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Unsteady currents over a continental shelf

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ABSTRACT

A barotropic β-plane model of unsteady coastal currents over a sloping continental shelf driven by surface winds is analysed. The equation for the alongshore velocity is solved numerically and includes the effects of Rossby, shelf and Kelvin waves. The solution is compared with observations of some upwelling events off Northwest Africa.

1. Introduction

The longshore currents produced in association with coastal upwelling over a sloping shelf by a steady equatorward wind stress have been discussed by Johnson (1976). The theory based on β-plane dynamics showed that counter currents offshore of the main coastal current are produced by steady winds if the wind stress varies with distance along the coast or with distance across the shelf.

Observations, however, suggest that the winds are often only favourable to upwelling for a few days at a time, usually referred to as an upwelling event. A good description of some upwelling events in the north Atlantic off Northwest Africa has been given by Mittelstaedt, Pilsbury and Smith (1975). Graphs are presented of the velocity components and surface temperature alongside the unsteady wind stress distribution. From these graphs it may be noted that longshore countercurrents often form after the wind changes direction. Also the onshore velocity at intermediate depths over the shelf is only weakly tied to the wind distribution.

Three features stand out as being important in the observations of coastal currents over continental shelves. They are topography, stratification and variability. Clearly topography is a basic ingredient of any model of shelf seas. The effect of stratification may be noticed in the variations with depth of the velocity field, and in the density and temperature distributions that are characteristic of upwelling areas. The variability of the driving wind stress has already been mentioned. It is not easy to produce analytical models that take accurate account of topography, stratification and variability. Consequently, in this paper the coastal currents produced over a sloping shelf by variable winds are examined on a β-plane and the flow is assumed to be essentially barotropic.

2. Equations of motion of the model

This unsteady model is a modification of the steady solution discussed in Johnson (1976) and the notation is similar. The dimensionless coordinate system has x eastward, y northward and z upward. The surface is at $z = 0$ and on the shelf the bottom is at $z = \alpha(x - 1)$ where the eastern boundary of the ocean model is at $x = 1$. The ocean over the continental shelf is divided into the regions indicated in Fig. 1.

The Ekman layers have thickness $0(E^{1/2})$ where E is an Ekman number based on vertical eddy viscosity and the maximum depth of the ocean. The bottom Ekman layer carries the upwelling to the surface to supply the offshore Ekman transport induced by the wind stress. In the shelf layer of thickness $0(E^{1/4})$ let

$$X = (x - 1)/E^{1/4} \quad \text{and} \quad \zeta = z/(E^{1/4}\,\alpha X)$$

so that the surface is $\zeta = 0$ and the bottom is $\zeta = 1$. The velocity components and pressure are scaled as follows in the shelf layer,

$$u = E^{1/4}\,U, \quad v = V, \quad w = E^{1/4}\,W, \quad p = E^{1/4}\,P.$$

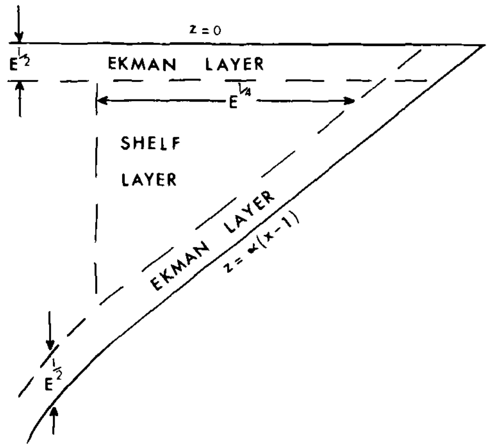

Fig. 1. Notation and geometry.

As $v \gg u$, the largest unsteady term in the equations of motion is expected to be $\partial v/\partial t$. The appropriate dimensionless equation is

$$(f_0 T)^{-1} v_t + R_0 v \cdot \nabla v + fu = -p_y$$
$$+ \text{ frictional terms,}$$

where R_0 is the Rossby number, f_0 is a scale for the Coriolis parameter and the dimensionless time is

$$t = (\text{dimensional time})/T$$

where T is a time scale. As $u/v = 0(E^{1/4})$ the unsteady term is important if $(f_0 T)^{-1} = 0(E^{1/4})$. For the typical values $E = 6.25 \times 10^{-6}$ and $f_0 = 10^{-4}$, this requires $T = 0(2 \times 10^5 \text{ s}) = 0(2.3 \text{ day})$. Therefore T is a suitable time scale to deal with wind distributions that produce upwelling events by changing over a period of a few days.

Choosing $f_0 T = E^{-1/4}$ and the above scales for the velocity components and the pressure, the equations for motion in the shelf layer are

$$fV = P_x, \quad V_t + fU = -P_y, \tag{1}$$

$$P_\zeta = 0, \quad \alpha X(U_x + V_y) + W_\zeta = 0, \tag{2}$$

which reduce to the equations in Johnson (1976) for the case of steady flow. In the strong coastal current along the coast the flow remains geostrophic. Eliminating the pressure from (1) and (2) leads to

$$V_{tx} + fU_x + fV_y + \beta V = 0 \tag{3}$$

and

$$U_\zeta = V_\zeta = 0, \quad W_{\zeta\zeta} = 0. \tag{4}$$

The horizontal velocity components are independent of depth as the model is assumed to be barotrophic. Use of the continuity eq. (2) with (3) gives

$$\alpha x(V_{tx} + \beta V) = fW_\zeta. \tag{5}$$

From (4) the vertical velocity is linear in ζ and hence may be determined from the suction conditions into the Ekman layers. The Ekman suction into the surface layer is

$$W = \tau_x^y / f \quad \text{on} \quad \zeta = 0 \tag{6}$$

and the bottom suction condition is

$$\frac{W - \alpha U}{(1 + \alpha^2)^{1/2}} = \frac{V_x}{(2f)^{1/2}} \quad \text{at} \quad \zeta = 1. \tag{7}$$

Condition (7) relates the normal suction into the Ekman layer on the left hand side of (7) to the gradient of the large shelf layer velocity just above the Ekman layer.

Using (6) and (7) to determine the coefficients in the linear expression for W gives

$$W = \frac{(1 - \zeta)}{f} \tau_x^y + \zeta \left\{ \alpha U + \left(\frac{1 + \alpha^2}{2f} \right)^{1/2} \times V_x \right\}. \tag{8}$$

Removing W from (5) and (8) produces

$$f\alpha U = \tau_x^y + \alpha x(V_{tx} + \beta V) - (f/2)^{1/2}(1 + \alpha^2)^{1/2} V_x. \tag{9}$$

Eliminating U from (3) and (9) yields the equation for V,

$$\alpha x V_{txx} + 2\alpha V_{tx} - (f/2)^{1/2}(1 + \alpha^2)^{1/2} V_{xx}$$
$$+ \alpha\beta x V_x + \alpha f V_y + 2\alpha\beta V = -\tau_{xx}^y. \tag{10}$$

The solution of (10) subject to suitable conditions gives the longshore flow V. Then (9) and (8) give the onshore flow U and the vertical velocity W respectively. The right hand side of (10) is zero if the wind stress does not vary with x across the shelf.

A similar equation to (10) is obtained by Hsueh and Peng (1978) for the flow over a continental shelf on a f-plane and after integrating vertically the equations of motion. They numerically integrate the steady form of their equation and the resulting geostrophic boundary currents compare favourably with data from the Coastal Upwelling Experiment 1 off Oregon.

3. The boundary conditions

To determine a solution of (10) requires conditions on $V(x, y, t)$ in both time and space. For the initial condition it is assumed that

$$V(x, y, 0) = 0 \tag{11}$$

so that the flow is spun up from rest at time $t = 0$ when the wind stress is first applied. The longshore current and upwelling are driven principally by the wind at the coast, which provides a boundary condition on V through the continuity of transport at the coast in which the Ekman fluxes into the corner at $X = 0 (x = 1)$ balance. This requires

$$V(0, y, t) = \left(\frac{2}{f}\right)^{1/2} \frac{\tau^y}{(1 + \alpha^2)^{1/2}}, \tag{12}$$

as given in Johnson (1976).

The matching condition with the interior is

$$V \to 0 \quad \text{as} \quad X \to -\infty \tag{13}$$

as the longshore current over the shelf is larger than the interior flow beyond the shelf. An additional condition is required as the model is on a β-plane and eq. (10) is parabolic in y. It is necessary to have a "starting" condition at some latitude $y = y_0$. It is assumed that

$$V(x, y_0, t) = V^*(x, t) \tag{14}$$

where V^* is prescribed and for simplicity in this paper V^* is chosen to be zero. The effect of the choice of V^* does not penetrate very far north from the chosen latitude y_0.

4. Some properties of the solution

The solution of (10) is the unsteady longshore velocity field over a sloping continental shelf on a *β-plane* forced by an unsteady wind stress distribution. As f varies linearly with the latitude y it is not possible to look for simple wave-like solutions and (10) must be solved numerically. However, included in (10) are a number of combinations of terms that represent wave motions. For example, if the first and fourth terms $(V_{txx}, \beta V_x)$ dominate, then (10) becomes the equation for Rossby waves, which leak energy westward out of the coastal boundary layer.

The second and fifth terms $(2V_{tx}, fV_y)$ are the modified Kelvin wave balance for the special case in which the onshore flow is zero. These waves propagate away from the equator along an eastern boundary. A combination of the above pairs of terms (first, second, fourth, fifth) with constant f is essentially the shelf wave equation on a f-plane. The third term comes from the bottom friction in the bottom Ekman layer and is the only dissipative term in (10).

In a study of the steady flow over a sloping shelf driven by a wind stress that is periodic in y, Csanady (1978) solves an equation that corresponds to the third and fifth terms $(V_{xx}, \alpha f V_y)$. He describes the solution as arrested topographic waves and finds that countercurrents form offshore of the main current.

In the numerical solution presented in the next section, soms of the properties of shelf, Rossby and Kelvin waves may be observed. In particular the propagation of energy both alongshore by Kelvin and shelf waves and offshore by Rossby waves form part of the solution. All these waves are described in detail by LeBlond and Mysak (1978).

5. The β-plane numerical solution

Equation (10) is solved numerically subject to conditions (11) to (14) using the following scheme. Starting from the rest state the solution is stepped forward in time. At each time step the solution is calculated for all x, t starting with latitude y_0 and stepping northward. The y-derivative in (10) is determined from the previous time step and then the values of V at each x can be found using the Crank–Nicolson method.

A number of different numerical solutions have been calculated and three are presented here. The first two have applied wind stresses that have fixed spatial distributions and vary with time to give a representation of upwelling events. The third example is driven by a pulse of wind that moves northward along the coast. In each case the solution is started at $y = y_0 = -0.75$, a latitude that is sufficiently far south of latitude $y = 0$ for the effect of the starting condition (14) to be negligibly small. To ensure a smooth spin-up of the flow, each wind stress satisfies $\tau = 0$ at $t = 0$. The constants used are $\alpha = 1$, $\beta = 0.7$.

(i) $\tau^y = + \sin 2\pi(y - y_0)\{\tanh 12t$

$$- \tanh 12(t - 1) + \tanh 12(t - 2)\}. \tag{15}$$

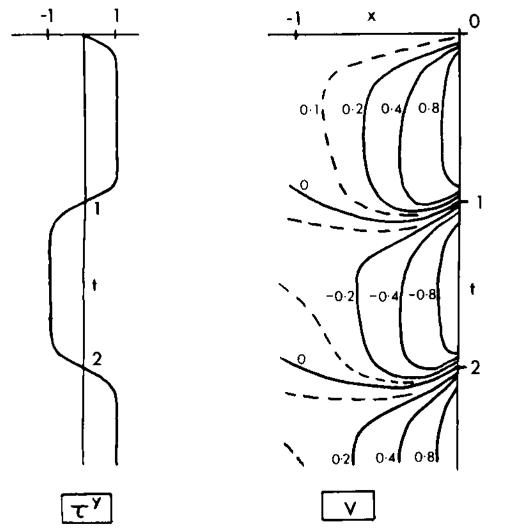

Fig. 2. The wind stress (15) and the corresponding contours of longshore current V in $x - t$ space, at latitude $y = 0$.

This distribution for the wind is shown in Fig. 2 at latitude $y = 0$. There is an initial spin up to a constant northward wind followd by a rapid change to a southward wind at times $t = 1$ and a further change in direction at $t = 2$. This provides upwelling favourable winds at $y = 0$ for $1 < t < 2$, corresponding to a short upwelling event. The $\sin \pi(y - y_0)$ is included to give smooth conditions at latitude y_0 at all times. Alongside the wind distribution in the figure are contours of V in $x - t$ space. At the times of changes in wind direction countercurrents form offshore of the coastal current. The slope in the contours near $x = 0$ show the propagation away from the coast of the wind forcing at the boundary.

(ii) $\tau^y = -\tanh 5(y - y_0)\{\tfrac{1}{2} + \sin \pi(t - \tfrac{1}{2}) + \tfrac{1}{3}\sin 3\pi(t - \tfrac{1}{2})\}$. (16)

In this example the $\tanh 5(y - y_0)$ merely provides a smooth start to the solution at latitude y_0 and a wind that is independent of latitude far away from y_0. The choice of time dependence is the

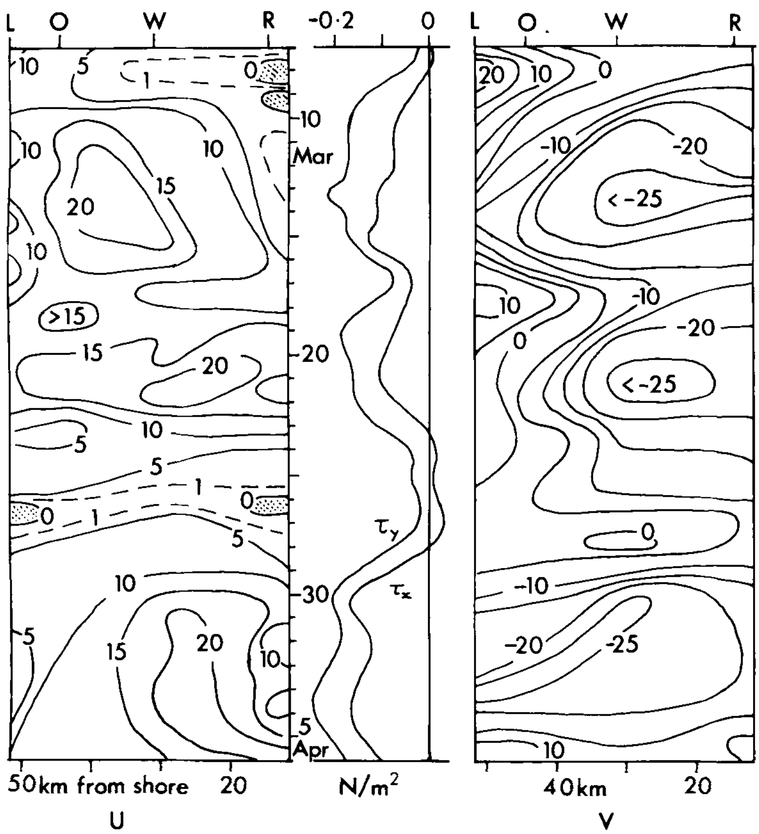

Fig. 3. Wind stress (τ^x, τ^y) and velocity components (U, V in cm s^{-1}) along the mooring line at the depth of maximum onshore flow at 21° 40′ N off Northwest Africa. (After Mittelstaedt, Pilsbury and Smith, 1975.)

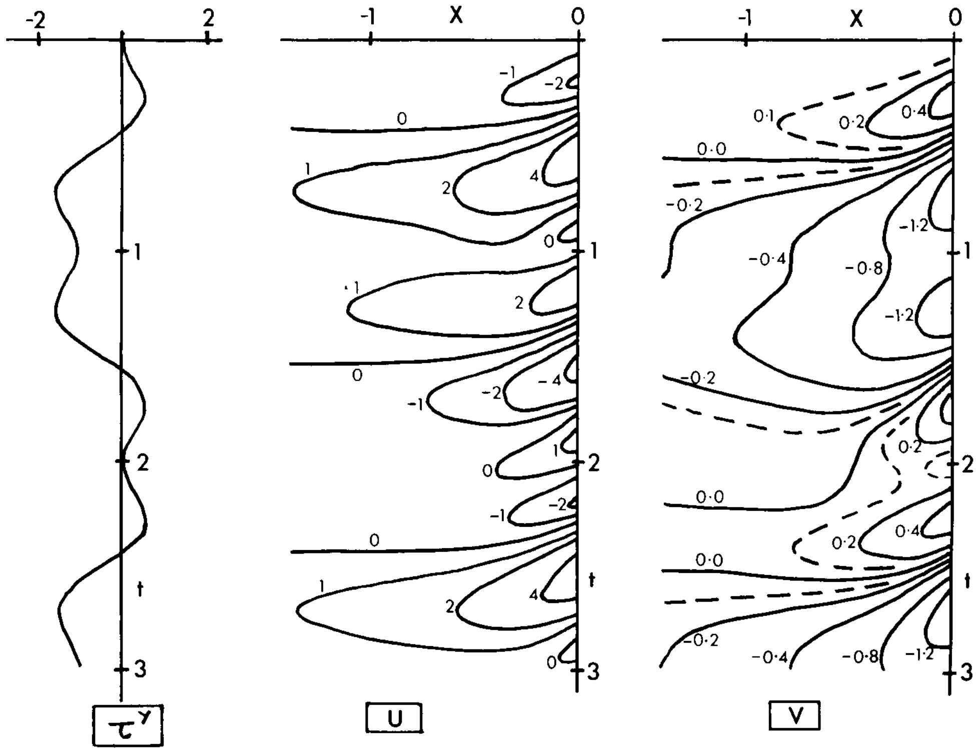

Fig. 4. The wind stress (16) and the corresponding contours of longshore current V and onshore flow U in $x - t$ space, at latitude $y = 0$. The onshore flow appears larger as it has been scaled by $E^{1/4}$.

minimum number of terms in a Fourier series that gives a fair comparison with the wind distribution measured by Mittelstaedt, Pilsbury and Smith (1975) during March and April 1974 as part of the Joint-1 experiment off Northwest Africa. Some of their observations are presented in Fig. 3. The distribution of the longshore velocity in the numerical solution driven by (16) is shown in Fig. 4 and has a number of similarities to the observations with both being closely related to the wind stress. The formation of countercurrents is more noticeable when the wind is light. The onshore flow U at intermediate depths is only weakly tied to the wind stress and is not always onshore during upwelling favourable winds, and vice versa. Comparison with the observations of U is less good as the lack of stratification in the model is more important here as the observations show considerable vertical structure in the onshore velocity.

(iii) $$\tau^y = \exp\{-60(y - y_0 - \tfrac{1}{4} - ct)^2\}\tanh 5t. \quad (17)$$

This expression for τ^y represents a pulse of wind that moves north with speed c. The tanh $5t$ ensures a smooth start at $t = 0$. The solution is presented in Fig. 5 for $c = 0.25$ and $c = 2$. In the former case the wind pulse moves slowly along the coast and the induced longshore flow forms ahead of the wind indicating the presence of waves propagating along the coast with phase speed greater than $c = 0.25$. However, for the faster moving wind pulse with $c = 2$, the distribution of longshore current lags behind the wind distribution at the coast showing that the wind pulse is moving faster than the coastal waves. After the pulse has passed, the velocity field decays to zero in response to the frictional term in (10).

6. Discussion

A variety of different wind stress patterns have been used to generate coastal currents over a sloping continental shelf. In the numerical solutions there is evidence of wave propagation both offshore and along the coast. Countercurrents form offshore of the principal coastal current at times when the wind changes direction. This is additional

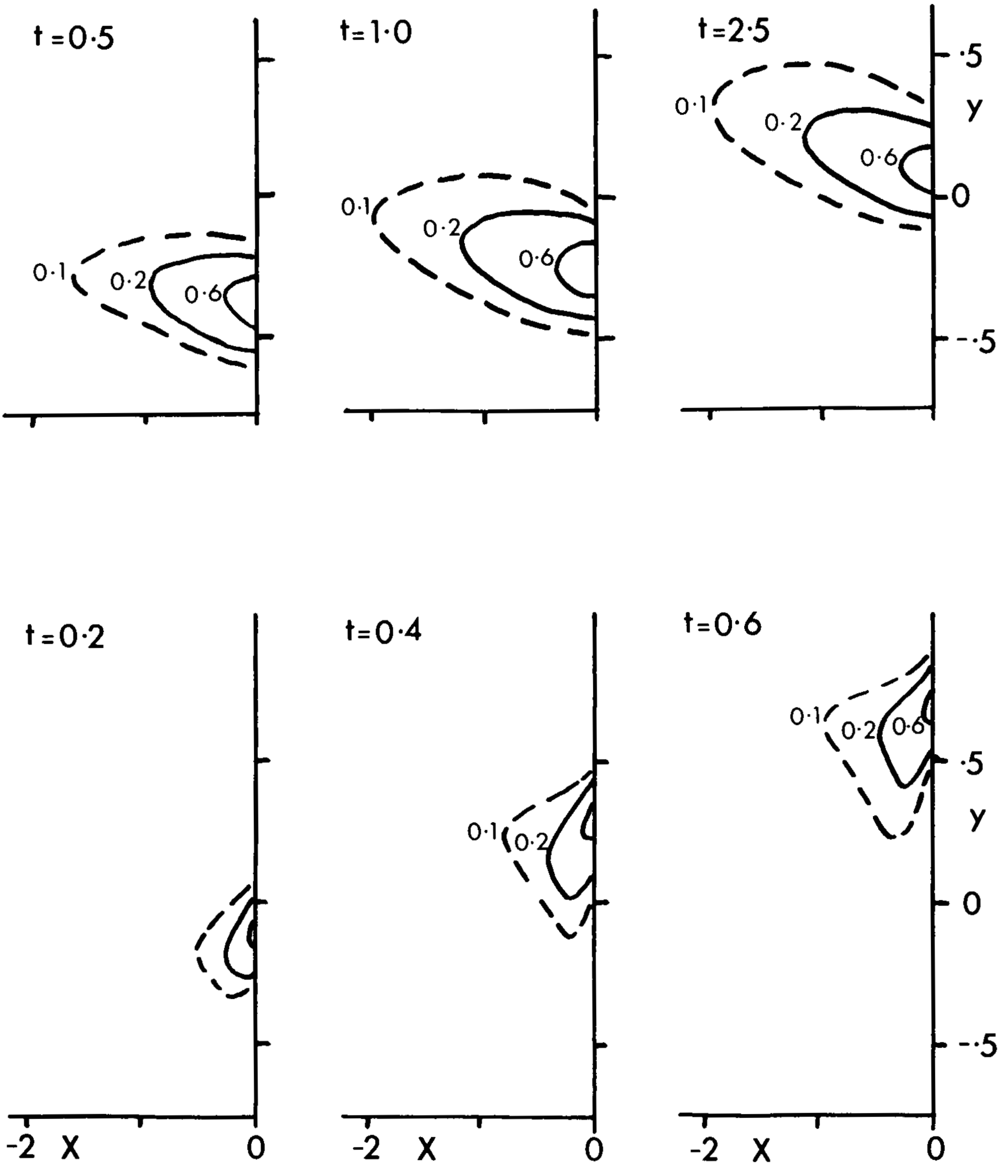

Fig. 5. The contours of longshore current in $x - y$ space at different times driven by the wind stress (17). (i) $c = 0.25$, (ii) $c = 2.0$.

to similar results in steady models when the wind varies spatially.

A small modification to the theory in Sections 2 and 3 allows account to be taken of continental slopes that change with latitude. A repeat of some of the above examples with $\alpha = \alpha(y)$ shows that the longshore current is reduced above submarine canyons and increased above transverse ridges.

In addition to the β-plane solutions described in this paper, the numerical integration has also been carried out for a f-plane (f constant, $\beta = 0$) and a quasi-beta plane (f constant, β constant but non-zero). For the f-plane solutions it is found that the longshore flow decays more rapidly with distance from the coast. This is a consequence of the absence of Rossby waves which propagate offshore in the β-plane model and help to spread out the coastal current. If f is constant, eq. (10) may be solved using Fourier series in y and a simpler single step numerical scheme can be adopted to find the contours of V in $x - t$ space. Some solutions have been calculated using the simpler method for both f and quasi-beta planes. When compared wtih solutions using the scheme described in Section 5, almost identical results were obtained for corresponding wind distributions apart from the initial spin-up phase of the more general solution.

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НЕСТАЦИОНАРНЫЕ ТЕЧЕНИЯ НАД КОНТИНЕНТАЛЬНЫМ ШЕЛЬФОМ

Анализируется баротропная модель на β-плоскости нестационарного прибрежного течения над наклонным континентальным шельфом, возбуждаемого ветром. Уравнение для компоненты скорости вдоль берега решается численно и включает эффекты волн Россби, Кельвина и шельфовой волны. Решение сравнивается с наблюдениями над апвеллингом у берегов северо-западной Африки.