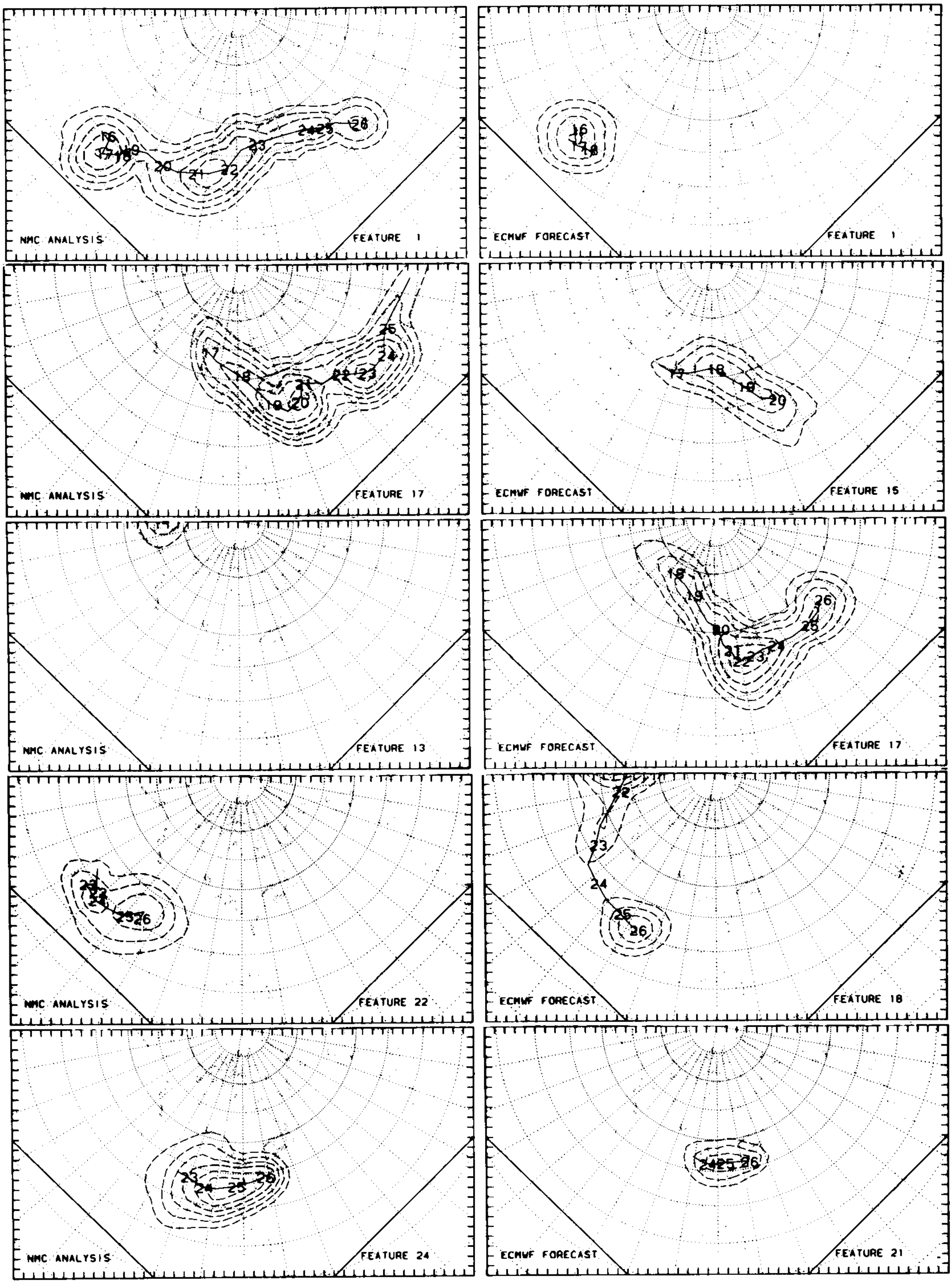

Fig. 9. Storm tracks for selected lows of ECMWF forecast and NMC analysis for January 16–26, 1979. Contour interval of 50 m, zero contour suppressed.

than Analysis Feature 24. However, once formed, the low moved off the east coast at about the right location and time. The forecast was not continued long enough to determine if its subsequent track across the Atlantic would be properly forecast.

5. Conclusions

The vortex representation presented here describes individual storms by a small number of parameters which have meaning to meteorologists. These parameters can be used as a basis for verification of properties of interest to meteorologists. The representation defines individual vortices in terms of an amplitude, radius, center, and two ellipticity parameters which specify the angle the major axis makes with the coordinate axis and the ratio of semimajor and semiminor axes. The parameters are determined by minimizing the squared difference between the observed field and the field specified by a combination of individual vortices. In this report the observed field which is represented by vortices is the deviation of the 500 mb height field from its zonal average. This definition causes one problem in that short wave troughs moving through larger scale ridges do not always appear as negative anomalies. When they do not they cannot be represented by vortices with negative amplitude. However, larger isolated lows are well represented by individual vortices and these are the features which generally produce the significant weather events.

Vortex representations for a sequence of maps allow storm tracks and storm intensities to be represented as a function of geographical location with the temporal aspects removed. The one forecast presented here indicates that the model might be able to forecast such storm tracks in certain instances and that such a representation might therefore be a useful forecast product. This one case, of course, is not a proof that the tracks are predictable but rather a demonstration of a procedure to represent the tracks and other features of the storm along the tracks. Undoubtedly there are cases or regions where the tracks are not well predicted by the model.

Many forecasts must be examined to provide a statistical base to establish criteria which indicate whether and when and where tracks are predictable. In order for the representation proposed here to be a useful forecast product, criteria must be determined which establish in advance which storm tracks in a forecast are more likely to be correct. Such criteria might be regional; for example, in the forecast considered above, the track across the Atlantic seems to be well predicted, but the one over western and central U.S. seems to be poorly predicted. Stability of the tracks over several sequential overlapping medium range forecasts might provide a criterion for predictability. Other criteria which might be examined are multiple storms along a single track in one forecast or stability of the parameters defining the storm along the track.

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ПРЕДСТАВЛЕНИЕ ПУТЕЙ ЦИКЛОНОВ И ЕГО ПРОВЕРКА

Излагается представление циклонов и антициклонов в виде вихрей, что позволяет математически описать индивидуальные локальные особенности малым числом параметров, воспроизводящих их величину, положение, размер и форму. Параметры, воспроизводящие конкретный набор данных, определяются путем минимизации интеграла от квадрата разности между математическим представлением и набором данных. Различные меры объективной проверки могут быть основаны на этих параметрах путем использования вихревого представления как для прогноза, так и для контрольных данных. Разность в величинах параметров обеспечивает меру ошибки в величине, положении, размере и форме индивидуальных особенностей. Разбирается пример применения этого представления к 10-ти суточному прогнозу ЕЦСПП и соответствующему анализу реальных данных НМЦ. Представление циклонических путей в прогнозе и анализе определяется путем нанесения на карту огибающих индивидуальных особенностей с течением времени, т.е. путем определения объема, заметаемого циклоном. Такое двумерное горизонтальное поле или карта указывает путь каждого циклона, так же как его интенсивность, величину вдоль его пути, в течение периода картирования. Иллюстрируется полезность такого картирования в качестве продукции прогноза и как средства проверки.