Tellus (1981), *33*, 444–452

Stratospheric volcanic aerosols: A model study of interactive influences upon solar radiation

By R. D. CESS[1] and J. A. COAKLEY, JR, *National Center for Atmospheric Research,*[2] *Boulder, Colorado 80307, U.S.A.*, and P. M. KOLESNIKOV, *Luikov Heat and Mass Transfer Institute, Byelorussian Academy of Sciences, Minsk 220728, U.S.S.R.*

(Manuscript received October 13, 1980; in final form February 3, 1981)

ABSTRACT

Considerable differences exist within the literature concerning the effect of volcanically induced stratospheric aerosols upon the earth's planetary albedo. In order to investigate these discrepancies, we have developed a two-stream approximation which is shown to be in good agreement with more detailed calculations. The model employs aerosol radiative properties which are determined from a separate Mie scattering calculation. We then make systematic changes within the model to illustrate the individual effects of Rayleigh scattering, the angular dependence of both Rayleigh and aerosol scattering, and full-flux versus single-wavelength calculations. When all interactive effects are included, the aerosol influence upon planetary albedo is reduced, relative to the usual single wavelength calculations, by more than a factor of two. This finding resolves the aforementioned discrepancies.

1. Introduction

In 1975/76 several model studies were published concerning the influence of volcanically induced stratospheric aerosols upon the earth's albedo. Russell and Hake (1977) summarized and compared several of these studies. Three of the models, due to Cadle and Grams (1975), Pinnick et al. (1976), and Harshvardhan and Cess (1976), were mutually consistent, suggesting that

$$d\alpha_p/d\tau_{vis} \simeq 0.2, \tag{1}$$

where α_p is the planetary albedo and τ_{vis} the aerosol optical depth at the wavelength of 0.55 μm. However, a fourth model (Pollack et al., 1976) produced the substantially lower result

$$d\alpha_p/d\tau_{vis} = 0.082. \tag{2}$$

In addition Luther (1976a) has reported yet another model study which is restricted solely to clear-sky conditions. For a surface albedo of 0.25, Luther also obtains eq. (2). This surface albedo is probably close to approximating actual global conditions, since the empirical expression of Budyko (1974) yields, for a surface albedo of 0.25, a clear-sky albedo of 0.27, which is not substantially different from the earth's planetary albedo ($\simeq$0.3).

There is a significant difference between the models that produce eq. (2) as opposed to eq. (1). Both Pollack et al. (1976) and Luther (1976a) performed detailed calculations which incorporated Rayleigh scattering, directional effects, and integration over solar wavelengths. On the other hand, single wavelength calculations were performed by Pinnick et al. (1976) at 0.53 μm and by Harshvardhan and Cess (1976) at 0.55 μm, whereas Luther (1976b) has indicated that single wavelength calculations near 0.55 μm significantly overestimate $d\alpha_p/d\tau_{vis}$. Although Cadle and Grams (1975) did perform a full-flux calculation for the aerosol layer, they did not incorporate the wavelength dependence due to Rayleigh scattering on the albedo of the underlying earth-atmosphere system.

[1] Visiting Scientist. Permanent address: Laboratory for Planetary Atmospheres Research, State University of New York, Stony Brook, New York 11794, U.S.A.

[2] The National Center for Atmospheric Research is sponsored by the National Science Foundation.

0040-2826/81/040444-09$02.50/0

To illustrate how Rayleigh scattering will reduce the full-flux calculation, consider for simplicity a nonabsorbing aerosol layer overlying a Rayleigh scattering atmosphere with a black underlying surface. We will ignore for the time being the angular dependence of Rayleigh scattering. Following the procedure discussed in Section 2, it is easily shown that

$$\frac{d\bar{\alpha}_\lambda}{d\tau_{\text{vis}}} = 2\bar{\beta}_\lambda(1 - \bar{r}_\lambda)^2 (\sigma_\lambda/\sigma_{\text{vis}}), \tag{3}$$

where $\bar{\alpha}_\lambda$ is the monochromatic albedo of the earth-atmosphere system, $\bar{r}_\lambda$ is the monochromatic spherical Rayleigh albedo, $\bar{\beta}_\lambda$ is the monochromatic backscattered fraction for isotropically incident radiation, σ_λ is the monochromatic scattering coefficient, and σ_{vis} denotes σ_λ at $\lambda = 0.55$ μm. Clearly the inclusion of Rayleigh scattering reduces $d\bar{\alpha}_\lambda/d\tau_{\text{vis}}$ at shorter wavelengths where $\bar{r}_\lambda$ becomes significant. Consequently the solar-wavelength averaged quantity $d\alpha_p/d\tau_{\text{vis}}$ is also reduced with respect to the single-wavelength calculations. We will later show that, as a consequence of Rayleigh scattering, rather than constituting a wavelength at which $d\bar{\alpha}_\lambda/d\tau_{\text{vis}}$ is representative of $d\alpha_p/d\tau_{\text{vis}}$, wavelengths of 0.53 and 0.55 μm are near the maximum value of $d\bar{\alpha}_\lambda/d\tau_{\text{vis}}$.

Another factor is the zenith-angle dependence of the reflected radiation interacting with the angular dependence of the scattering by aerosols. This factor could further influence model predictions of $d\alpha_p/d\tau_{\text{vis}}$. Indeed Harshvardhan (1979), employing parameterizations for the zenith-angle dependence of both clear-sky and cloudy-sky albedos, finds that eq. (1) is reduced to $d\alpha_p/d\tau_{\text{vis}} \simeq 0.14$ for a single-wavelength calculation at 0.55 μm.

The present paper systematically investigates the various factors which could be responsible for differences between eqs. (1) and (2). For this investigation we employ a two-steam model which is discussed in the next section. We then present in Section 3 a description of the aerosol models which we adopt together with the relevant radiative transfer properties of the aerosols as computed from Mie theory. Next, for purposes of illustrating interactive effects, specific results are presented for global mean conditions.

2. Radiation model

In this section we develop a simple prescription for calculating the effect of stratospheric aerosols upon the earth's albedo. In accordance with the previous section, we specifically focus on the incorporation of the wavelength dependence for Rayleigh scattering and of the angular dependence for scattering within the atmosphere.

We first consider the simple model of an aerosol layer over a diffusely reflecting earth-atmosphere system. With μ denoting the cosine of the solar zenith angle, the monochromatic planar albedo of the combined system is conventionally expressed as (Chandrasekhar, 1960, p. 273)

$$\alpha_\lambda(\mu) = R_\lambda(\mu) + \frac{\bar{\alpha}_{0\lambda}\bar{T}_\lambda T_\lambda(\mu)}{1 - \bar{\alpha}_{0\lambda}\bar{R}_\lambda} \tag{4}$$

where $R_\lambda(\mu)$ is the reflectivity of the aerosol layer, $T_\lambda(\mu)$ is the transmissivity of the layer, and $\bar{\alpha}_{0\lambda}$ is the albedo of the underlying diffusely reflecting system; i.e., the earth-atmosphere albedo in the absence of the stratospheric aerosol layer. Overbarred quantities represent reflectivities, transmissivities and albedos for diffuse radiation and are equivalent to "globally averaged" quantities. They are obtained by averaging over the surface of a sphere while keeping the position of the sun fixed. Thus the average is

$$\bar{\alpha}_\lambda = 2\int_0^1 \alpha_\lambda(\mu)\mu\, d\mu, \tag{5}$$

irrespective of whether it is albedo, reflectivity, or transmissivity that is being averaged. The quantity $\bar{\alpha}_\lambda$ is also referred to as the spherical albedo.

In the above we have considered the earth-atmosphere system to be a diffuse reflector. It is not. The albedo of the system depends upon the angle of incident radiation which, with the aerosol present, depends on the optical properties of the aerosol. We approximately allow for this angular dependence by applying the following assumptions. Because the stratospheric aerosol layer is optically thin, and since a sizeable fraction of the radiation it scatters is confined to a forward peak, we assume that most of the radiation passing through the layer maintains a path near that of the direct beam. If we further assume that once scattered by the earth-atmosphere system the radiation becomes diffuse, then in place of eq. (4) we obtain

$$\alpha_\lambda(\mu) = R_\lambda(\mu) + \frac{\alpha_{0\lambda}(\mu)\bar{T}_\lambda T_\lambda(\mu)}{1 - \bar{\alpha}_{0\lambda}\bar{R}_\lambda}. \tag{6}$$

That is, $\bar{\alpha}_{0\lambda}$ has been replaced by $\alpha_{0\lambda}(\mu)$ within the numerator of the second term in eq. (4).

That the earth-atmosphere system diffusely scatters radiation, yet maintains an albedo dependent upon the angle of the incident radiation, is a violation of reciprocity. The justification for this assumption rests not in its description of physical reality, but rather in its ability to produce reasonably good results, as will be shown shortly.

In order to compare eqs. (4) and (6), as well as for the subsequent application of eq. (6), it is necessary to describe $\alpha_{0\lambda}(\mu)$ and $\bar{\alpha}_{0\lambda}$. For this purpose we will neglect the reduction of upward reflected solar radiation due to absorption by atmospheric water vapor. This effect is significant only longward of about 0.9 μm, where our model results show that only 20% of the influence upon $d\alpha_p/d\tau_{\text{vis}}$ occurs. Assuming the planetary surface to be a diffuse reflector, an assumption which we will later relax, and following the same procedure as employed in arriving at either eq. (4) or (6), we obtain

$$\alpha_{0\lambda}(\mu) = r_\lambda(\mu) + \frac{\alpha_s(1 - \bar{r}_\lambda)[1 - r_\lambda(\mu)]}{1 - \alpha_s \bar{r}_\lambda}, \tag{7}$$

while from eq. (5)

$$\bar{\alpha}_{0\lambda} = \bar{r}_\lambda + \frac{\alpha_s(1 - \bar{r}_\lambda)^2}{1 - \alpha_s \bar{r}_\lambda}. \tag{8}$$

Here α_s denotes the surface albedo, while $r_\lambda(\mu)$ and $\bar{r}_\lambda$ are the reflectivities due to Rayleigh scattering. From Coakley and Chýlek (1975), it may be shown that

$$r_\lambda(\mu) = [1 + 2\mu/\tau_{R\lambda}]^{-1}, \tag{9}$$

where $\tau_{R\lambda}$ is the Rayleigh optical depth which we take from Hoyt (1977). It in turn follows from eq. (5) that

$$\bar{r}_\lambda = \tau_{R\lambda}\left\{1 + \frac{\tau_{R\lambda}}{2}\ln\left(\frac{\tau_{R\lambda}}{2 + \tau_{R\lambda}}\right)\right\}. \tag{10}$$

For use within eqs. (4) and (6), we employ the two-stream $R_\lambda(\mu)$ and $T_\lambda(\mu)$ expressions of Coakley and Chýlek (1975, their eqs. 13 and 14). The corresponding spherical averages, $\bar{R}_\lambda$ and $\bar{T}_\lambda$, are in turn evaluated from eq. (5) through numerical quadrature. For the practically important case on a nonabsorbing aerosol, $T_\lambda(\mu) = 1 - R_\lambda(\mu)$, $\bar{T}_\lambda = 1 - \bar{R}_\lambda$, and

$$R_\lambda(\mu) = [1 + \mu/\tau_\lambda \beta_\lambda(\mu)]^{-1}, \tag{11}$$

where $\beta_\lambda(\mu)$ is the backscattered fraction for monodirectional incident radiation (Coakley and Chýlek, 1975), and τ_λ is the aerosol optical depth.

To illustrate the accuracy of the above two-stream approximation, we choose an asymmetry factor, $g = \langle \cos\theta \rangle$, of 0.7, and then evaluate $\beta_\lambda(\mu)$ employing a Henyey-Greenstein phase function (Hansen, 1969). Fig. 1 illustrates the resulting $\beta_\lambda(\mu)$, together with $R_\lambda(\mu)$ and $T_\lambda(\mu)$

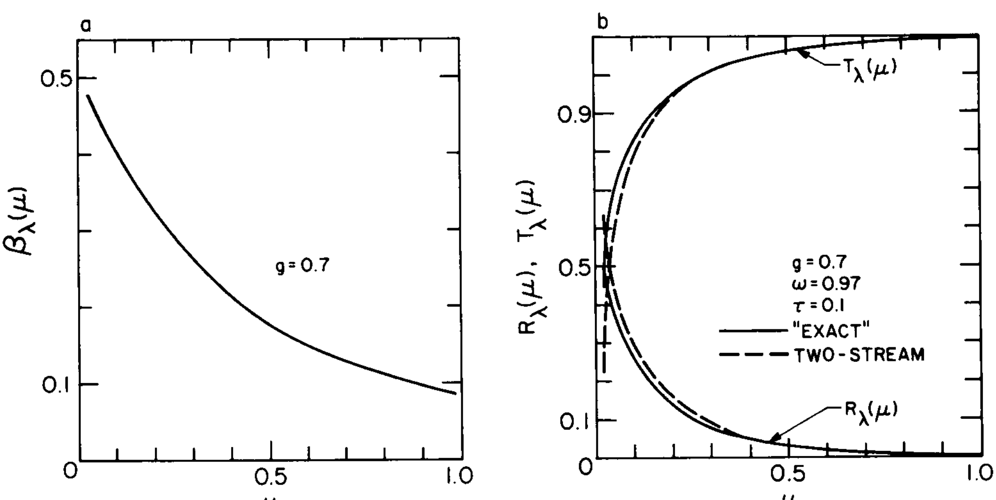

Fig. 1. (a) $\beta_\lambda(\mu)$ for Henyey-Greenstein phase function with asymmetry factor $g = 0.7$. (b) $R_\lambda(\mu)$ and $T_\lambda(\mu)$ for an aerosol layer with optical depth $\tau_\lambda = 0.1$ and single scattering albedo $\omega = 0.97$.

as evaluated from the two-stream approximation and compared with "exact" results. In this comparison the aerosol layer has an optical depth of $\tau_\lambda = 0.1$ and a single scattering albedo $\omega = 0.97$. Note that Fig. 1 illustrates monochromatic (as opposed to solar averaged) quantities. The "exact" results are obtained using a doubling algorithm as described by Hunt (1971). Fig. 2 shows a similar comparison for the Rayleigh reflectivity $r_\lambda(\mu)$. For Rayleigh scattering $\beta_\lambda(\mu) = 1/2$, so that eq. (9) is equivalent to eq. (11). The figures reveal that, except for large zenith angles, the two-stream approximation yields accurate numerical values for the reflectivities and transmissivities. For global average quantities, the effect of errors at large zenith angles is reduced by the cosine weighting in eq. (5).

Because of the product $\alpha_{0\lambda}(\mu)T_\lambda(\mu)$ within eq. (6), changes in the global average albedo,

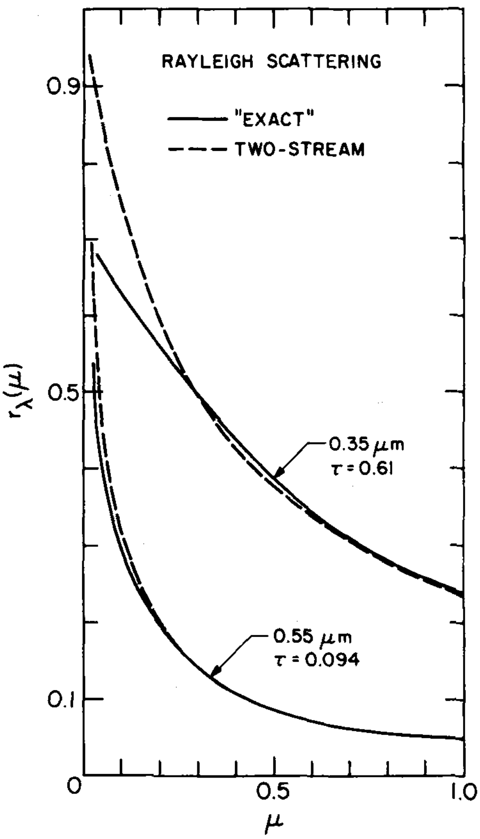

Fig. 2. Reflectivity $r_\lambda(\mu)$ for Rayleigh scattering by the earth's atmosphere over a surface with the albedo $\alpha_s = 0$.

due to the presence of a stratospheric aerosol layer, are bound to differ from those obtained using eq. (4). To assess this difference as well as the applicability of eq. (6), we again employ the previously described two-stream approximation. The results presented in Table 1 show how the aerosol's impact is affected by the wavelength and angular dependencies of scattering by the earth-atmosphere system. The table includes the results of calculations with an aerosol layer over a Rayleigh atmosphere with optical depth $\tau_{R\lambda} = 0.61$ to simulate conditions at $\lambda = 0.35$ μm, and with $\tau_{R\lambda} = 0.094$ at $\lambda = 0.55$ μm. Note that these wavelengths represent different degrees of interaction of Rayleigh scattering upon aerosol scattering. The results are presented for several surface albedos; recall that the surface is taken to be diffusely reflecting.

Table 1 shows that at 0.35 μm and with $\alpha_s = 0$, the change in the global average albedo, $\bar{\Delta}\alpha_\lambda$, is less than half that calculated for 0.55 μm. Also for $\alpha_s = 0$, the table shows that allowing for the angular dependent reflectivity of the atmosphere reduces the impact of the aerosol by 10–15% at 0.55 μm and by 50–100% at 0.35 μm. The angular dependence of the reflectivity becomes less significant as the optical depth of the underlying atmosphere becomes small.

Thus, through these simple calculations, we have demonstrated the need for including in assessments of the aerosol's impact the wavelength and angular dependencies of scattering by the underlying system. That these simple calculations are adequate for evaluating global average changes is judged through comparison with the results of "exact" radiative transfer calculations also presented in Table 1. For the "exact" calculations the mass mixing ratio of the aerosol is constant between 17 km (88 mb) and 23 km (35 mb) and zero elsewhere. For the cloud case the aerosol layer is placed over the cloud and Rayleigh scattering is omitted. Cloud scattering is modeled using a Henyey-Greenstein phase function with $g = 0.844$, a single scattering albedo $\omega = 1.0$, and a cloud optical depth of 8.0 (Hansen, 1969). The cloud reflectivity obtained by the "exact" calculation is also used in eqs. (4) and (6) for the approximate results. The radiative properties for the resulting inhomogeneous atmospheric models are obtained by using a combination of adding and doubling algorithms as described by Hunt (1971).

Table 1. *Change in the global average albedo $\Delta\bar{\alpha}_\lambda$ due to the presence of an aerosol layer of optical depth $\tau_\lambda = 0.1$ over Rayleigh and cloudy atmospheres*

	Aerosols over Rayleigh atmosphere $\lambda = 0.35\ \mu m$					
	Nonabsorbing aerosol $\omega = 1.0$			Absorbin aerosol $\omega = 0.97$		
Surface albedo	Atmosphere as diffuse reflector (eq. (4))	Atmosphere as angular reflector (eq. (6))	"Exact"	Atmosphere as diffuse reflector (eq. (4))	Atmosphere as angular reflector (eq. (6))	"Exact"
0.0	0.0156	0.0102	0.0107	0.0110	0.0054	0.0062
0.1	0.0135	0.0081	0.0093	0.0086	0.0033	0.0044
0.2	0.0115	0.0068	0.0079	0.0060	0.0012	0.0025
0.4	0.0076	0.0038	0.0052	0.0010	−0.0030	−0.0014
0.6	0.0040	0.0012	0.0027	−0.0040	−0.0069	−0.0053
	$\lambda = 0.55\ \mu m$					
0.0	0.0297	0.0264	0.0253	0.0276	0.0240	0.0231
0.1	0.0246	0.0207	0.0209	0.0217	0.0188	0.0179
0.2	0.0198	0.0173	0.0168	0.0161	0.0135	0.0130
0.4	0.0116	0.0097	0.0098	0.0062	0.0041	0.0042
0.6	0.0054	0.0041	0.0046	−0.0020	−0.0033	−0.0029
	Aerosols over cloud					
Cloud	0.0087	0.0046	0.0054	0.0025	−0.0017	−0.0014

This completes the formulation of the two-stream approximation for application to stratospheric aerosols. The change in planetary albedo, associated with a given aerosol optical depth τ_{vis}, is in turn given by

$$\Delta\alpha_p = \int_{\lambda_0}^{\lambda_1} (\bar{\alpha}_\lambda - \bar{\alpha}_{0\lambda})(S_\lambda/S_0)\, d\lambda \qquad (12)$$

where $\bar{\alpha}_\lambda$ and $\bar{\alpha}_{0\lambda}$ are given by eqs. (6) and (7), with subsequent averaging in accordance with eq. (5). Here $\lambda_0 = 0.3\ \mu m$ and $\lambda_1 = 2.6\ \mu m$; S_λ is the wavelength dependent solar flux at the top of the atmosphere and $S_0 = 1370\ W\ m^{-2}$ is the solar constant. The 0.3 μm cutoff simulates ozone absorption, while there is negligible solar radiation beyond 2.6 μm. Values of $(S_\lambda/S_0)\Delta\lambda$ have been taken from Thekaekara (1972).

3. Aerosol models

We assume, as have Pollack et al. (1976), Harshvardhan and Cess (1976), and Harshvardhan (1979), that the stratospheric aerosol is composed of 75% aqueous sulfuric acid (Rosen, 1971). The real refractive index is taken from Palmer and Williams (1975), while solar absorption by the aerosol is neglected, since Palmer and Williams' value for the imaginary refractive index is $0(10^{-7})$. Two particle size distributions are employed; the zero order logarithmic distribution (Zold distribution) of Pollack et al. (1976), and a gamma distribution corresponding to Deirmendjian's (1969) haze *H* distribution as used by Luther (1976a). Comparison of these two size distributions by Toon and Pollack (1976) shows that the Zold distribution has a greater number of droplets with radii less than 0.1 μm, while the two distributions have similar concentrations of larger droplets.

Employing a Mie scattering program at the B.S.S.R. Academy of Sciences (Kolesnikov and Cess, 1980), we have evaluated the backscattered fraction for isotropic irradiation $\bar{\beta}_\lambda$ (see Coakley and Chýlek, 1975; and Wiscombe and Grams, 1976, for further details), the scattering coefficient

σ_λ, and the asymmetry factor g for both particle size distributions. These are illustrated in Fig. 3. Our results for $\sigma_\lambda/\sigma_{\text{vis}}$ and g are consistent with similar calculations for the Zold distribution by Toon and Pollack (1976).

Additionally we require the backscattered fraction for monodirectional incident radiation $\beta_\lambda(\mu)$. Following Wiscombe and Grams (1976), we have numerically evaluated $\beta_\lambda(\mu)$ by employing a Henyey-Greenstein phase function together with an appropriate mean value for the asymmetry factor g. We find that 80% of the contribution to $d\alpha_p/d\tau_{\text{vis}}$ results from wavelengths less than 0.9 μm, such that from Fig. 3 we have chosen g = 0.7. The resulting $\beta_\lambda(\mu)$ is that shown in Fig. 1a. Subject to the constraint that (Wiscombe and Grams, 1976)

$$\bar{\beta}_\lambda = \int_0^1 \beta_\lambda(\mu)\, d\mu, \tag{13}$$

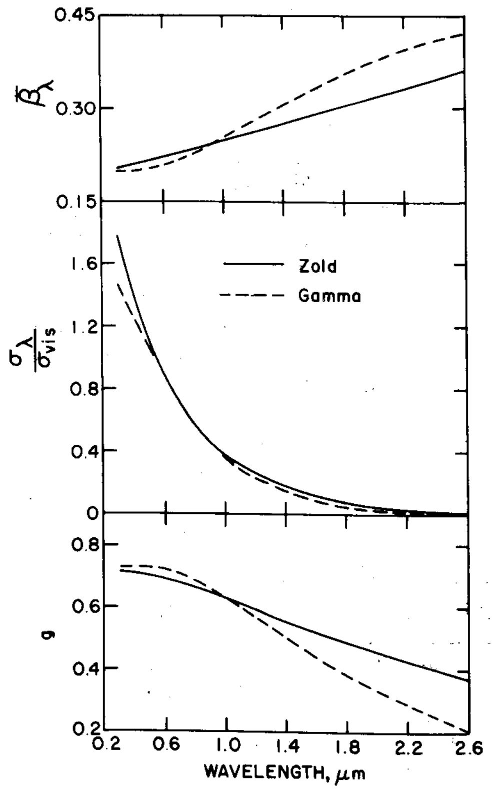

Fig. 3. The aerosol parameters $\bar{\beta}_\lambda$, $\sigma_\lambda/\sigma_{\text{vis}}$ and g, as computed from Mie theory for gamma and Zold size distributions.

a fit to our numerical $\beta_\lambda(\mu)$ results yields

$$\beta_\lambda(\mu)/\bar{\beta}_\lambda = [0.415 + 1.070\mu + 1.047\mu^2]^{-1}. \tag{14}$$

This fit is used in the two-stream approximation for the aerosol radiative properties.

4. Results

To illustrate the various interactive effects upon $d\alpha_p/d\tau_{\text{vis}}$, as defined by eq. (12), we initially consider clear-sky conditions with $\alpha_s = 0$. Table 2 shows a number of results in which we make systematic changes within our model to illustrate interactive effects. The columns with the headings r_λ and β_λ simply refer to the manner in which either Rayleigh or aerosol scattering is incorporated into the angular and wavelength integrations. Thus with 0 for r_λ, we delete Rayleigh scattering, while with the use of $\bar{r}_\lambda$ the angular dependence of Rayleigh scattering is ignored, as is the case with aerosol scattering when $\bar{\beta}_\lambda$ is employed. When we incorporate $r_\lambda(\mu)$ and $\beta_\lambda(\mu)$, then all angular effects are included.

Consider first the bottom row in Table 2. It gives the results for a single-wavelength calculation at 0.55 μm with no Rayleigh scattering. The $d\alpha_p/d\tau_{\text{vis}}$ values of 0.402 and 0.436 are consistent with eq. (1), since to convert a $d\alpha_p/d\tau_{\text{vis}}$ value for α_s = 0 to one for which the aerosol-free planetary albedo is α_{0p} amounts to multiplying by the factor $(1 - \alpha_{0p})^2$. Thus for α_{0p} = 0.3, the 0.55 μm values of Table 2 convert to $d\alpha_p/d\tau_{\text{vis}}$ = 0.20 and 0.21 for the respective gamma and Zold distributions. All other results in Table 2 refer to full-flux calculations. Moving up to the next row in

Table 2. *Comparison of $d\alpha_p/d\tau_{\text{vis}}$ for $\alpha_s = 0$ and $\omega = 1.0$. The columns labeled r_λ and β_λ denote the manner in which Rayleigh and aerosol scattering has been incorporated into the wavelength and angular integrations (see text for further details)*

r_λ	β_λ	Gamma	Zold
$r_\lambda(\mu)$	$\beta_\lambda(\mu)$	0.188	0.205
$r_\lambda(\mu)$	$\bar{\beta}_\lambda$	0.205	0.224
$\bar{r}_\lambda$	$\beta_\lambda(\mu)$ or $\bar{\beta}_\lambda$	0.220	0.242
0	$\beta_\lambda(\mu)$ or $\bar{\beta}_\lambda$	0.276	0.308
0.55 μm value with $r_\lambda = 0$		0.402	0.436

Table 2, we again ignore Rayleigh scattering. Thus the planetary (or spherical) albedo is equivalent to the planar albedo with isotropic irradiation from above. Providing the aerosol layer is optically thin, as in the case here, then there is no distinction between using $\beta_\lambda(\mu)$ or $\bar{\beta}_\lambda$. Note that the full-flux calculation reduces $d\alpha_p/d\tau_{\rm vis}$ by roughly 30%, for both size distributions, relative to the 0.55 μm results. Thus even in the absence of Rayleigh scattering the 0.55 μm value of $d\bar{\alpha}_\lambda/d\tau_{\rm vis}$ is not representative of $d\alpha_p/d\tau_{\rm vis}$.

We next include Rayleigh scattering, but use its spherical reflectivity $\bar{r}_\lambda$. Again, for an optically thin aerosol layer, there is no distinction between the use of $\beta_\lambda(\mu)$ or $\bar{\beta}_\lambda$, since the upward reflected radiation, due to Rayleigh scattering, is isotropic. The inclusion of Rayleigh scattering produces a 20% further reduction in $d\alpha_p/d\tau_{\rm vis}$. Finally we incorporate the directional effects first of Rayleigh scattering and then of aerosol scattering. Collectively these produce an additional 15% reduction of $d\alpha_p/d\tau_{\rm vis}$. The point of Table 2, of course, is that when all the separate factors are accounted for, the net reduction in $d\alpha_p/d\tau_{\rm vis}$ is by more than a factor of two. Moreover, there is no significant difference between the gamma and Zold distribution results.

In Fig. 4 we illustrate the wavelength dependence of $d\bar{\alpha}_\lambda/d\tau_{\rm vis}$ for the gamma distribution and for conditions corresponding to the top and next to last rows in Table 2; i.e., with and without Rayleigh scattering. Clearly the inclusion of Rayleigh scattering substantially reduces $d\bar{\alpha}_\lambda/d\tau_{\rm vis}$ at the shorter wavelengths. Interestingly enough, the often-used 0.55 μm reference wavelength coincides with the *maximum* value for $d\bar{\alpha}_\lambda/d\tau_{\rm vis}$.

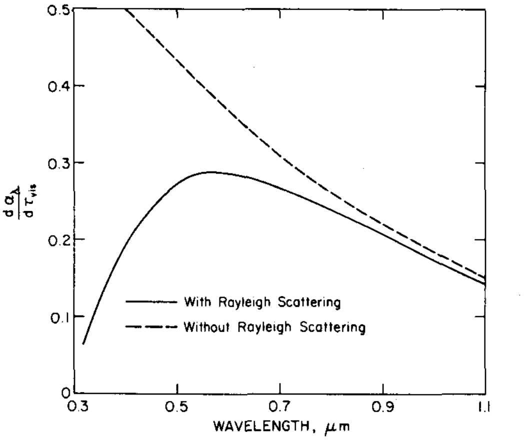

Fig. 4. The wavelength dependence of $d\bar{\alpha}_\lambda/d\tau_{\rm vis}$, with and without the incorporation of Rayleigh scattering, for the gamma size distribution, $\alpha_s = 0$ and $\omega = 1.0$.

Since Luther's (1976a) model results are for clear-sky conditions, his and our models may be directly compared. Luther employed the gamma distribution, a real refractive index of 1.45, which is close to that of Palmer and Williams (1975), but an imaginary refractive index of 0.005, well above $0(10^{-7})$ from Palmer and Williams. Thus Luther's model does include aerosol absorption. Luther has kindly supplied us with a detailed description of his model, from which we find that his model corresponds to the single scattering albedo $\omega \simeq 0.97$. In Table 3 we compare our results of $\Delta\alpha_p/\tau_{\rm vis}$, for $\tau_{\rm vis} = 0.109$ which is the value employed by Luther. The agreement between the two models is quite satisfactory. For comparative purposes we have also included $\omega = 1.0$ results for both the gamma and Zold distributions. Furthermore, comparing the $\omega = 1.0$ results of Table 3 with the top row of Table 2 shows that there is a slight reduction in $\Delta\alpha_p/\tau_{\rm vis}$ for $\tau_{\rm vis} = 0.109$, as compared with $d\alpha_p/d\tau_{\rm vis}$. In the present paper $d\alpha_p/d\tau_{\rm vis}$ is defined as $\Delta\alpha_p/\tau_{\rm vis}$ for $\tau_{\rm vis} = 10^{-5}$.

To incorporate clouds within our model, we adopt here, in contrast to Section 2, an empirical prescription for the angular dependence of cloud reflectivity. Since clouds are negligible absorbers of solar radiation at wavelengths for which Rayleigh scattering is significant, where we imbed the cloud within the atmosphere is immaterial (Konyukh et al., 1979), and the easiest procedure is to replace the clouds by an equivalent reflecting surface. To incorporate the zenith-angle

Table 3. *Comparison of $\Delta\alpha_p/\tau_{\rm vis}$ results, as a function of surface albedo, for $\tau_{\rm vis} = 0.109$ and clear-sky conditions. The columns labeled Gamma and Zold refer to the present model calculations*

α_s	Luther (1976a) $\omega \simeq 0.97$	Gamma $\omega = 0.97$	Gamma $\omega = 1.0$	Zold $\omega = 1.0$
0	0.158	0.154	0.170	0.183
0.10	0.127	0.119	0.138	0.150
0.15	0.111	0.102	0.124	0.134
0.25	0.082	0.072	0.097	0.105
0.50	0.017	0.003	0.042	0.046

dependence of the albedo of this equivalent surface cloud, we let

$$\alpha_s(\mu) = \bar{\alpha}_s + \frac{d\alpha_s}{d\mu}(\mu - 2/3) \tag{15}$$

which satisfies the relationship between planar and spherical albedos. The formulation of Section 2 is then modified to accommodate $\alpha_s(\mu)$ by replacing α_s within the numerator of the second term of eq. (7) by $\alpha_s(\mu)$, whereas in the denominator it is replaced by $\bar{\alpha}_s$. In turn

$$\bar{\alpha}_{0\lambda} = 2\int_0^1 \alpha_{0\lambda}(\mu)\mu\, d\mu$$

replaces eq. (8).

For this cloud-albedo model we employ $\alpha_s = 0.44$ (Cess, 1976) and $d\alpha_s/d\mu = -0.55$ (Lian and Cess, 1977). For the Zold distribution, and for a clear-sky surface albedo of 0.1, which coincide with the model of Pollack et al. (1976), we find

$$d\alpha_p/d\tau_{\text{vis}} = 0.151 \text{ for clear skies} \tag{16a}$$

$$= 0.033 \text{ for cloudy skies and } d\alpha_s/d\mu = -0.55 \tag{16b}$$

$$= 0.058 \text{ for cloudy skies and } d\alpha_s/d\mu = 0. \tag{16c}$$

Assuming, as did Pollack et al. (1976), 50% cloud cover, and employing eqs. (16a) and (16b), we find that on a global average $d\alpha_p/d\tau_{\text{vis}} = 0.092$, in good agreement with $d\alpha_p/d\tau_{\text{vis}} = 0.082$ obtained from the results reported by Pollack et al. Equations (16) illustrate that stratospheric aerosols produce an effect upon clear-sky regions that is five times that produced upon cloudy sky regions. This result may have important regional consequences.

An interesting by-product of the above calculation concerns the possible importance of including the angular dependence of the albedo of some types of highly reflecting surfaces. Equations (16b) and (16c) illustrate that incorporation of the angular dependence of the albedo of the effective surface cloud nearly halves the effect of stratospheric aerosols upon the cloudy sky $d\alpha_p/d\tau_{\text{vis}}$. To incorporate the angular dependence of surface reflectivity, we rephrase the preceding surface-cloud model for application to cloud-free conditions over a "global" desert. From Lian (1978) $d\alpha_s/d\mu = -0.7$ for typical desert conditions. Upon choosing $\alpha_s = 0.35$ for a desert albedo, our model, employing the Zold distribution, produces

$$d\alpha_p/d\tau_{\text{vis}} = 0.058 \text{ for } d\alpha_s/d\mu = -0.7 \tag{17a}$$

$$= 0.098 \text{ for } d\alpha_s/d\mu = 0. \tag{17b}$$

The obvious conclusion is that for regional applications, the angular dependence of the surface reflectivity could have a significant influence.

5. Concluding remarks

The primary goal of the present study is to illustrate the importance of performing full-flux calculations, together with incorporating the angular dependence of scattering within the earth-atmosphere system, when modeling the influence of volcanically induced stratospheric aerosols upon the earth's albedo. The incorporation of these interactive effects thus explains the substantial differences between existing aerosol-albedo models. Although the present study was restricted to global mean conditions, the results nevertheless suggest that aerosol-induced albedo modifications could be highly regionally dependent.

While we have considered here only effects upon solar radiation, stratospheric aerosols also produce a greenhouse effect. The net climatological influence of increased stratospheric aerosols will then be the difference between cooling due to an enhanced albedo and heating resulting from the increased greenhouse effect. Thus, the error in the net effect will be greater than the error in just $d\alpha_p/d\tau_{\text{vis}}$.

6. Acknowledgements

We thank Dr F. M. Luther (Lawrence Livermore Laboratory) for supplying us with a detailed description of his aerosol model, and Dr A. P. Prishivalko (B.S.S.R. Academy of Sciences) for programming assistance. This work was jointly supported by the Byelorussian Academy of Sciences and the United States National Science Foundation, with the latter support stemming in part from NSF Grant ENG-77-09124 to SUNY Stony Brook. One of us (R. D. C.) appreciates the hospitality shown to him during visits to both the National Center for Atmospheric Research and the B.S.S.R. Academy of Sciences, and for travel funds supplied through DOE Agreement 79-EV0220.00 to the National Center for Atmospheric Research.

REFERENCES

Budyko, M. I. 1974. *Climate and life.* New York: Academic Press, 508 pp.

Cadle, R. D. and Grams, G. W. 1975. Stratospheric aerosol particles and their optical properties. *Rev. Geophys. Space Phys. 13*, 475–501.

Cess, R. D. 1976. Climate change: An appraisal of atmospheric feedback mechanisms employing zonal climatology. *J. Atmos. Sci. 33*, 1831–1843.

Chandrasekhar, S. 1960. *Radiative transfer.* New York: Dover, 393 pp.

Coakley, J. A., Jr and Chýlek, P. 1975. The two-stream approximation in radiative transfer: Including the angle of the incident radiation. *J. Atmos. Sci. 32*, 409–418.

Deirmendjian, D. 1969. *Electromagnetic scattering on spherical polydispersions.* New York: American Elsevier Publishing Co., 290 pp.

Hansen, J. 1969. Exact and approximate solutions for multiple scattering by cloudy and hazy atmospheres. *J. Atmos. Sci. 26*, 478–487.

Harshvardhan 1979. Perturbations of the zonal radiation balance by a stratospheric aerosol layer. *J. Atmos. Sci. 36*, 1274–1285.

Harshvardhan and Cess, R. D. 1976. Stratospheric aerosols: Effect upon atmospheric temperature and global climate. *Tellus 28*, 1–10.

Hoyt, D. V. 1977. A redetermination of the Rayleigh optical depth and its application to selected solar radiation problems. *J. Appl. Meteorol. 16*, 432–438.

Hunt, G. E. 1971. A review of computational techniques for analyzing the transfer of radiation through a model cloudy atmosphere. *J. Quant. Spectrosc. Radiat. Transfer 11*, 655–690.

Kolesnikov, P. M. and Cess, R. D. 1980. Investigation of radiation scattering by sulfuric acid droplets. *J. Engr. Phys. 38*, 863–870 (in Russian).

Konyukh, L. A., Yurevich, F. B. and Cess, R. D. 1979. Tropospheric aerosols: Effects upon the surface and surface-atmosphere radiation budgets. *J. Quant. Spectrosc. Radiat. Transfer 22*, 483–488.

Lian, M. S. 1978. Energy balance climate models and climate sensitivity. Ph.D. Thesis, State University of New York at Stony Brook, 109 pp.

Lian, M. S. and Cess, R. D. 1977. Energy balance climate models: A reappraisal of ice-albedo feedback. *J. Atmos. Sci. 34*, 1058–1062.

Luther, F. M. 1976a. Relative influence of stratospheric aerosols on solar and longwave radiative fluxes for a tropical atmosphere. *J. Appl. Meteorol. 15*, 951–955.

Luther, F. M. 1976b. Aerosols and the radiation budget: One-wavelength versus full-flux calculations. *Proceedings of the symposium on radiation in the atmosphere.* Princeton: Science Press, 475–476.

Palmer, K. F. and Williams, D. 1975. Optical constants of sulfuric acid; application to the clouds of Venus? *Appl. Optics 14*, 208–219.

Pinnick, R. G., Rosen, J. M. and Hoffman, D. J. 1976. Stratospheric aerosol measurements III: Optical model calculations. *J. Atmos. Sci. 33*, 304–314.

Pollack, J. B., Toon, O. B., Sagan, C., Summers, A., Baldwin, B. and Van Camp, W. 1976. Volcanic explosions and climatic change: A theoretical assessment. *J. Geophys. Res. 81*, 1071–1083.

Rosen, J. M. 1971. The boiling point of stratospheric aerosols. *J. Appl. Meteorol. 10*, 1044–1046.

Russell, P. B. and Hake, R. D., Jr 1977. The post-Fuego stratospheric aerosol: Lidar measurements, with radiative and thermal implications. *J. Atmos. Sci. 34*, 163–177.

Thekaekara, M. P. 1972. Solar energy outside the earth's atmosphere. *Solar Energy 14*, 109–127.

Toon, O. B. and Pollack, J. B. 1976. A global average model of atmospheric aerosols for radiative transfer calculations. *J. Appl. Meterol. 15*, 225–246.

Wiscombe, W. J. and Grams, G. W. 1976. The backscattered fraction in two-stream approximations. *J. Atmos. Sci. 33*, 2440–2451.

СТРАТОСФЕРНЫЕ ВУЛКАНИЧЕСКИЕ АЭРОЗОЛИ; МОДЕЛЬ ДЛЯ ИЗУЧЕНИЯ ВЛИЯНИЯ ВЗАИМОДЕЙСТВИЯ НА СОЛНЕЧНУЮ РАДИАЦИЮ

В литературе имеются серьезные расхождения по вопросу воздействия стратосферных аэрозолей вулканического происхождения на альбедо Земли. Для исследования этих расхождений в данной статье разработано двухпотоковое приближение, хорошо согласующееся с уточненными расчетами. Модель использует радиационные свойства аэрозоля, определяемые по дифракционной теории рассеяния Ми.

В работе сделана систематическая модификация модели для иллюстрации отдельных эффектов Рэлеевского рассеивания, угловой зависимости Рэлеевского и аэрозолъного рассеивания и расчеты полного потока в зависимости от отдельной длины волны. Если все эффекты взаимодействия включены, влияние аэрозоля на альбедо Земли уменьшается в соответствии с обычными расчетами по отдельной длине волны более чем в два раза, этот результат устраняет вышеуказанные несоответствия в литературе.