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SHORT CONTRIBUTION

Note on stratocumulus instability

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1. Introduction

With the exception of a few numerical modeling studies (e.g. Oliver et al., 1978), investigations of the stratocumulus-topped planetary boundary layer have evolved from Lilly's (1968) mixed-layer approach (e.g. Schubert, 1976; Deardorff, 1976; Kraus and Schaller, 1978; Schubert et al., 1979a,b). In a discussion of the stability of these cloud decks, Lilly (1968) introduced the concept of moist convective instability, which occurs when very dry air is entrained into the cloud's top, and becomes negatively buoyant (with respect to the cloud) while becoming saturated. Lilly asserted that this would happen when the difference across the cloud top of moist static energy is negative; i.e., when $h_c > h_u$, where subscripts "c" and "u" refer to the cloud and upper (clear) air respectively. (Lilly (1968) actually discussed the problem in terms of wet-bulb potential temperature, which is equivalent to moist static energy for purposes of this discussion.)

Randall (1980) discussed the mechanism in terms of a "convective instability of the first kind upside down" and showed that Lilly's criterion should be corrected for the liquid water contribution to the buoyancy. Randall approached the problem in terms of the amount of cloud-top entrainment, and showed that the instability leads to a kinetic energy-generating convective flux at the cloud's top. Randall also gave a parcel-theory interpretation of the instability.

Deardorff (1980) referred to the mechanism as a "cloud-top entrainment instability" and produced a set of mixing diagrams to illustrate the process graphically. He also showed the connection between the graphical parcel mixing and Randall's (1980) entrainment derivation. Deardorff's (1980) paper contains some results from his three-dimensional numerical model which vindicate Lilly's (1968) original simplifications and show that the instability mechanism indeed causes breakup of a stratocumulus deck.

It is my purpose to contribute to this discussion by, first, showing how the mixing of two air parcels can create the instability, and second, presenting a mixing diagram similar to Deardorff's which shows more generally when this will occur. In doing so, the liquid water correction to Lilly's (1968) criterion is confirmed, and the linearization whereby the buoyancy flux is related to the moist static energy and total water fluxes is shown to introduce virtually no error in the mixed-layer model.

2. Mixing of two parcels

To simplify this first analysis, it is assumed that the mixing occurs at constant height and that a parcel's buoyancy is given by its dry static energy. Moist static energy (h) and total water content (r) are conserved for phase changes, and the mixing of a volume of cloud air, α_c, with a volume of upper air, α_u; produces

$$\alpha_c r_c + \alpha_u r_u = (\alpha_c + \alpha_u) r_m \qquad (1)$$

$$\alpha_c h_c + \alpha_u h_u = (\alpha_c + \alpha_u) h_m \qquad (2)$$

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where the final mixture is denoted by the subscript "m". The expressions (1) and (2) are homogenous in the volumes α_c and α_u, so that the non-trivial solution is given by the determinant of the coefficients.

It is convenient to define

$$\Delta r = r_u - r_c; \quad \delta r = r_c - r_m;$$
$$\Delta h = h_u - h_c; \quad \delta h = h_c - h_m.$$

The solution to (1) and (2) is given by

$$\delta r \Delta h - \delta h \Delta r = 0. \tag{3}$$

The Δ-quantities depend on the (upper-cloud) differences, i.e., the inversion strength, while the δ-quantities are the (cloud-final mixture) differences. Presumably, the latter are much smaller than the former, since the final mixture is part of the cloud. This being the case, the mixing proceeds such that $q_m = q^*(T_m)$ where q is water vapor and q^* the saturation value. Since δq and δT are small, it is reasonable to set

$$\delta q = \zeta \delta T,$$

where ζ is positive.

With this definition, $\delta h = (c_p + \zeta L)\delta T$, and (3) becomes

$$\delta T = \frac{\delta r}{(c_p + \zeta L)\Delta r} \Delta h. \tag{4}$$

For conditions of stratocumulus-topped mixed layers, $\Delta r < 0$; i.e., the upper air is drier than the cloud. Moreover, $\delta r > 0$, since the final mixture cannot contain more water than the cloud. Thus,

$$\delta T \propto -\Delta h. \tag{5}$$

Since it has been assumed here that buoyancy depends only on dry static energy (i.e., temperature), the new parcel will be stable with respect to the cloud top only if $\delta T < 0$. Thus, this simple analysis shows that the criterion for stratocumulus stability is $\Delta h > 0$, confirming Lilly's (1968) assertion. Next the effect of water vapor and liquid on the buoyancy are examined.

3. Mixing diagrams

Parcel buoyancy is dependent on the virtual dry static energy:

$$s_v = c_p T_v + gz = c_p T(1 + 0.61q - l) + gz. \tag{6}$$

(This analysis also applies to virtual potential temperature.) A mixing diagram is constructed via the following procedure:

(i) specify the pressure, temperature and total water ($r = q + l$);
(ii) calculate the saturation water vapor content, $q^*(T, p)$;
(iii) denote a "cloud top" by the curve $r = q^*$; for $r < q^*$ conditions are undersaturated and for $r > q^*$ conditions are cloudy;
(iv) obtain h and s_v, and construct a cross-plot of s_v vs. (h, r).

Such a diagram, for p = 900 mb, is shown in Fig. 1. The contours of s_v (solid lines) were

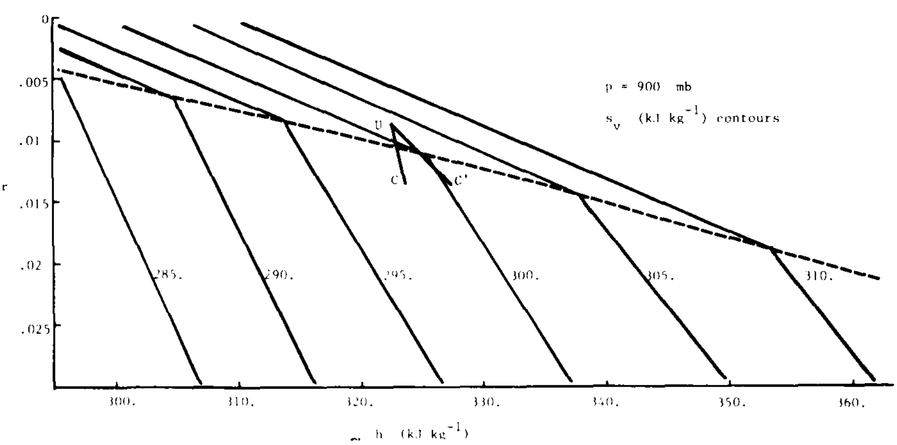

Fig. 1. Contours of s_v(kJ kg^{-1}) on h–r plane, illustrating moist convective instability. U–C is stable; U–C' is unstable. Dashed line is saturation.

obtained by the non-linear formula (6). Conditions above and below the dashed line ($r = q^*$) denote, respectively, unsaturated and saturated conditions. This diagram is similar to Deardorff's (1980) results, but moist static energy is used as an independent quantity rather than liquid potential temperature.

Since r and h are conservative with respect to water vapor phase changes, mixing between a saturated parcel and an unsaturated parcel occurs on a line connecting the two, and the intermediate states occur along that line, for example the heavy line U–C in Fig. 1. In particular, nowhere along line U–C is the virtual dry static energy lower (i.e. less buoyant) than the cloud value. Thus, it appears that mixing between parcels U and C is stable with respect to the maintenance of a stratocumulus deck.

The instability occurs for mixing between U–C', where intermediate values of s_v are less than the cloud virtual static energy. Clearly, the criterion for the stability depends on the elbow in the s_v contours around saturation and on the slope

$$\left(\frac{dr}{dh}\right)_{s_v=\mathrm{const}}$$

in the saturated conditions. From Fig. 1 I estimate that:

$$\left(\frac{dr}{dh}\right)_{s_v=\mathrm{const}} = 1.6 \cdot 10^{-3}\,(\mathrm{kJ\ kg^{-1}})^{-1}$$

along the 300 kJ kg^{-1} contour.

In terms of the earlier parcel theory, the stability criterion is thus

$$\Delta h - 625.0\Delta r > 0.$$

This confirms Randall's (1980) and Deardorff's (1980) results that a negative Δh can exist atop a stable stratocumulus deck (since $\Delta r < 0$).

4. Discussion

In all of the stratocumulus models which follow Lilly's (1968) approach, the buoyancy fluxes are found from a linear combination of the conservative fluxes (of r and h, here). The contours of s_v in Fig. 1—which were obtained from the non-linear formula (6)—show the validity of this linearization for a wide range of temperature and water content. For saturated conditions at constant height the linearization takes

$$s_v' = T_r(0.61q' - 1') + (1 + 0.61q_r - 1_r)T'$$

where $q' = q^*(T')$, primes denote differentials and subscript "r" is a reference value. Using the Clausius-Claperyon relation, and defining

$$\varepsilon = \frac{c_p T_r}{L};$$

$$\eta \equiv (1 + 0.61\,q_r - 1_r);$$

$$\gamma \equiv \frac{0.622\,L^2 q_r^*}{R c_p T_r^2} = \frac{L}{c_p}\left(\frac{\partial q^*}{\partial T}\right)_p$$

the result is that

$$s_v' = \left(\frac{\eta + 1.61\varepsilon\gamma}{1+\gamma}\right)h' - \varepsilon L r'.$$

This duplicates the derivations of Deardorff (1976), Schubert et al. (1979a), Randall (1980) and Deardorff (1980). Along contours of constant virtual dry static energy ($s_v' = 0$), therefore

$$\left(\frac{r'}{h'}\right)_{s_v=\mathrm{const}} = \frac{\eta + 1.61\varepsilon\gamma}{\varepsilon L(1+\gamma)} \equiv \frac{\beta}{\varepsilon L}.$$

Contours of $\beta/(\varepsilon L)$ are shown in Fig. 2 for the saturated part of the (h–r) plane in Fig. 1. Over the range of temperature used here (273–300 K) there is nearly a factor of two variation in this slope. The stability criterion,

$$\beta\Delta h - \varepsilon L \quad \Delta r > 0$$

therefore exhibits enough sensitivity to the linearization constants that they should not be prescribed, but rather optimized with respect to the actual conditions.

5. Conclusion

I have shown how Lilly's (1968) criterion for moist convective instability depends on the assumption that buoyancy is dependent on temperature, and that correcting for liquid water effects allows stability in some cases where $\Delta h < 0$. This argument has confirmed the discussions of Randall

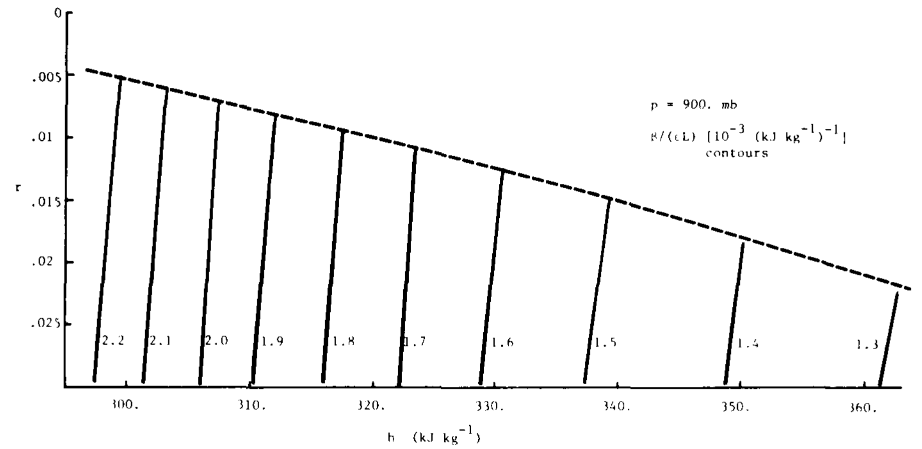

Fig. 2. Contours of $\beta/(\varepsilon L)$ $[10^{-3}(\mathrm{kJ\ kg^{-1}})^{-1}]$ on h–r plane.

(1980) and Deardorff (1980). The mixing diagram (Fig. 1) gives a straightforward interpretation of the moist convective instability process, and validates the use of a linear combination of moist static energy and total water fluxes to find the virtual dry static energy flux. However, it also shows (Fig. 2) that the linearization constants are sensitive to the actual conditions and should be optimized.

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