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A numerical comparison between Lagrangian and Eulerian rainfall statistics

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ABSTRACT

Air trajectories and precipitation data are used to estimate certain parameters characterizing the duration and intensity of precipitation events and the length of the intervening dry periods. The results seem to indicate that the magnitudes of these parameters differ only moderately whether they be estimated in an Eulerian or a Lagrangian frame of reference.

1. Introduction

We address the problem of estimating some statistical parameters characterizing the precipitation experienced by a parcel of air as it is carried along by the winds. This problem is of considerable interest in connection with studies of the transport and dispersion through the atmosphere of compounds for which precipitation scavenging constitutes an important sink mechanism (for example, sulphate particles formed in the atmosphere by oxidation of SO_2 from anthropogenic emissions).

The average rate of removal of such compounds by precipitation is determined both by chemical and microphysical processes occurring in and below the cloud and by the frequency with which the parcel of air is experiencing clouds and precipitation. Estimates of this rate for a certain pollution particle thus require a knowledge about the "weather" experienced by the air parcel in which the particle is contained and which is carried along by the winds. Data of this kind are often referred to as *Lagrangian data*. On the other hand, almost all meteorological observations are obtained at fixed locations, yielding *Eulerian data*. A fundamental problem of the relation between Lagrangian and Eulerian data arises. Statistical models of the distribution of residence times, such as those developed by Rodhe and Grandell (1972, 1981), are of little use in calculations of transport and diffusion unless the parameters characterizing the Lagrangian precipitation statistics can be properly estimated.

In the theory of atmospheric turbulence a similar problem has been considered for several years (see, e.g., Pasquill, 1974). It was shown by Taylor (1921) that the rate of expansion of a cluster of particles subjected to turbulent diffusion could be related to the statistical distribution of Lagrangian velocity fluctuations. The difficulty in obtaining such data has prevented the use of this relation in most practical applications. Many attempts have been made to relate Lagrangian velocity fluctuations to the more readily measurable Eulerian velocity fluctuations. Empirical data seem to suggest that the Lagrangian and Eulerian spectra of turbulent velocity fluctuations are similar in shape, but that the time scale in the Lagrangian frame of reference is longer than the Eulerian counterpart by about a factor of 4 (Pasquill, 1974).

A similar comparison, of more relevance in the present context, is between the Lagrangian and Eulerian properties of large-scale horizontal wind patterns. Murgatroyd (1969) used geostrophic trajectories at 700 and 30 mbar at mid-latitudes to estimate the Lagrangian time scale of velocity fluctuations. He found this time scale to be, on the

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average, 60% of the corresponding time scale estimated from Eulerian data.

The question of the relation between Lagrangian and Eulerian rainfall statistics has received little attention so far. A preliminary study based on trajectories and precipitation data from North America was reported by Vickers and Slinn (1979).

In the present study we use a fairly large data base to estimate the distribution of precipitation events in both Lagrangian and Eulerian reference frames. As a starting point we have taken the models for the distribution of residence times formulated by Rodhe and Grandell (1981). The parameters of these models include the average length of the dry and precipitation periods. The precise definitions of the parameters which we have estimated are given in Section 3.

2. The data

The air trajectories, which have been kindly provided by the Norwegian Meteorological Institute, are estimated from observed winds at the 850 mbar level. Details about the computational procedures are given in OECD (1977). The area under study extends from about 40° W to 60° E and from the North Pole down to 40° N. The trajectories have been calculated backwards in time from a given point of arrival for a period of 96 h or, alternatively, until they reach the limit of the area. Thirty-six arrival points in northern Europe have been considered in the analysis (cf. Figs. 1 and 2).

For each station, trajectories have been initiated every 6 h during a one-year period (1978). This gives 52,560 trajectories in all. A certain seasonal variability has been allowed for by grouping the data into one summer season (April–September) and one winter season (October–March).

It is wise to bear in mind the substantial uncertainties associated with trajectory estimates. Sources of error include

errors in the wind observations

uncertainties introduced by the interpolation of these observations to the grid points, particularly in regions where data are sparse (over the sea)

uncertainties introduced by the finite difference approximation in the calculations

errors due to the neglect of vertical displacements.

The first three of these sources of error make individual trajectories, extending over several days, quite uncertain. If only the statistical distribution of the trajectories is considered, however, this uncertainty is probably much reduced. The fourth source is likely to be the most serious one in a study like ours. This is due to the systematic correlation between precipitation and vertical air movements. It is clearly unrealistic to assume that a trajectory forced to remain at the 850 mbar level will be representative for the movement of a parcel of air as it is carried up through a precipitating cloud system, be it a small scale cumulonimbus cloud or a larger scale nimbostratus.

The implication of this difficulty is that the accuracy of the trajectories is probably good enough for assessing the statistical distribution of the length of the dry periods. However, the length of the precipitation periods as well as the amount of precipitation in such periods may not be well estimated with this kind of trajectory data.

Another serious limitation of our analysis is the use of backward trajectories. For the estimate of residence times of particles emitted at a particular source it would have been more correct to use forward trajectories initiated at the source. Unfortunately, such forward trajectories were not available to us. One might argue that if the conditions were horizontally homogeneous there would be no difference in the derived statistical quantities. However, the presence of mountains in conjunction with an anisotropic distribution of wind directions will give rise to definite differences between the characteristics of backward and forward trajectories initiated from a certain point. Stations on the windward side of a mountain ridge will have forward trajectories with a much shorter average period until the first precipitation event than the backward trajectories.

Because of this limitation the values of the Lagrangian quantities obtained for individual stations should not be uncritically adopted. We believe that the uncertainties introduced by the use of backward trajectories instead of forward trajectories are substantially reduced by the averaging which is performed over the whole geographical area.

The precipitation data which we have used consist of objectively analysed fields for every sixth hour. The underlying data are observations at regular meteorological stations of the amounts of precipitation that have fallen during the 6 h preceding the time of observation. The observed values have been extrapolated to the grid points (the grid distance is about 150 km) with the aid of the objective analysis procedure described by Cressman (1959). A procedure of this kind entails considerable spatial smoothing particularly in regions with few observations (e.g. over the sea). The result is that the frequency of occurrence of low precipitation intensities is exaggerated whereas high precipitation intensities are suppressed. This is a considerable bias in the precipitation data. As a result the length of the precipitation periods will be overestimated and the length of the dry periods underestimated.

We have not attempted to quantify this bias. However, since we have used the same interpolated rainfall data for our estimates of both Lagrangian and Eulerian statistics we hope that the relative magnitude of these estimates remains little affected by this bias.

3. Methods of calculation

By combining the trajectory and precipitation data the "precipitation history" of each trajectory is calculated. Each trajectory position is associated with the precipitation value for the corresponding time and for the nearest grid point.

The statistical parameters which we have estimated are listed below. The names in brackets refer to those models of residence times as given in Rodhe and Grandell (1981) in which the parameter is needed.

- τ_d — Average length of dry periods (Markov model, renewal model)
- $\tilde{\tau}_d$ — Average time from an arbitrary moment in a dry period to the end of that period (of interest in comparison with τ_d)
- τ_p — Average length of the precipitation periods (Markov model, renewal model)
- $\tilde{\tau}_p$ — Average time from an arbitrary moment in a precipitation period to the end of that period (of interest in comparison with τ_p)
- m_p — Average amount of precipitation in a precipitation period (models which assume proportionality between scavenging intensity and precipitation intensity)
- R_0 — Precipitation intensity averaged over precipitation and dry periods (models which assume proportionality between scavenging intensity and precipitation intensity)
- $\Gamma/(2R_0)$ — Measure of the long-term variability of the precipitation intensity (cf. Rodhe and Grandell, 1981) (long-life approximation).

Let us now consider the estimation of the different parameters.

In order to estimate $\tilde{\tau}_d$ we only use the trajectories "starting" (strictly speaking: ending) with a dry period. Furthermore we must distinguish between the trajectories which are lost before they encounter precipitation and those which are not. For the ith trajectory, let

- t_i be the time until precipitation is encountered
- T_i be the time until the trajectory is lost (either at the boundary of the region or after 96 h)
- a_i be a parameter with a value of 1 if precipitation is encountered before the trajectory is lost and 0 otherwise
- n be the total number of trajectories used in the estimate
- k be the number of trajectories where precipitation is encountered before the trajectory is lost.

As the estimate $\tilde{\tau}_d^*$ of $\tilde{\tau}_d$ we have used

$$\tilde{\tau}_d^* = \frac{1}{k} \sum_{i=1}^{n} (a_i t_i + (1 - a_i) T_i)$$

The estimate is the maximum likelihood estimate of $\tilde{\tau}_d$ if all t_i are independent and exponentially distributed (Bartholomew, 1957, eq. 4).

In order to indicate that this estimate is reasonable we consider the case where all T_i are equal to T. Let F be the distribution function of t_i and write $\tilde{\tau}_d^*$ in the form

$$\tilde{\tau}_d^* = \frac{\frac{1}{n} \sum_{i=1}^{n} (a_i t_i + (1 - a_i) T)}{\frac{1}{n} \sum_{i=1}^{n} a_i}$$

Under general assumptions about the correlation between the t_i for different values of i it can be shown that

$$\tilde{\tau}_d^* \to \frac{E(a_i t_i) + TE(1 - a_i)}{E(a_i)}$$

$$= \frac{\int_0^T x\, dF(x) + T(1 - F(T))}{F(T)}$$

$$= \frac{\int_0^T (1 - F(x))dx}{F(T)} \quad \text{as } n \to \infty$$

If all t_i are assumed to be exponentially distributed with mean $\bar{\tau}$, i.e. $F(x) = 1 - e^{-x/\bar{\tau}}$, then $\tilde{\tau}_d^* \to \tilde{\tau}_d$ as $n \to \infty$. This shows that the estimate is consistent. Although the assumption of exponentially distributed t_i is not quite fulfilled we still believe this estimate to be reasonable. Since as much as 25% of all trajectories are lost before precipitation is encountered these trajectories must be taken into account.

In order to estimate τ_d we use those trajectories where a dry period is started after a precipitation period. We then use an estimate τ_d^* of τ_d constructed as $\tilde{\tau}_d^*$ but where t_i and T_i are measured from the beginning of that dry period.

For τ_p, $\tilde{\tau}_p$ and m_p we only consider those trajectories where a dry period is encountered before the trajectory is lost. Since precipitation periods are short compared to dry periods this only causes a slight underestimate.

The estimates R_0^* of R_0 and Γ^* of Γ are based on the total amount of precipitation M_i for those trajectories which are observed during the whole period of 96 h. We have used the estimates

$$R_0^* = \frac{1}{96} \cdot \frac{1}{n} \sum_{i=1}^{n} M_i = \frac{1}{96} \bar{M}$$

$$\Gamma^* = \frac{1}{96} \frac{1}{n-1} \sum_{i=1}^{n} (M_i - \bar{M})^2$$

The last estimate is natural since $\Gamma \approx \frac{1}{96}$ Var (M_i) (see Rodhe and Grandell, 1981).

The fact that our estimates of τ_d and $\tilde{\tau}_d$ are quite different (cf. Table 1) indicates that, in reality, the distributions are not quite exponential. Had the distribution been exponential, τ_d would have been equal to $\tilde{\tau}_d$ (and τ_p to $\tilde{\tau}_p$).

4. Results and discussion

In Table 1 we have summarized the results of our estimates of Lagrangian and Eulerian precipitation characteristics. The values in the table represent averages over the 36 stations for which data were analysed. In the calculation of τ_d^*, $\tilde{\tau}_d^*$, τ_p^* and $\tilde{\tau}_p^*$ the intensity defining the limit of precipitation has been set at 0.1 mm h^{-1}.

It would have been desirable to estimate the variance of these estimates. However, since the trajectories are correlated both in time and space, this would be very difficult and we have made no attempt to do so.

The most notable result is the small differences between the Lagrangian and the Eulerian values. A slight tendency for lower rainfall amounts and lower average intensity of precipitation in the Lagrangian frame of reference is hardly of importance. We wish to re-emphasize (cf. section on data) that the spatial smoothing applied to the precipitation data implies a significant underestimate of the absolute magnitudes of τ_d and an overestimate of the magnitude of τ_p. The possibility cannot be ruled out that this bias may also have affected the relative magnitudes of the estimates of the Lagrangian and the Eulerian quantities.

The differences between the two seasons are also quite small. It is only for the intensity of precipitation (R_0) that a systematic seasonal variation is evident. This is somewhat surprising in view of the results of earlier estimates of seasonal values of $\tilde{\tau}_d^{\mathrm{E}}$ and $\tilde{\tau}_p^{\mathrm{E}}$ (Rodhe and Grandell, 1972), which indicated that summer values of $\tilde{\tau}_d^{\mathrm{E}}$ were more than twice as large as winter values whereas summer values of $\tilde{\tau}_p^{\mathrm{E}}$ were only half as large as the winter values. (Superscripts "E" and "L" have been used to denote Eulerian and Lagrangian estimates.)

The reason for the lack of seasonality in the present data is most likely the fact that a very substantial spatial smoothing of the precipitation data has taken place prior to the analysis (cf. Section 2). As was pointed out earlier this smoothing will have the effect of reducing the frequency of occurrence of high precipitation intensities and enhance the frequency of low precipitation intensities. This process will modify summertime precipitation data, with its higher intensities and shorter durations, to look more like the wintertime data, which generally have lower intensities and longer durations in this area.

Table 1. *Comparison between Lagrangian and Eulerian estimates of precipitation parameters for the two seasons*

Quantity	Unit	Summer		Winter	
		Lagrangian	Eulerian	Lagrangian	Eulerian
τ_d	h	51	44	53	50
$\tilde{\tau}_d$	h	61	57	60	71
τ_p	h	11	13	11	13
$\tilde{\tau}_p$	h	11	14	12	13
m_p	mm	5.6	6.5	4.9	5.2
R_0	mm h^{-1}	0.10	0.13	0.08	0.09
$\Gamma/(2R_0)$	kg m^{-2}	70	63	73	88

The absolute values of the parameters characterizing the distribution of dry and precipitation periods will depend on the lower limit that is chosen for the intensity in the definition of precipitation. The lower the intensity, which is counted as precipitation, the longer will be the precipitation periods and the shorter the intervening dry periods. This is illustrated in Table 2. Because of the greater uncertainties inherent in the estimates of $\tilde{\tau}_p$ (cf. Section 2) we have not included those values in the table.

The temporal smoothing of the data implied by the use of 6 h precipitation totals is probably less critical for our result than the spatial smoothing. It is likely that, as a consequence of the temporal smoothing, τ_p and $\tilde{\tau}_p$ will be overestimated and τ_d and $\tilde{\tau}_d$ underestimated by a few hours. It is only for τ_p and $\tilde{\tau}_p$ that the difference could be of any importance.

In order to study the spatial variations of the parameters we have attributed the Lagrangian quantities to the locations of the receptor stations. When comparing Lagrangian and Eulerian values for a particular station it must be borne in mind that whereas Eulerian values are truly representative for that station (except for the spatial smoothing employed) the Lagrangian values are affected by the whole area over which the trajectories have moved. It follows that Eulerian values are somewhat more spatially variable than the Lagrangian values.

Table 2. *Comparison between Lagrangian and Eulerian estimates of* $\tilde{\tau}_d$*, unit: hours*

Limit of precipitation intensity (mm h^{-1})	Summer		Winter	
	L	E	L	E
0	29	33	24	28
0.05	50	47	47	55
0.1	61	57	60	71
0.15	69	66	69	87

The geographical variations of the estimates of $\tilde{\tau}_d^E$ and $\tilde{\tau}_d^L$ are shown in Figs. 1 and 2. In view of the small number of stations considered the indicated isolines are uncertain and should not be

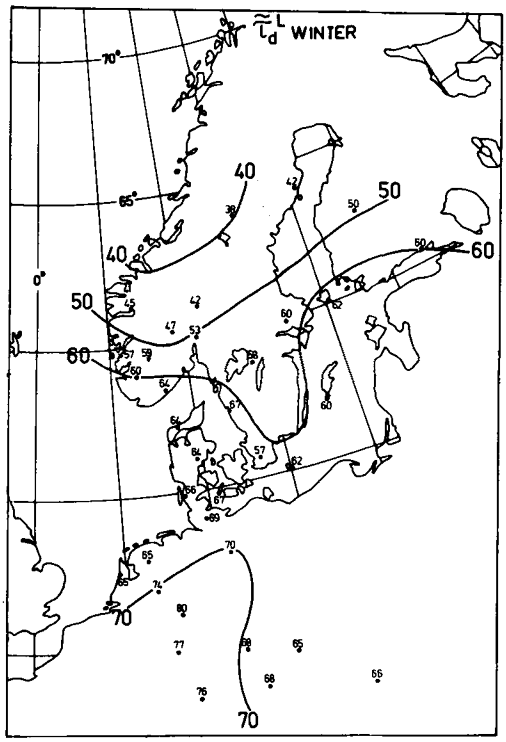

Fig. 1. Areal distribution of $\tilde{\tau}_d$ estimated in a Lagrangian frame of reference (winter season). Unit: hours.

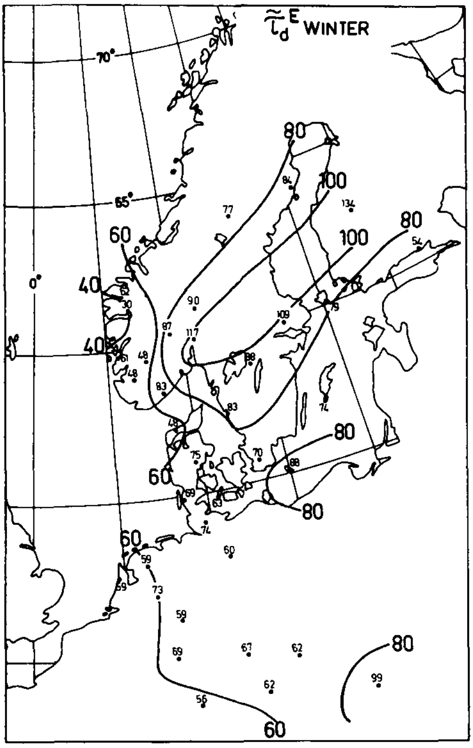

Fig. 2. Areal distribution of $\tilde{\tau}_d$ estimated in a Eulerian frame of reference (winter season). Unit: hours.

given much weight. It is seen that $\tilde{\tau}_d^E$ varies from 30 h at the Norwegian coast with its abundant precipitation to well over 100 h in central Finland. This pattern is broadly consistent with the climatological distribution of precipitation amounts for each season. The estimates of $\tilde{\tau}_d^L$ exhibit a smoother pattern, increasing gradually from about 40 h in northern Scandinavia to around 80 h in central Europe. The estimates of $\tilde{\tau}_p^L$ and $\tilde{\tau}_p^E$ (not shown here) vary only moderately over the area. Naturally the distribution of m_p^E resembles the climatological distribution of precipitation amounts. Again, m_p^L varies relatively less.

5. Conclusions

We have used 850-mbar air trajectories over northern Europe in conjunction with precipitation data to derive parameters characterizing Lagrangian precipitation statistics. Comparisons with similar Eulerian statistics—spatially averaged over the area—indicate no significant differences between these two frames of reference. In view of the limitations of our study, the major ones being associated with a substantial spatial smoothing of the precipitation data and uncertainties in the trajectories, it is uncertain how generally valid the results are. At least, until further studies have given a different result, it would seem reasonable to assume as a first approximation an equality between the Lagrangian and the Eulerian estimates of the parameters which we have studied.

It should be stressed that if Eulerian precipitation data are used to estimate parameters characterizing the average turn-over time for particles with respect to precipitation scavenging (Henmi and Reiter, 1978), the parameter values must be integrated over a sufficiently large geographical area. "Sufficiently" here implies an area comparable in size to the characteristic distance over which the particles are transported.

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ЧИСЛЕННОЕ СРАВНЕНИЕ ЛАГРАН ЖЕВОЙ И ЭЙЛЕРОВОЙ СТАТИСТИК ОСАДКОВ

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