

Measured and numerically-simulated autumn cooling in the Bay of Bothnia

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ABSTRACT

Vertical temperature profiles are analysed for the case of cooling of brackish sea water around the temperature of maximum density. Field measurements in the Bay of Bothnia cover a period of 52 days, during which the surface water was cooled from 6 °C to 1.5 °C. A mathematical model, which is based on the conservation equations for momentum, heat and salt in their one-dimensional forms, is presented. The equation of state is linear with respect to salinity and quadratic with respect to temperature. Turbulent exchange coefficients are calculated with a two-equation model of turbulence.

The measured temperature profiles clearly demonstrate the importance of both temperature and salinity gradients in the mixed-layer dynamics. The mathematical model describes these and the general development of the temperature profile in a most satisfactory way.

1. Introduction

Cooling of brackish sea water around the temperature of maximum density brings about some complicated phenomena regarding the mixing processes. To explain these, it should first be noted that the relation between temperature and density is strongly non-linear in this temperature interval. Further, cooling of brackish sea water above the point of maximum density will cause the stratification to be unstable with respect to temperature, while cooling below will have a stabilizing effect. The most spectacular effects, however, are related to the combined effect of salinity and temperature stratification. One such effect, which will be discussed further later on, is the local temperature maximum just below the well-mixed layer. This maximum can only exist if density stratification depends on both salinity and temperature gradients. It is also worth mentioning that the

relative importance of salinity gradients will grow when approaching the point of maximum density. At the point of maximum density, the gradient of density with respect to temperature is zero, which means that buoyancy effects are due to salinity alone.

As is well-known from other studies of mixed-layer dynamics, see below, the meteorological forcing has a strong and direct effect on the vertical distribution of velocity, temperature, etc. The present study concerns itself with “autumn conditions”, which implies strong winds and a net heat flux from the water to the atmosphere.

One of the purposes of this paper is to present measurements of the vertical temperature profile obtained in the Bay of Bothnia (Fig. 1). These measurements cover a period of 52 days during which the surface layer was cooled from 6 °C to 1.5 °C. However, the main objective of the study is to evaluate whether or not a one-dimensional mathematical model can be used for describing the development of the temperature profile. An attempt will therefore be made to formulate an adequate model and apply it to the Bay of Bothnia. By

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Fig. 1. Map of Scandinavia with surrounding waters.

comparing measured and predicted temperature profiles for the 52-day period, some conclusions about the usefulness of a one-dimensional model, as applied to the Bay of Bothnia, are drawn.

To the authors' knowledge, the numerical modeling of cooling of brackish sea water around the temperature of maximum density has not been discussed in the literature. Several studies, however, do provide useful pieces of information for the problem addressed. To be mentioned are the study of salinity effects in the ocean mixed layer by Miller (1976), the numerical simulation of thermohaline convection by Delnore (1980) and the numerical investigation of mixed-layer dynamics by Kundu (1980). Furthermore, the non-linearity and pressure dependence in the equation of state have been discussed by Farmer and Carmack (1981) in connection with cooling of deep lakes. The review by Niiler and Kraus (1977) gives an account of one-dimensional models, and the reader is therefore referred to this paper for details about the assumptions involved. Niiler and Kraus (1977) discuss the so-called closure models of turbulence. This is the kind of representation of turbulence that will be used in the present study. An excellent review of closure models of turbulence, has recently been presented by Rodi (1980). Of particular significance for the present study are the two papers by Svensson (1979, 1981), where the

mathematical model used in this study was verified for situations relevant to the present one.

2. Model equations

2.1 Basic assumptions

This study will restrict its attention to horizontally homogeneous flows, which means that terms containing gradients in the horizontal plane are neglected. It will further be assumed that there is no mean vertical velocity. Short-wave radiation is known to penetrate the water body following an exponential decay. During the period considered, the solar radiation, however, is of minor importance and will therefore be treated as a surface flux of heat. Gravitational effects are assumed to obey the Boussinesq approximation.

A schematic representation of the mathematical model as applied to the Bay of Bothnia is given in Fig. 2.

2.2. Mean flow equations

Primarily it is the temperature distribution that is of interest, but a few more variables are needed to describe the problem, as discussed in the introduction. Salinity is one of these, and the velocity distribution also needs to be considered, since

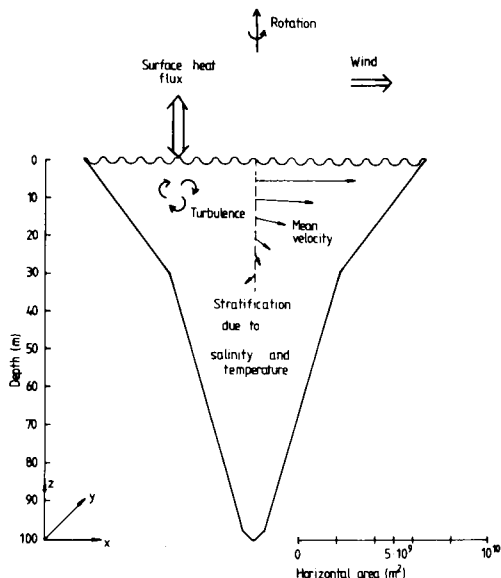


Fig. 2. Schematic representation of the one-dimensional model of the Bay of Bothnia.

turbulence to a large extent is produced by shear. Within the assumptions made, the equations for these variables read:

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial Z} \left(-\overline{w\theta} + \kappa \frac{\partial T}{\partial Z} \right), \quad (1)$$

$$\frac{\partial S}{\partial t} = \frac{\partial}{\partial Z} \left(-\overline{ws} + \kappa_s \frac{\partial S}{\partial Z} \right), \quad (2)$$

$$\frac{\partial U}{\partial t} = \frac{\partial}{\partial Z} \left(-\overline{wu} + \nu \frac{\partial U}{\partial Z} \right) + fV, \quad (3)$$

$$\frac{\partial V}{\partial t} = \frac{\partial}{\partial Z} \left(-\overline{wv} + \nu \frac{\partial V}{\partial Z} \right) - fU, \quad (4)$$

where Z is the vertical space coordinate, positive upwards, t time coordinate, f Coriolis parameter, U and V mean velocities in horizontal directions, T mean temperature, S mean salinity, ν molecular kinematic viscosity, κ molecular thermal diffusivity and κ_s molecular diffusivity for salt. The correlation terms $\overline{wu}$, $\overline{wv}$, $\overline{w\theta}$ and $\overline{ws}$ represent Reynolds stresses and turbulent transport of heat and salt, respectively.

Boundary conditions for the mean flow equations at the surface are specified according to:

$$\left(-\overline{wu} + \nu \frac{\partial U}{\partial Z} \right) = \frac{\tau_x(t)}{\rho_0}, \quad (5)$$

$$\left(-\overline{wv} + \nu \frac{\partial V}{\partial Z} \right) = \frac{\tau_y(t)}{\rho_0}, \quad (6)$$

$$\left(-\overline{ws} + \nu_s \frac{\partial S}{\partial Z} \right) = 0, \quad (7)$$

$$\left(-\overline{w\theta} + \kappa \frac{\partial T}{\partial Z} \right) = \frac{Q(t)}{\rho_0 C_p}, \quad (8)$$

where ρ_0 is the water density, $\tau_x(t)$ and $\tau_y(t)$ wind stresses, C_p specific heat of water and $Q(t)$ net heat flux. $Q(t)$ is of course a sum of several processes, as sensible heat flux, latent heat flux and radiation fluxes. These will be discussed further in Section 3. The zero-flux condition for salinity is an approximation since advection from rivers, precipitation and evaporation in effect generate a non-zero flux. At the lower boundary, a zero-flux condition is used for all variables, the reason being that this boundary is positioned well below the depth of surface influence. This boundary condition of

course is also an approximation, especially when strong bottom currents are present, but is nevertheless regarded as the most relevant one within the one-dimensional framework. An equation of state is also needed. This equation should reproduce the almost quadratic relation between temperature and density around the point of maximum density, and also the linear dependency on salinity. The equation of state employed reads:

$$\rho = \rho_0(1 - \alpha(T - T_M)^2 + \beta S), \quad (9)$$

where α and β are constants and T_M is the temperature of maximum density. The constants α and β should be chosen with respect to the temperature interval under consideration. In the present study they are set to $5.57 \cdot 10^{-6}$ and $8.13 \cdot 10^{-4}$, respectively. The temperature of maximum density, T_M , is strictly a function of salinity and pressure, Caldwell (1978). In this study T_M , however, is set to a constant value of 3.2°C .

Since the mean-flow equations are exact, within the assumptions made, they need no further discussion. However, due to the turbulence correlations present, the system is not closed; a turbulence model is therefore needed.

2.3. Turbulence model

A fundamental process in mixed-layer dynamics is the conversions of energy. Turbulent kinetic energy can be converted into potential energy or vice versa. A budget equation for k , the turbulent kinetic energy, is needed. With the introduction of a kinematic eddy viscosity, ν_T , a modelled form of the k -equation, see Launder and Spalding (1972) for details, reads:

$$\frac{\partial k}{\partial t} = \frac{\partial}{\partial Z} \left(\frac{\nu_T}{\sigma_k} \frac{\partial k}{\partial Z} \right) + \nu_T \left(\left(\frac{\partial U}{\partial Z} \right)^2 + \left(\frac{\partial V}{\partial Z} \right)^2 \right)$$

rate of change diffusive transport production by shear

$$+ \frac{g}{\rho} \nu_T \frac{\partial \rho}{\partial Z} - \epsilon \quad (10)$$

buoyant viscous
production/ dissipation
destruction pation

where σ_k is a Prandtl/Schmidt number, g gravitational acceleration and ϵ dissipation rate of k . The rate of change is thus balanced by the diffusive transport, the production by mean-velocity gradients, the production or destruction by

buoyancy, and the dissipation of k by viscous action. The relative importance of different terms was discussed in Svensson (1979) for the case of unstratified conditions. It was found that close to the surface, there is a balance between shear production and dissipation, while the diffusional transport contributes mainly in the deeper part of the boundary layer. When buoyancy effects are added, this balance will be modified according to the magnitude of the buoyancy flux. For the problem of autumn cooling in the Bay of Bothnia, the mere existence of sharp temperature and salinity gradients indicates that the buoyancy term enters the balance in a significant way.

From this discussion, it is clear that the relative importance of terms in the turbulent kinetic energy equation will change with time for the period considered. During conditions of strong surface cooling, $T > T_M$, and negligible windshear, production of turbulence will be due to buoyancy and vertical transport by turbulent diffusion. This balance will be dominant throughout the boundary layer. Conditions of strong wind and weak surface cooling will, on the contrary, produce a balance between production due to shear and dissipation.

By solving an equation for k , a velocity scale, $k^{1/2}$, is available for the determination of v_T . From physical reasoning and dimensional analysis it is expected that v_T is the product of a velocity scale and a length scale. Given the velocity scale, a length scale is thus needed. If once again dimensional analysis is used, it can be shown that a length scale, l , can be obtained from k and ϵ as:

$$l \sim k^{3/2}/\epsilon. \quad (11)$$

Recalling that the velocity scale is $k^{1/2}$, the Prandtl/Kolmogorov relation for v_T is obtained:

$$v_T = C_\mu k^2/\epsilon, \quad (12)$$

where C_μ is an empirical constant. It remains for us to formulate an equation for ϵ , if v_T is to be calculated according to (12).

This equation is the weak point of most turbulence models presented to date, and some workers argue that a prescribed scale is as good as any dynamical equation presented. The success met by the ϵ -equation in predicting a wide range of shear flows, see Rodi (1980) and Singhal and Spalding (1981), does indicate a certain degree of

generality and the form of the ϵ -equation given by these workers is therefore employed:

$$\begin{aligned} \frac{\partial \epsilon}{\partial t} = & \frac{\partial}{\partial Z} \left(\frac{v_T}{\sigma_\epsilon} \frac{\partial \epsilon}{\partial Z} \right) + C_{1\epsilon} v_T \cdot \frac{\epsilon}{k} \left(\left(\frac{\partial U}{\partial Z} \right)^2 + \left(\frac{\partial V}{\partial Z} \right)^2 \right) \\ & + C_{3\epsilon} \frac{g}{\rho} \frac{\epsilon}{k} v_T \frac{\partial \rho}{\partial Z} - C_{2\epsilon} \frac{\epsilon^2}{k}. \end{aligned} \quad (13)$$

rate of diffusive production
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In Svensson (1979) it was shown that this equation gives a vertical distribution of the dissipation rate which is in general agreement with field measurements. The calculations to be presented gave further confidence. From eq. (11) it was found that a realistic length scale distribution was produced during the whole cooling period.

Boundary conditions for k and ϵ at the surface are specified by using empirical information on the relation between shear stress and k and ϵ (for details see Rodi (1980)). At the lower boundary a zero-flux condition is used.

Having introduced the eddy-viscosity concept, it is now possible to relate the turbulence correlations in the mean-flow equations to their appropriate gradients through the following expressions:

$$-\overline{wu} = v_T \frac{\partial U}{\partial Z}, \quad (14)$$

$$-\overline{wv} = v_T \frac{\partial V}{\partial Z}, \quad (15)$$

$$-\overline{w\theta} = \frac{v_T}{\sigma_T} \frac{\partial T}{\partial Z}, \quad (16)$$

$$-\overline{ws} = \frac{v_T}{\sigma_s} \frac{\partial S}{\partial Z}, \quad (17)$$

where σ_T and σ_s are turbulent Prandtl/Schmidt numbers for temperature and salinity, respectively. At present, σ_T and σ_s are assumed to be constant, and equal to 1.0, even if a dependence on stratification is known to exist.

The empirical constants appearing in the turbulence model are treated as being universal. For complex flows, i.e. recirculating, curved, etc., this universality may be questioned, but for boundary layer flows, the standard values used in the present

Table 1. *Constants in the turbulence model*

C_u	C_{1r}	C_{2r}	C_{3r}	σ_k	σ_ϵ
0.09	1.44	1.92	0.8	1.0	1.3

study and shown in Table 1, may be used with confidence.

Eqs. (1)–(4), (9), (10) and (13)–(17) form a closed system and thus constitute the formulation of the mathematical model. This set of equations, in their finite difference form, were integrated forward in time using an implicit scheme and a standard tri-diagonal matrix algorithm.

2.4. *Considerations of applicability*

The set of equations employed is capable of describing inertial Ekman dynamics and vertical diffusion. If realistic predictions are to be expected, an assessment of the importance of these processes in the Bay of Bothnia is needed. From the classical observations by Gustafson and Kullenberg (1933), it is known that inertial motion is a characteristic feature of the dynamics of the Baltic and it can be expected that this also applies to the Bay of Bothnia. The importance of vertical diffusion should be considered by a comparison with horizontal transport and other modes of vertical transport. According to Walin (1972), the response of a system like the Baltic or the Bay of Bothnia to meteorological forcing is mainly baroclinic in a narrow coastal region, while the response of the main basin is essentially barotropic. This implies that the vertical transport is more important than the horizontal.

These considerations indicate that one-dimensional analysis, which includes Ekman dynamics, may be suitable for the Bay of Bothnia. The comparison with field measurements, Section 4, will serve as a quantitative test of the applicability.

3. The Bay of Bothnia

The Bay of Bothnia is the northern extension of the Baltic (see Fig. 1). Climatically it is situated in the northern part of the westerlies. Consequently the weather is influenced by the meandering polar front and the disturbances on it during late autumn which can cause strong winds. During late autumn

and winter, the low-pressure systems normally pass south of the area and cold air masses cover the whole bay. Ice formation starts in the northern archipelago in the beginning of November and the horizontal growth continues until the whole basin is ice covered. This normally occurs in the middle of January.

The Bay of Bothnia (cf. Figs. 1 and 3) has an area of 36,260 km² and a volume of 1481 km³. The maximum depth is 126 m while the mean depth is 41 m. The Northern Quark, with a sill depth of 20 m connects the Bay of Bothnia with the Sea of Bothnia. The north-going water transport through the Quark of saltier water, around 5‰, is approximately 400 km³ yr⁻¹, while the south going transport is approximately 500 km³ yr⁻¹. This higher figure is due to inflows of fresh water mainly by rivers. In late autumn and in winter, the water is two-layered with respect to salinity. The depth of the upper layer varies between 30–50 m and has a salinity of 3.0–3.5‰, while the bottom layer salinity is in the range 4.0–4.5‰. A typical residence time for the Bay of Bothnia is 3 years.

On October 22, 1979, a water-temperature measuring system was placed outside Skellefteå (see Fig. 3). This system contained 2 thermistor chains, type Aanderaa, with 11 thermistors in each. Measurements were made from a depth of 1 m below the sea surface to a depth of 75 m, the vertical resolution being 2 m for the upper chain and 5 m for the lower one. Data were sampled

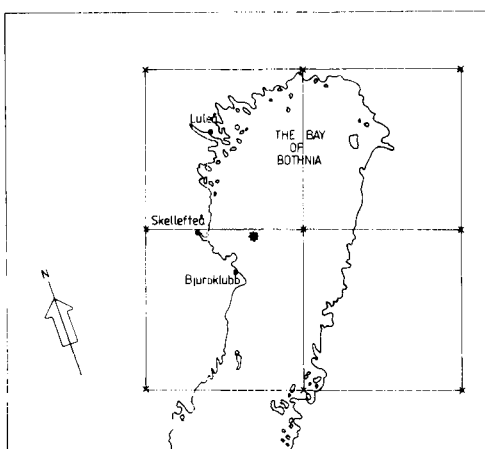


Fig. 3. Map of the Bay of Bothnia with the 150 km grid points marked as crosses. The star indicates where temperature measurements were made.

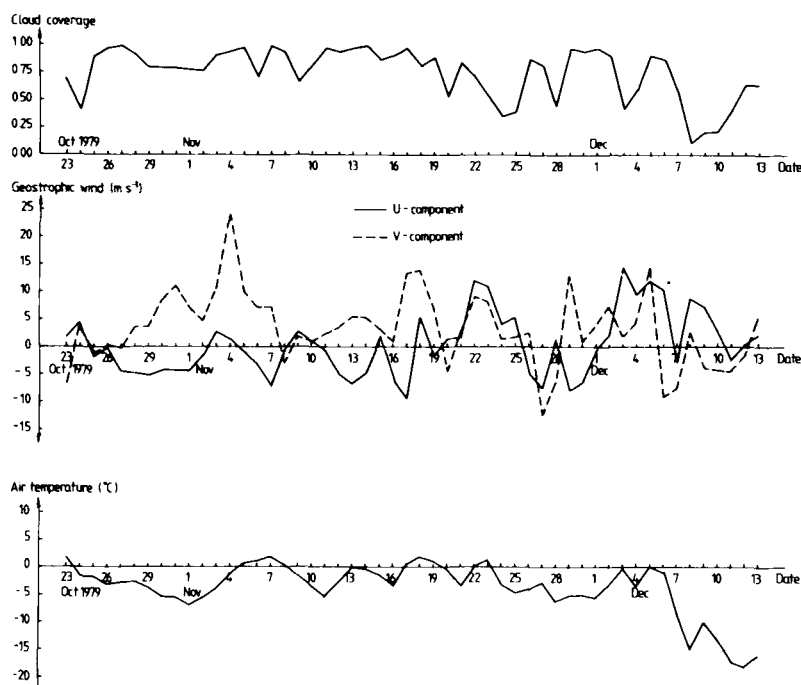


Fig. 4. Meteorological input data.

every 30 min and stored on a magnetic tape. The relative accuracy of the data is better than 0.05°C . Due to the ice formation risk, the measurements ended December 14. The recording system had then been working for 52 consecutive days and fortunately no data were lost.

To calculate the heat and radiation fluxes and the wind stress, weather data have been extracted from analysed weather charts. This was carried out for every third hour according to a 150 km cartesian grid (see Fig. 3).

Bulk formulae were used for the sensible and latent heat calculations with exchange coefficients C_H and C_E equal to $1.42 \cdot 10^{-3}$ and $1.32 \cdot 10^{-3}$, respectively, according to Friehe and Schmitt (1976). As the short-wave radiation is of minor importance during the studied time period, it is simply included in the boundary condition. Calculations of the net long-wave radiation is in accordance with Washington et al. (1976). They used a Brunt-type formulation modified with a cloud factor. From the extracted air-pressure data, the geostrophic wind speed was calculated which, together with the cloud coverage data, represent areal mean values over the Bay of Bothnia. These

data together with the areal mean air temperatures are shown in Fig. 4. The wind-stress calculation reduced and turned the geostrophic wind in order to obtain the speed 10 m above the surface. A quadratic law with C_D equal to $1.3 \cdot 10^{-3}$ was used for the stress calculation at 10 m.

4. Results and discussion

In this section, the results of the field measurements and the calculations with the mathematical model will be presented. Measured temperature profiles are shown as dashed lines in Figs. 6 and 7. Boundary conditions for the mathematical model were obtained as described in Section 3, and are presented in Fig. 6. Initial conditions are also needed for the model: these are shown in Fig. 5. The salinity profile in Fig. 5 was measured close to the temperature chains. Calculated temperature profiles are shown as solid lines in Fig. 6. In Fig. 7 calculated profiles of salinity, density and dynamical eddy viscosity are displayed. These are useful when discussing particular details in the temperature profiles. After this general information

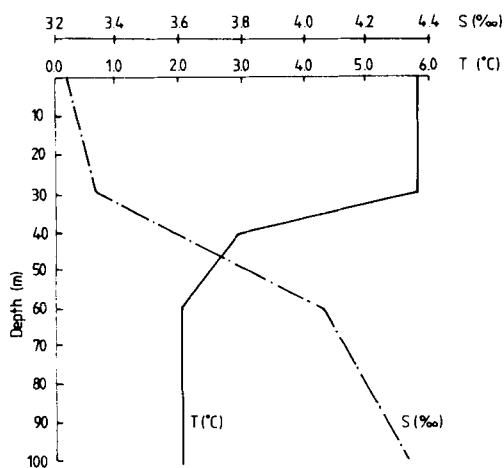


Fig. 5. Initial profiles of salinity and temperature.

about the results, interest is now focused on the details of the measured and predicted temperature profiles.

The first profile to be examined is the one of October 28. It is clear that both the measured and predicted profile have a maximum below the well-mixed layer. This is explained by the combined effect of temperature and salinity gradients. Surface cooling reaches down only to 20 m, where a net stable stratification is produced by the salinity gradient. This leaves warmer water trapped at an intermediate level. During the next few days the model seems to be more sensitive to the increased wind stress and cooling than that displayed in the measured profiles. On November 2, the depth of the well-mixed layer is overpredicted but two days later there is a strong wind and the depth of the mixed layer is again in good agreement with measurements. Details of these two days may be found in Fig. 7. A less encouraging period starts on November 9 and lasts for about 5 days. The cause of the disagreement can be traced by noting the rapid change of total heat content of the water column from November 7 to 9. This change is not due to the surface heat flux. Advective transports, not modelled, are thus the only explanation. In Fig. 8, this explanation is firmly supported. It is seen that the change in heat content by surface heat flux does not correspond to the actual change in heat content during this period. Actually, a diagram like this is very useful since it clearly points out periods when one-dimensional models are inadequate. The next date to pay particular attention to is Novem-

ber 30, since the mixed-layer temperature now passes the temperature of maximum density, 3.2 °C. Further cooling will cause stable stratification with respect to temperature. In Fig. 7 it is also noted that the mixed-layer depth is now almost 50 m, as compared to 20 m on October 28. The temperature of the mixed layer is still correctly predicted even if the measured profile is not as straight as the one predicted. The development of the profiles in December are strongly influenced by the fact that both temperature and salinity gradients now act to reduce the mixing. In Fig. 7, December 8 and 12 are displayed in more detail. From the eddy-viscosity profiles, it is found that the fully turbulent layer is much shallower than one could conclude from the σ_T -profiles. It is the weak stable stratification due to the temperature that gives the effect. Interestingly enough, these conditions are once again favourable for production of a maximum in the temperature profile below the mixed layer. During the final days of the period, this maximum is also found, both in measurements and predictions.

Presumably this examination of the particular profiles has demonstrated that cooling of brackish sea water brings out a number of interesting and complicated phenomena. Of special significance is the effect of salinity gradients close to the point of maximum density. When judging the performance of the mathematical model, one ought to bear this in mind in addition to the inevitable uncertainty in the boundary and initial conditions. In the light of this, the agreement obtained after 52 days of integration is, in the authors' view, most satisfactory. Further testing is planned in order to ensure the significance of the results obtained.

5. Summary and conclusions

The objective of this study has been to examine the autumn cooling in the Bay of Bothnia. For this purpose the development of the temperature profile was measured during a suitable period. These measured profiles have been compared to simulated ones, obtained through a mathematical model.

The conclusions emerging from this study can be summarized as follows:

(i) The measured temperature profiles clearly demonstrate the importance of both temperature and salinity gradients in the mixed-layer dynamics

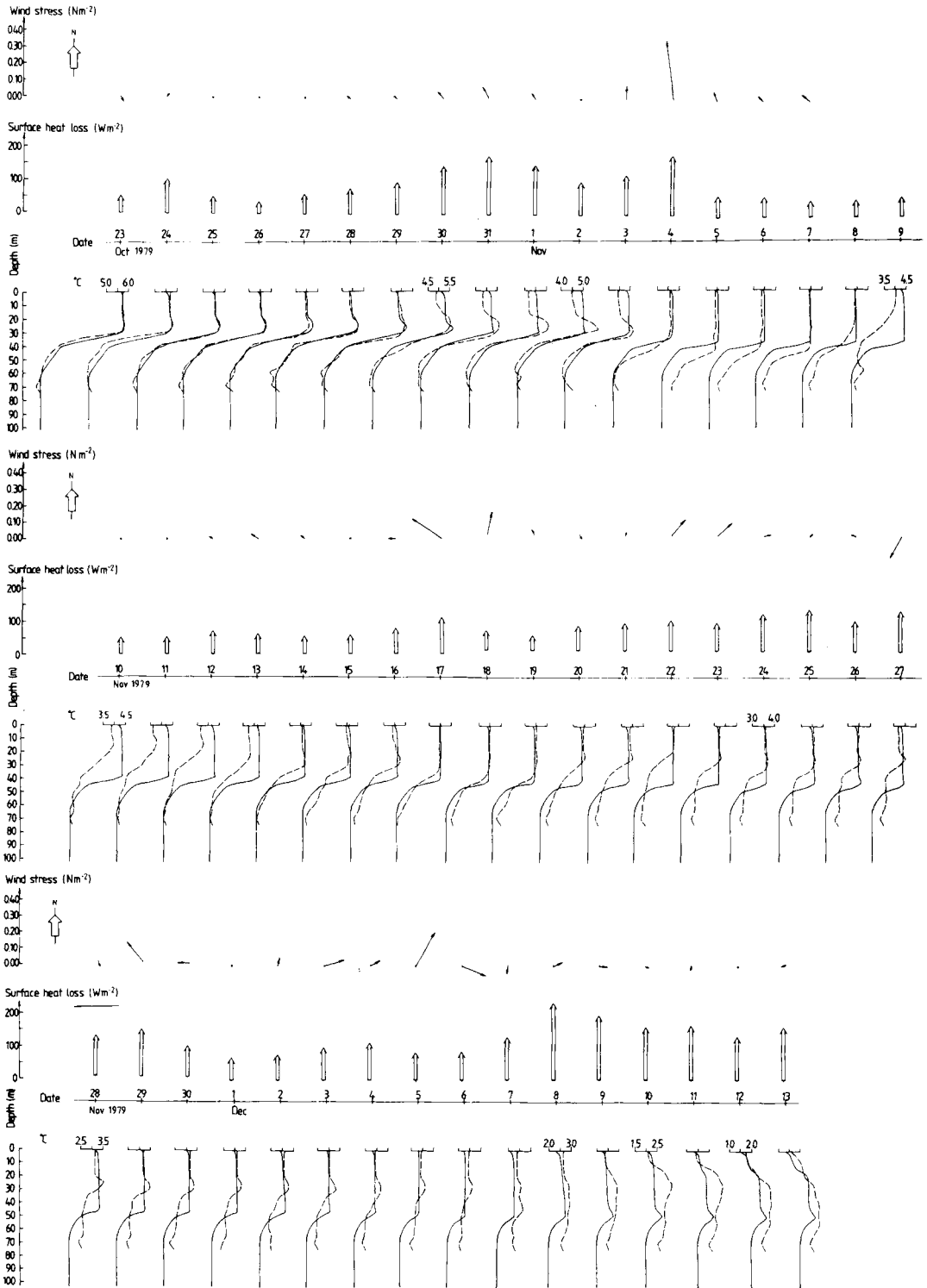


Fig. 6. Surface boundary conditions and calculated temperature profiles (solid lines). Measured profiles are shown as dashed lines. All data are 24 h averages.

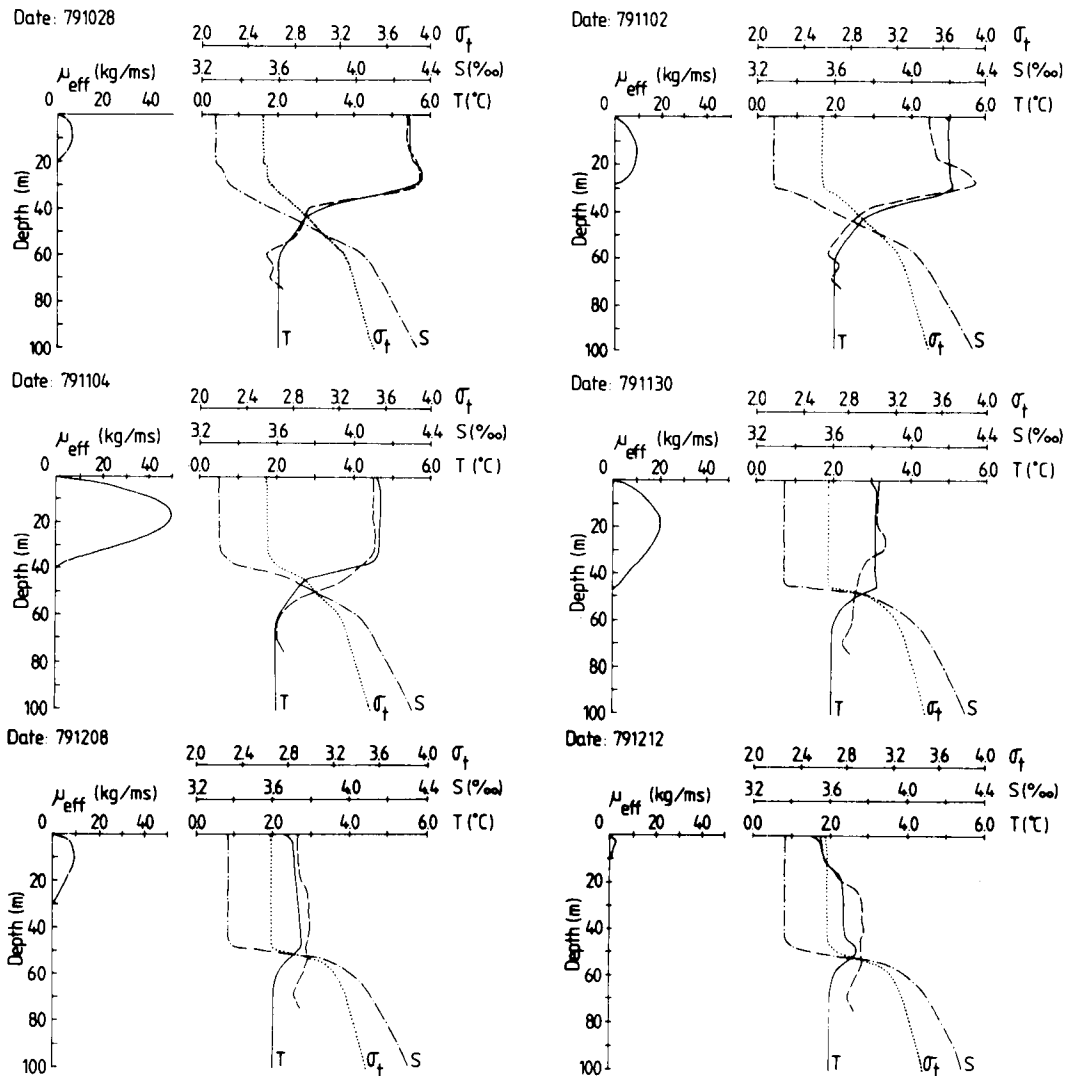


Fig. 7. Model calculations of dynamic eddy viscosity (μ_{eff}), temperature (T), salinity (S) and density (σ_t) from 6 occasions during the cooling period. The dashed lines are observed temperature profiles. All data are 24 h averages.

during autumn cooling. The mathematical model describes these and the general development of the temperature profile in a most satisfactory way.

(ii) The observed temperature maximum below the mixed layer is successfully predicted by the mathematical model.

(iii) Advection, not represented in the mathematical model, was found to influence the temperature profiles in the Bay of Bothnia during a 5-day period. Such events show the weakness of one-

dimensional models, which of course cannot handle horizontal transport processes.

6. Acknowledgement

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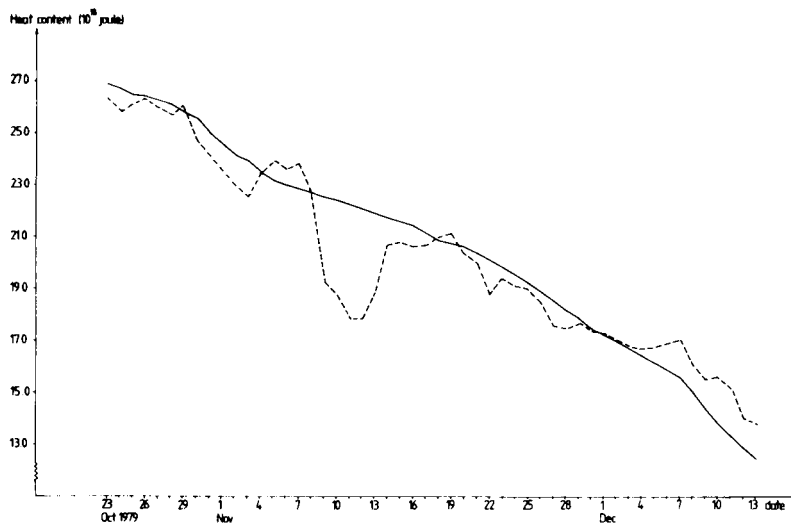


Fig. 8. Heat content of the water column as given by the thermistor chain (dashed line), and expected variation due to surface flux of heat (solid line).

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