Tellus (1981), *33*, 340–350

A study of the sensitivity of the winter mean meridional circulation to sources of heat and momentum

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(Manuscript received September 23; in final form December 19, 1980)

ABSTRACT

The time and zonally averaged momentum and thermodynamic equations on a spherical earth are combined to formulate an equation for the mean meridional circulation based on the quasi-static assumption discussed in Eliassen (Astrophys. Nor. 1951, 19–60). The forcing terms and coefficients in the equation are taken from previously published zonally and seasonally averaged data, which include the net radiative heating, condensation heating, boundary layer friction and sensible heating, tropical cumulus friction, the mean zonal wind and large-scale eddy transports of momentum and heat in winter. The computed meridional circulation in the Northern Hemisphere compares well to the observations with a three-cell structure of Hadley, Ferrel and polar circulations. To estimate the relative importance in maintaining the meridional circulation a series of solutions is computed by removing one of the individual sources at a time and the solutions are compared to the standard solution. It is found that the Hadley cell maintains the intensity level at only one-third of the standard solution if the condensation heating is absent. Although there is virtually no source in the tropics due to eddy momentum and heat transports, the mid-latitude eddy transport affects the Hadley circulation significantly, suggesting that the cooling due to eddy heat transport in the subtropics is an important component balancing the adiabatic heating in the downward branch of the Hadley cell. The individual effects of radiation, surface sensible heating and surface friction are all comparable, but the contributions in the balance of meridional circulation is, on the global average, not more than one-third of the condensation and resultant eddy effect, although the effects are different for the individual Hadley, Ferrel and polar circulations.

1. Introduction

The primary source of available potential energy for atmospheric circulation is the solar differential heating due to the sphericity of the earth. The seasonal mean insolation is a smooth function of latitude. The atmospheric response is, however, very different in low and high latitudes, primarily due to the different Coriolis force and redistribution of heating by hydrological and other thermal and dynamical processes. The equator-to-pole pressure gradient maintained by the differential heating produces poleward horizontal motions in low latitudes, leading to a horizonal mass divergence which is balanced by the upward vertical motion (Fig. 1). The poleward meridional motions, however, lose the momentum by increasing Coriolis deflection force in the mid-latitudes, causing meridional mass convergence which forces the air to subside against the stable stratification in the subtropics. The closed Hadley cell thus formed in low latitudes characterizes the circulation responding to the meridional differential heating, but the actual intensity of the circulation is largely enhanced by the additional heating associated with the upward convection in the cell due to release of latent energy (Schneider and Lindzen, 1977). In this way the Hadley cell equalizes the horizontal temperature in the vast area in subtropics but, at the same time, it maintains the horizontal tempera-

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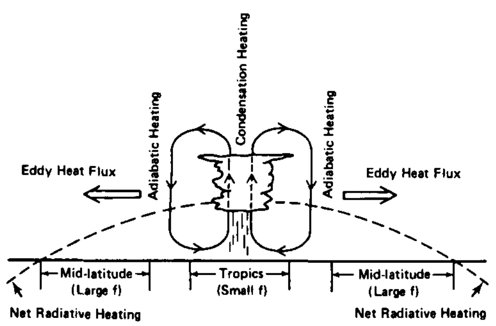

Fig. 1. A schematic picture of Hadley circulation.

ture gradient in the mid-latitudes at the expense of potential energy associated with the downward motions.

In addition to the condensation heating in the intertropical convergent zone, several other sources have been considered as important in the past studies. Kuo (1956) analyzed the meridional circulation driven by momentum sources generated by large-scale eddies in mid-latitudes. He obtained a hemispheric three-cell structure from the momentum source alone. On the other hand, a smooth hemispheric diabatic heating produced only a weak single cell in the hemisphere which is hardly comparable to the observations. Thus, he concluded that the meridional circulation is not driven by the heat source but by the eddy momentum source. But notable discrepancies in his result are that the three cells are all equally weak and that the assumed heating is quite different from that of recent observations. Dickinson (1971) followed the approach by Kuo but used an updated heating distribution confined within the equatorial region, and obtained an analytical solution which compares well with observation. The radiational cooling proportional to the temperature perturbation is considered as an important dissipation in his model. However, since eddy heat transport from the subtropics was not considered, the question has been left unresolved as to whether the Newtonian cooling in his model does actually represent the radiative cooling. More recently Schneider and Lindzen (1977) and Schneider (1977) made a series of analyses using a model of zonally symmetric atmospheric motions in which the zonal motions and zonal mean temperature both are dependent variables forced by the meridional circulation. The new result is that the frictional and heating effects of cumulus convection are most important sources which produce the major three-cell meridional circulation, although some discrepancy in the model is attributed to the eddy heat transport neglected in the study.

In numerical simulation experiments (e.g., Manabe et al., 1965) the energy budget diagnosis shows that the adiabatic heating due to downward vertical motion of the Hadley cell tends to balance the cooling due to poleward heat transport by mid-latitude eddies. This balance compares well with the statistics by Oort and Rasmusson (1971) which were used in Sasamori (1978, Fig. 1).

The mathematical basis of Kuo (1956) and Dickinson (1971) is found in the earlier study by Eliassen (1951), in which it is assumed that the meridional circulation is essentially an ageostrophic motion forced by the frictional and heating sources in quasi-stationary geostrophic zonal mean flow. By combining the westerly momentum equation and the thermodynamic equation through the geostrophic relation, a Poisson equation in the meridional plane is formulated, which is basically the same as the ω-equation currently being used for the quasi-geostrophic motions in mid-latitudes.

The equation indicates that the vertical static stability and the Coriolis horizontal stability defines the depth and width of the meridional circulation, while the horizontal differential of heating and vertical differential of momentum sources determines the intensity. However, Kuo (1956) and Dickinson (1971) both have modified Eliassen's model, assuming that a strict stationary state exists, and combined the momentum and thermodynamic equations through the linear Rayleigh friction and Newtonian cooling. Although their subsequent assumption that the ratio between two linear coefficients be unity has turned out to be equivalent to the quasi-stationary assumption by Eliassen the two approaches involve a basic conceptual difference. In the present paper we base our analysis on Eliassen's original model: the zonal wind and temperature are assumed to be known and remain in geostrophic balance everywhere on the globe. The forcing due to heating and friction is evaluated using the observations and no attempt is made to parameterize the sources in terms of the meridional circulation.

2. The governing equation

The momentum, thermodynamic, and continuity equations for hydrostatic motions on the spherical earth averaged zonally and in time are (see Boyd, 1976):

$$\overline{\left(\frac{\partial u}{\partial t}\right)} + \bar{w}\frac{\partial \bar{u}}{\partial z} - \hat{f}\bar{v} = \bar{M} \tag{2.1}$$

$$\overline{\left(\frac{\partial \phi_z}{\partial t}\right)} + \bar{v}\tilde{f}\frac{\partial \bar{u}}{\partial z} + N^2\bar{w} = \bar{H} \tag{2.2}$$

$$L_\theta(\bar{v}) + L_z(\bar{w}) = 0 \tag{2.3}$$

where z is $-h\log(p/p_0)$, p denotes the pressure, p_0 the standard reference pressure (1000 mb), and h is the scale height of a standard atmosphere, which is assumed to be 8 km. N is the Brunt-Väisälä frequency, u the zonal wind, w the vertical velocity, v the meridional velocity, and ϕ_z the partial derivative of the geopotential height with respect to z. The overbar $(\bar{\ })$ above any quantity indicates that it has been averaged both zonally and over a winter defined by three months, December through February. The terms $\hat{f}$ and $\tilde{f}$ denote the modified Coriolis parameters in spherical coordinates, which are:

$$\hat{f} = f + \frac{\bar{u}\tan\theta}{a} - \frac{\partial\bar{u}}{a\partial\theta}$$
$$\tilde{f} = f + 2\frac{\bar{u}}{a}\tan\theta \tag{2.4}$$

where $f = 2\Omega\sin\theta$, with Ω being the rate of angular rotation of the earth, a is the radius of the earth, θ is the latitude, operators $L_z(\ )$ and $L_\theta(\ )$ are defined as:

$$L_z(\ ) = \frac{\partial}{\partial z}(\ ) - \frac{(\ )}{h}$$
$$L_\theta(\ ) = \frac{\partial}{a\partial\theta}(\ ) - \frac{\tan\theta}{a}(\ ) \tag{2.5}$$

The terms for the momentum and heat sources, $\bar{M}$ and $\bar{H}$, are respectively:

$$\bar{M} = \bar{M}_e + \bar{d} \quad \text{and} \quad \bar{H} = \bar{H}_e + \frac{R\bar{Q}}{c_p h}$$

with

$$\bar{M}_e = -L_\theta^1(\overline{u'v'} + \overline{u^*v^*}) - L_z(\overline{u'w'} + \overline{u^*w^*})$$
$$\bar{H}_e = -L_\theta(\overline{v'\phi_z'} + \overline{v^*\phi_z^*}) - L_z(\overline{w'\phi_z'} + \overline{w^*\phi_z^*}) \tag{2.6}$$

where $L_\theta^1(\ )$ denotes differential operator

$$L_\theta^1(\ ) = \frac{\partial}{a\partial\theta}(\ ) - \frac{2}{a}\tan\theta(\ ) \tag{2.7}$$

d the subsynoptic-scale friction term, Q the diabatic heating, R the gas constant for air, and c_p the specific heat of air at a constant pressure. Primed variables (′) indicate the transient eddy component, and asterisked (*) variables denote the stationary standing eddy component of any quantity.

In the derivation of (2.1), (2.2), and (2.3) it has been assumed that the time and zonally averaged temperature is composed of a basic state component $\bar{T}_1$ which is a function of height alone, and a deviation from the basic state temperature, $\bar{T}_2$, which is a function of both height and latitude. Thus,

$$\bar{T} = \bar{T}_1(z) + \bar{T}_2(z,\theta) \tag{2.8}$$

Using (2.8) and the hydrostatic equation, the geopotential may be defined as,

$$\frac{d\bar{\phi}_1}{dz} = \frac{R}{h}\bar{T}_1$$
$$\frac{\partial\bar{\phi}_2}{\partial z} = \frac{R}{h}\bar{T}_2 \tag{2.9}$$

The total geopotential is written as

$$\bar{\phi} = \bar{\phi}_1(z) + \bar{\phi}_2(z,\theta) \tag{2.10}$$

where $\bar{\phi}_1$ amd $\bar{\phi}_2$ are the basic state and deviation components respectively. The vertical static stability is then

$$N^2 = \frac{R}{h}\left(\frac{d\bar{T}_1}{dz} + \frac{R}{c_p h}\bar{T}_1 + \frac{\partial\bar{T}_2}{\partial z}\right) \tag{2.11}$$

The term $(R\bar{T}_2/(c_p h))$ has been neglected in order to allow the zonally averaged equations to be conservative with respect to the sum of kinetic and available potential energy.

The quasi-static approximation is applied to the zonally averaged equations by assuming that the zonal wind and the meridional gradient of the geopotential are in geostrophic balance:

$$\tilde{f}\frac{\partial u}{\partial z} = -\frac{\partial\phi_z}{a\partial\theta} \tag{2.12}$$

and therefore

$$\overline{\frac{\partial}{\partial t}\left(\tilde{f}\frac{\partial u}{\partial z}\right)} = -\overline{\frac{\partial}{\partial t}\left(\frac{\partial \phi_z}{a\partial \theta}\right)} \tag{2.13}$$

From (2.3) the meridional streamfunction χ is defined

$$\begin{aligned} \bar{v} &= L_z(\chi) \\ \bar{w} &= -L_\theta(\chi) \end{aligned} \tag{2.14}$$

By eliminating $\bar{u}$ and $\bar{\phi}$ between (2.1) and (2.2) by use of (2.13) we obtain

$$\begin{aligned} &A\frac{\partial^2 \chi}{\partial z^2} + B\frac{\partial^2 \chi}{\partial y^2} + C\frac{\partial^2 \chi}{\partial y\, \partial z} + D\frac{\partial \chi}{\partial z} \\ &\quad + E\frac{\partial \chi}{\partial y} + F\chi \\ &= G - \underline{\left(\frac{\partial \bar{u}}{\partial z}\frac{2}{a}\tan\theta\right)\frac{\partial \bar{u}}{\partial t}} \end{aligned} \tag{2.15}$$

where $dy = ad\theta$. The coefficients A through G are

$$\begin{aligned} A &= \tilde{f}\hat{f} \\ B &= N^2 \\ C &= 2\tilde{f}\frac{\partial \bar{u}}{\partial z} \\ D &= \frac{\partial}{\partial y}\left(\tilde{f}\frac{\partial \bar{u}}{\partial z}\right) + \tilde{f}L_z(\hat{f}) - \frac{\tan\theta}{a}\frac{\partial \bar{u}}{\partial z}\tilde{f} \\ E &= \tilde{f}L_z\left(\frac{\partial \bar{u}}{\partial z}\right) - \frac{N^2}{a}\tan\theta - \frac{\partial N^2}{\partial y} \\ F &= -h^{-1}\left(\tilde{f}\frac{\partial \hat{f}}{\partial z}\right) - \frac{\tan\theta}{a}\tilde{f}\frac{\partial^2 \bar{u}}{\partial z^2} \\ &\quad - \frac{\partial}{\partial y}\left(\tilde{f}\frac{\partial \bar{u}}{\partial z}\right)h^{-1} - \frac{N^2}{a^2}\sec^2\theta \\ &\quad + \frac{\tan\theta}{a}\frac{\partial N^2}{\partial y} \\ G &= -\tilde{f}\frac{\partial \bar{M}}{\partial z} - \frac{\partial \bar{H}}{\partial y} \end{aligned} \tag{2.16}$$

It is found by scale analysis that the order of magnitude of all coefficients in (2.16), except C, is determined by terms which do not involve the mean zonal wind. Since the term involving C in (2.15) is one order of magnitude smaller than the largest terms on the left hand side of (2.15), it is implied that the mean zonal wind may be neglected without seriously affecting the solution. The underlined term in (2.15) is at least three orders of magnitude smaller than the other terms in the equation. The result is due in part to the observation that the mean zonal wind changes a maximum of only 0.5 m s^{-1} over a period of a month. With the underlined term neglected, (2.15) is the time independent, governing equation for the meridional circulation.

We assume that the lower surface at $z = 0$ is featureless and the disturbances with the source in the troposphere vanish in the middle stratosphere which is set at 22 km. Together with the lateral boundary condition at the poles we set the boundary condition as

$$\chi = 0 \quad \text{at } z = 0 \text{ and } 22 \text{ km} \quad \text{and } \theta = \pm 90°$$

3. Standard solution of winter circulation

The data sources for the coefficients A through G in (2.16) are summarized in Table 1.

For those data which are not available in the Southern Hemisphere (S.H.), we have substituted the summer data of the Northern Hemisphere (N.H.). The eddy forcing terms are summarized in Figs. 2 and 3. Both eddy sensible heating and momentum sources are large in middle and high latitudes and larger in winter hemisphere than summer hemisphere. There is a region in the winter hemisphere where the eddy momentum source tends to cancel the eddy heating source. The net radiative heating (Fig. 4) and the condensation heating rate (Fig. 5) are given by a conventional

Table 1. *Data source for the coefficients A through G in (2.16)*

1. $\bar{u}, \bar{\phi}, N^2, \bar{H}_e, \bar{M}_e$	Oort and Rasmusson (1971)
2. Radiative heating	London (1957)
3. Condensation heating	Newell et al. (1972)
4. Sensible heating	Vincent (1969)
5. Surface friction	Kung (1967) and Oort and Rasmusson (1971)
6. Cumulus friction	Yanai et al. (1973) and Schneider and Lindzen (1976)

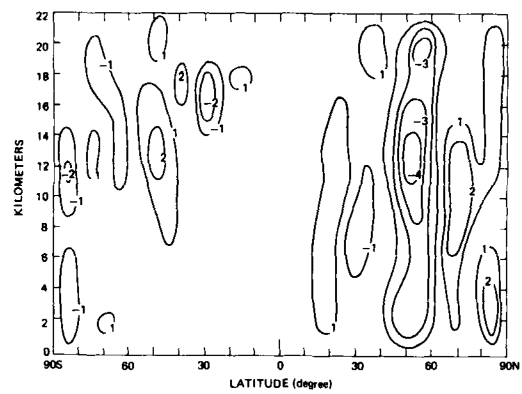

Fig. 2. The forcing term $-\partial H_e/\partial y$, arising from eddy heat transport in units of $10^{-13}\ s^{-3}$ (Oort and Rasmusson, 1971). The data in the S.H. beyond 20 °C are substituted by the summer data in the N.H.

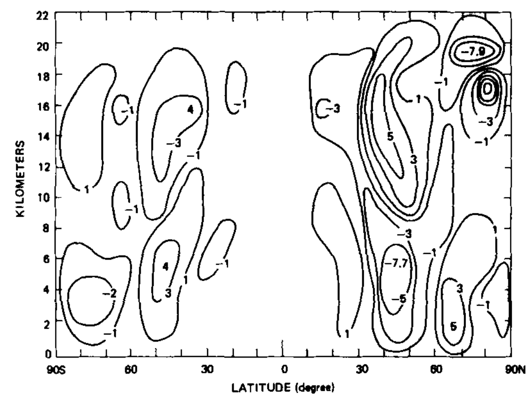

Fig. 3. Same as Fig. 2, except the forcing term $-\bar{f}\,\partial M_e/\partial z$, arising from eddy momentum transport in units of $10^{-13}\ s^{-3}$.

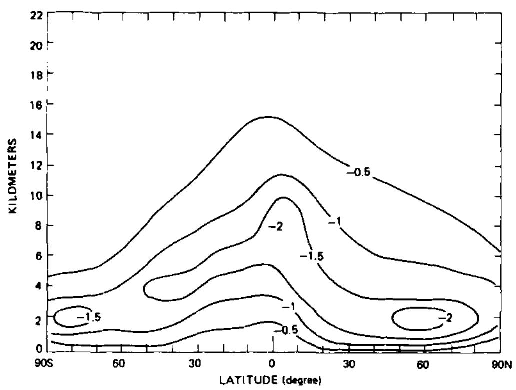

Fig. 4. Net radiative heating (deg/day) by solar and infrared radiation (London, 1957).

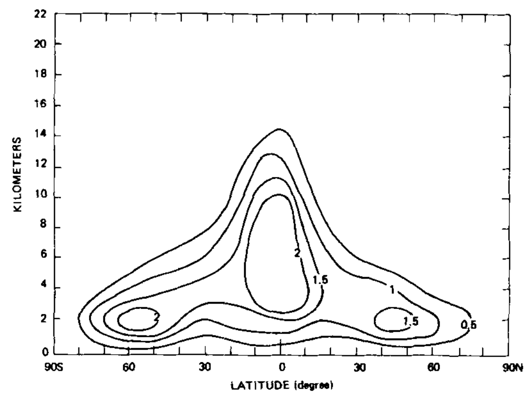

Fig. 5. Rate of heating by water vapor condensation (deg/day), Vincent (1969).

unit in deg/day. The radiative cooling is large throughout the troposphere, but its horizontal differentials appear to be smaller than those of condensation heating, which shows large meridional change in the low latitudes at about 10 km level. The sensible heating in the lower layer (Vincent, 1969) is shown in Table 2. In the model we have set these heating rates at 2 km level.

We consider two components in subsynoptic friction, the boundary layer surface friction and the cumulus friction: $d = F_b + F_c$. The surface friction is computed using the linear drag for the zonal mean motion:

$$F_b = -\beta \bar{u}_b$$

where $\bar{u}_b$ is the zonal wind at the top of the boundary layer which is assumed at 2 km and β is a constant which is approximately equal to 1/day (Kung, 1967). Vertical momentum transport by cumulus clouds is incorporated in the tropical rain belt using a formula by Schneider and Lindzen (1976):

$$F_c = -\frac{1}{\rho}\frac{\partial}{\partial z}\left[M_c(\bar{u} - \bar{u}_c)\right]$$

where $\bar{u}_c$ is the velocity of the cloud, which is assumed as constant and equal to $\bar{u}_b$, ρ is the air density, and M_c is the mass flux in the cumulus clouds. We take the numerical value of M_c estimated by Yanai et al. (1973). The vertical distribution is shown in Table 3. The horizontal distribution of F_c is assumed to vanish at neighboring latitudes $\pm 10°$.

Table 2. *The sensible heating at the top of the boundary layer (2 km), based on January average data obtained from Vincent (1969). Units are in °C day*$^{-1}$

Latitude	Sensible heating	Latitude	Sensible heating
90° N	0.06	5°	0.42
85°	0.06	10°	0.38
80°	0.08	15°	0.47
75°	0.10	20°	0.56
70°	0.12	25°	0.62
65°	0.14	30°	0.69
60°	0.16	35°	0.36
55°	0.28	40°	0.04
50°	0.42	45°	−0.13
45°	0.56	50°	−0.29
40°	0.70	55°	−0.13
35°	0.76	60°	0.03
30°	0.81	65°	0.02
25°	0.79	60°	0.01
20°	0.76	75°	0.01
15°	0.71	80°	0.05
10°	0.66	85°	0.05
5°	0.56	90° S	0.05
0°	0.47		

Table 3. *The cumulus friction at 5 °S latitude in units of* 10^{-5} *m s*$^{-2}$

Height (km)	Cumulus friction (F_c)
0	0.0
2	−1.03
4	−0.78
6	−0.68
8	−0.41
10	1.06
12	1.49
14	1.96
16	1.86
18	−0.43
20	0.0
22	0.0

The coefficients A through G are computed at grid points with $\Delta z = 2$ km and $\Delta\theta = 5°$ and (2.15) is solved using the method by Lindzen and Kuo (1969). The solution is shown in Fig. 6. The three-cell circulation in the N.H. compares well with the analysis by Newell et al. (1972, p. 152, Fig. 4.13), except that the polar circulation is calculated only in the upper troposphere. The intensity ratio between the Hadley and Ferrel cells at the individual maxima is approximately 6:1, which is compared to 10:1 of the analysis by Newell et al. (1972). The strength of the Hadley cell in the S.H. is only 1/10 of the cell in the N.H. This ratio is too small by factor 2 as compared to observation. Also, the core of the Hadley cell calculated at 10 km is about 2.5 km too high compared to the analysis by Newell et al. (1972). The agreement of the calculation with the observation in the S.H. is not as good as in the N.H. partly due to the crude approximation of input data taken from the summer in the N.H.

4. Sensitivity calculations

In this section the numerical solutions of the meridional circulation are compared with the standard solution shown in the previous section when one of the individual source terms is removed at a time. There are several problems in this sort of sensitivity study. First of all, there is a possibility in the real atmosphere that the change in the source producing the different meridional circulation may affect the zonal mean wind and temperature. Fortunately, in (2.1) the acceleration of the zonal wind by the vertical advection of the zonal momentum is smaller than the effect of the Coriolis force, and in (2.2) the rate of temperature change by the horizontal advection is smaller than the adiabatic temperature change by the vertical motion. In fact, a solution obtained with the coefficients in which $\bar{u} = 0$ (shown in Fig. 7) is approximately equal to the standard solution (Fig. 6). Secondly, in the real atmosphere, it may be possible that change in the heating source affects the mean vertical static stability. We have made a calculation using the standard $\bar{u}$ and sources except a constant N^2 which is equal to 10^{-4} s^{-2} everywhere. The solution is shown in Fig. 8. The Hadley cell reduces the intensity by approximately 30%, the Ferrel cell increases intensity by 35%, and the Hadley cell shrinks in depth significantly, showing that the vertical static stability is one of the parameters needed to be specified accurately. Since the present model is incapable of calculating the vertical static stability we assume invariant static stability independent of the changing source.

Strictly speaking, the variational solutions for the individual sources are physically meaningful if the sources are independently variable. In the real atmosphere, for example, the condensation heating

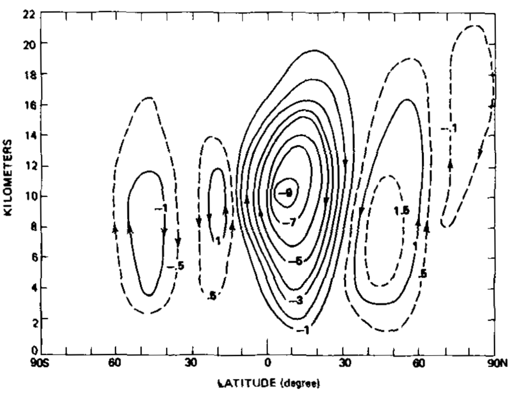

Fig. 6. The standard solution for the meridional streamfunction in units of 10^3 m^2 s^{-1}.

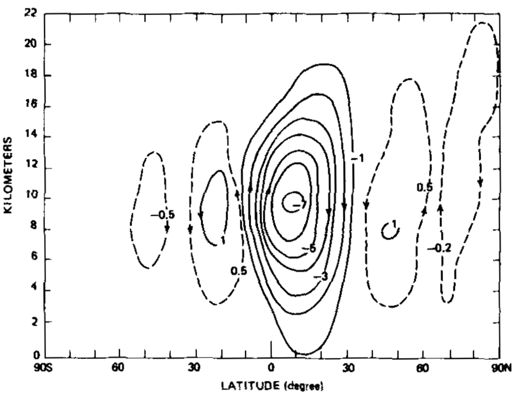

Fig. 7. The streamfunction obtained by setting $\bar{u} = 0$ in the coefficients A through F in (2.16), unit: 10^3 m^2 s^{-1}.

is associated with the cloud distribution and therefore correlates with the radiative heating (Ramanathan and Coakley, 1978). The coupling and feedback among the individual sources are, however, not within the scope of the present analysis.

4.1. *Condensation heating*

Fig. 9 shows the streamfunction obtained without condensation heating. The variation from the standard solution is largest for the Hadley cell. The intensity is reduced to about 1/3 of the standard solution and the axis of the cell being tilted towards S.H. The Ferrel cell increases intensity slightly, but the three cells remain in the N.H. It is interesting to compare this solution with the earlier result by Kuo (1956) which shows that the eddy momentum source alone produces a weak three-cell structure. The enhanced intensity in the Ferrel cell suggests that the circulation may actually be a composite circulation made up from the condensation heating and other sources whose effects tend to partially cancel each other (Sasamori, 1978).

4.2. *Eddy forcing*

The variational solutions without eddy heating and eddy momentum sources are respectively shown in Figs. 10 and 11. In both solutions, the Hadley and Ferrel cells are decreased in intensity by about the same magnitude. Since the absolute intensity of the Ferrel cell is much weaker, we see that the effect of eddies is more pronounced for the Ferrel cell, as expected from the distribution of the eddy sources shown by Figs. 2 and 3. These diagrams also indicate that there is only a weak eddy source in the region of the Hadley cell. Nevertheless, the Hadley cell changes its intensity *when the eddy sources are removed.* This result seems noteworthy in conjunction with a recent finding by Stone (1978) that the mean meridional *temperature gradient in mid-latitudes is only* slightly larger than the critical gradient which he defines as the temperature gradient at which baroclinic Rossby waves are marginally stable. The fast restoring tendency of the horizontal mean temperature gradient toward the critical gradient by virtue of efficient horizontal eddy heat transport is compared by Stone to the vertical counterpart, convective adjustment of vertical temperature gradient toward the moist adiabatic lapse rate (Manabe and Strickler, 1964). Let us suppose that the variational solution (Fig. 10) without horizontal eddy heat transport does represent a critical Hadley circulation which would be observed at a moment when the mid-latitude horizontal temperature gradient is marginally stable for baroclinic Rossby waves. It is then tempting to speculate that the actual mean Hadley cell averaged over a winter season, which presumably is approximated by the standard solution (Fig. 6), maintains its intensity level slightly above the critical value.

4.3. *Other sources*

The individual variational solutions without radiative heating, surface sensible heating, surface friction and cumulus friction are shown in Fig. 12 through Fig. 15.

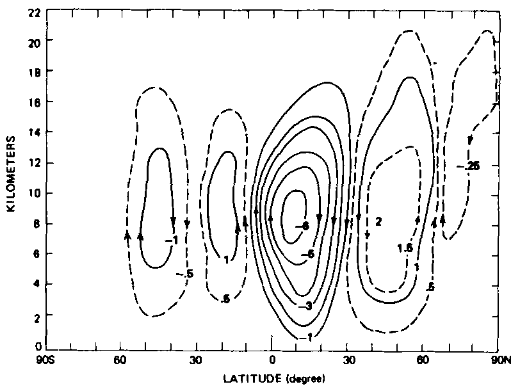

Fig. 8. The streamfunction obtained by setting $N^2 = 10^{-4}\ s^{-1}$ everywhere; unit: $10^3\ m^2\ s^{-1}$.

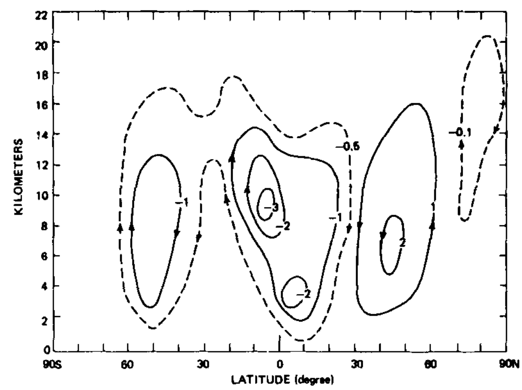

Fig. 9. The streamfunction produced without condensation heating, unit: $10^3\ m^2\ s^{-1}$.

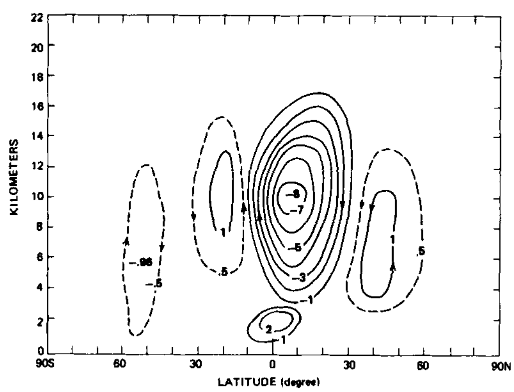

Fig. 10. The streamfunction produced without eddy heat transport, unit: $10^3\ m^2\ s^{-1}$.

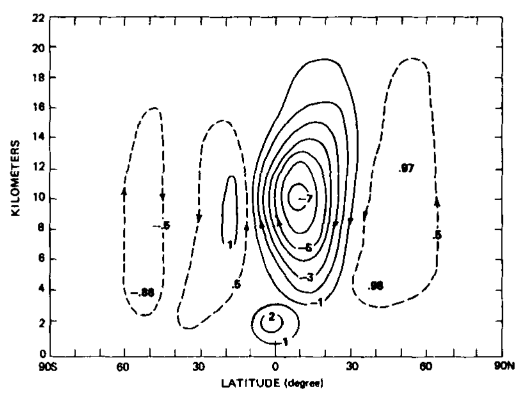

Fig. 11. The streamfunction produced without eddy momentum transport, unit: $10^3\ m^2\ s^{-1}$.

The only noticeable effect of the absence of radiative heating is seen in the slightly deeper Hadley cell (Fig. 12). As shown in Fig. 4, the radiative heating is relatively uniform over the globe, so that it does not influence the meridional circulation very significantly.

The effect of no surface sensible heating is seen only in the lowest layer of the model and the meridional circulation appears to be unaffected in most parts of the atmosphere (Fig. 13). The effect of the absent surface friction is seen in the significant reduction of the intensity for both Hadley and Ferrel cells (Fig. 14). Since the friction in the standard solution is confined in the lowest layer, the large vertical differential is also confined near the surface. As compared to the standard solution, the Ferrel cell in Fig. 14 reduces the intensity by factor 2, probably due to larger frictional dissipation in mid-latitude. This result indicates that the frictional dissipation of westerly geostrophic winds is an important source generating the ageostrophic meridional circulation in mid-latitudes (Eliassen, 1951).

Finally, the effect of absent cumulus friction is shown in Fig. 15. The intensity of the Hadley cell is reduced slightly, but this result which is apparently different from Schneider (1977) may, in part, be due to that the cumulus friction is assumed in the present study only at the intertropical convergent zone.

5. Conclusion

The gross magnitude of the contributions from various sources in the balance of meridional

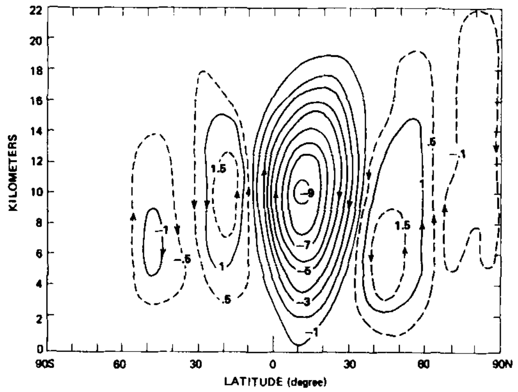

Fig. 12. The streamfunction produced without net radiative heating, unit: 10^3 m^2 s^{-1}.

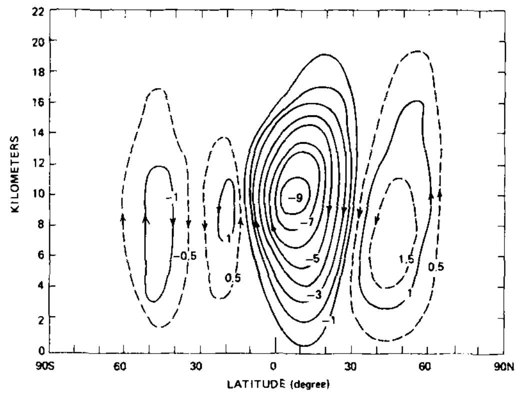

Fig. 13. The streamfunction produced without surface sensible heating, unit: 10^3 m^2 s^{-1}.

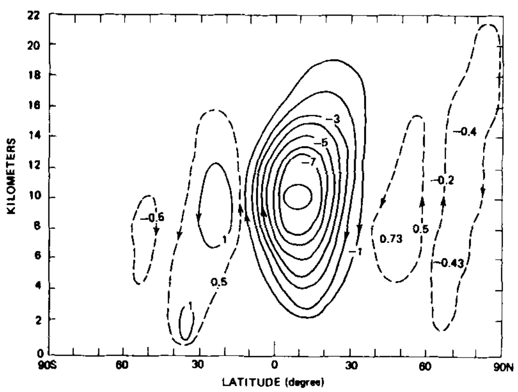

Fig. 14. The streamfunction produced without surface friction, unit: 10^3 m^2 s^{-1}.

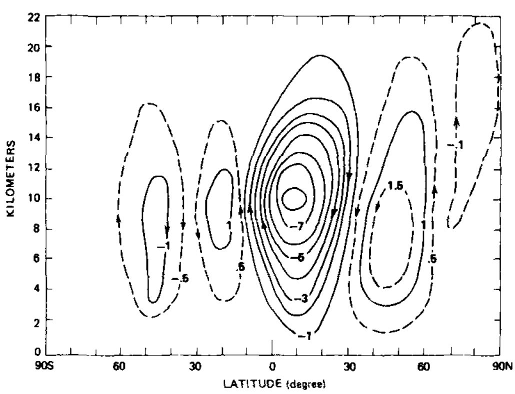

Fig. 15. The streamfunction produced without cumulus friction, unit: 10^3 m^2 s^{-1}.

circulation is summarized in Table 4, using the standard deviation of the variational solution from the standard solution.

The standard deviation is defined by the square root of the integral sum of the squares of the difference between two stream function solutions at latitude-height grid in the model domain. The single largest deviation is found for the solution without condensation heating, indicating that the condensation heating is indeed indispensable in maintaining the Hadley cell (Schneider and Lindzen, 1977).

If we add the two eddy effects the standard deviation becomes comparable to the latent heat effect, suggesting a strong coupling between the Hadley circulation and eddy activity in the troposphere. The individual effects of radiation, surface sensible heating and surface friction are all comparable, but the contributions in the balance of meridional circulation is, on the global average, not more than 1/3 of the condensation and resultant eddy effect, although the effects are largely dif-

Table 4. *The standard deviation from the standard solution streamfunction is calculated for each case study and listed in order of size*

Case	Standard deviation (m^2 s^{-1})
No latent heat release	67.3
No eddy heat flux	30.1
No eddy momentum flux	28.1
No boundary layer friction	19.6
No boundary layer sensible heating	17.0
No radiative heating	16.5
No cumulus friction	4.8

ferent for the individual Hadley, Ferrel and polar circulations.

With aid of the recent analysis by Stone (1978), an attempt was made to depict a physical picture on the interplay of the mid-latitude eddy heat transport and the intensity of the Hadley cell. The troposphere in mid-latitudes is highly convective in both vertical and horizontal directions. The vertical convections are maintained by the heat stored in the lithosphere at the direct expense of the insolation at the surface, and the convective motions are largely subsynoptic disturbances. On the other hand, the horizontal eddy convections are maintained by the low-latitude heat source (Sasamori and Melgarejo, 1978), which owes its origin to the water vapor supplied from the lithosphere, and consists largely of synoptic-scale disturbances.

The vertical convections take the heat from the surface and eventually radiate it into the stratosphere and space while the convections maintain the vertical temperature gradient near the critical value (Manabe and Strickler, 1964, Ramanathan and Coakley, 1978). The horizontal counterpart of convection in the mid-latitudes removes the heat out of the subtropics and hands it over to the polar region while maintaining the mid-latitude temperature gradient near critical gradient, a process referred to as baroclinic adjustment by Stone (1978). The present analysis on the meridional circulation appears to be compatible with this view.

6. Acknowledgement

The funds for this study were provided by the Grant ATM78-09159, National Science Foundation, Climate Dynamics Program and by the Research Board of the University of Illinois.

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ИЗУЧЕНИЕ ЧУВСТВИТЕЛЬНОСТИ СРЕЛНЕЙ ЗИМНЕЙ МЕРИДИОНАЛЬНОЙ ЦИРКУЛЯЦИИ К ИСТОЧНИКАМ ТЕПЛА И ИМПУЛЬСА

Для вывода уравнения для средней меридиональной циркуляции, основанного на квазистатическом соотношении, проанализированного Элиассеном (1951), комбинируются осредненные по времени и по долготе уравнения для импульса и термодинамическое уравнение. Возбуждающие члены и коэффициенты взяты из ранее опубликованных данных, осредненных по долготе и по озону. Эти члены включают чистое радиационное нагревание, нагревание за счет конденсации, трение в пограничном слое, тепловое нагревание, трение за счет тропических кучевых облаков, средний зональный ветер и крупномасштабные вихревые переносы импульса и тепла зимою. Вычисленная меридиональъная циркуляция в северном полушарии хорошо совпадает с наблюдаемой трехячеистой структурой ячеек Хэдли, Феррела и полярной. Для оценки относительного вклада различных факторов в поддержание меридиональной циркуляции вычислялся ряд решений с удалением одного из источников и сравнением со стандартным решением. Найдено, что интенсивность ячейки Хэдли составляет только одну треть от стандартного решения, если нагревание за счет конденсации отсутствует, хотя по-видимому в тропиках нет источника, обусловленного вихревым переносом импульса и тепла, вихревой перенос в средних широтах оказывает значительное воздействие на циркуляцию Хэдли, подтверждая, что охлаждение из-за вихревого переноса тепла в субтропиках является важным фактором, уравновешивающим адиабатическое нагревание в нисходящей ветви ячейки Хэдли. Индивидуальные эффекты радиации, теплообмена с поверхностью и поверхностного трения сравнимы, но их вклад в баланс меридиональной циркуляции при глобальном осреднении не превосходит одной трети от вклада конденсации и результирующего вихревого эффекта, хотя баланс различен для отдельных циркуляций Хэдли, Феррела и полярной.