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Turbulent flux of heat and water vapor from the ocean

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ABSTRACT

Observations from the weather ship station M (66 °N, 2 °E) are used in this study. For each month of the 19-year period 1948–1967 the data are divided into 18 groups according to the wind direction. For each wind speed interval we calculate the mean value of the difference between the sea surface temperature and the surface air temperature, and the difference between the saturation water vapor pressure for the sea surface temperature and the water vapor pressure in the surface air. We find that these differences are independent of the wind speed. These observational facts are shown to be in agreement with the classical turbulence theory, which includes the use of wind-speed independent transfer coefficients for heat and water vapor.

Also a differential equation is derived from which we may calculate the surface temperature of an air column when the temperature profile of the initial air mass and the sea surface temperature along the surface trajectory of the air column are known.

1. Introduction

For parametric computations of vertical fluxes of heat and water vapor over the open ocean, one commonly makes use of the so-called bulk aerodynamic formulae (see for example Krauss, 1972, p. 152):

$$f_H = C_z \rho c_p V_a (T_s - T_a) \tag{1.1}$$

$$f_v = C_z \rho V_a [q_w(p_s, T_s) - q_a] \approx C_z \rho V_a \frac{\varepsilon}{p_s} \times [e_w(T_s) - e_a] \tag{1.2}$$

The turbulent transfer coefficients for heat and water vapor are put equal to the drag coefficient C_z. z is the height where the local mean wind speed V_a, air temperature T_a, specific humidity q_a, and water vapor pressure e_a are measured. Furthermore c_p denotes specific heat of the air at constant pressure, ρ air density, T_s sea surface temperature, $q_w(p_s, T_s)$ the saturation specific humidity at the sea surface, p_s surface pressure, $e_w(T_s)$ saturation water vapor pressure, and ε the ratio between the gas constants for dry air and water vapor. These formulae for the vertical flux densities of heat, f_H, and water vapor, f_v, are supposed valid under nearly neutral equilibrium conditions. Fortunately, this is a rather common occurrence in the air over water.

The drag coefficient may be expressed as $C_z = (\kappa/\ln z/z_0)^2$ (see Krauss, 1972, p. 140), where κ is the von Karman's constant and z_0 the roughness length. At a given height C_z will vary with z_0. From dimensionality arguments one obtains for the roughness length over sea an equation of the form $z_0 = b(u_*^2/g)$ (see for example Monin and Yaglom, 1971, p. 295). Here g is the acceleration of gravity, u_* the friction velocity, and b a constant. According to Monin and Yaglom (1971) observations reported give rather different values for b, for example Charnock (1958) found $b \approx \frac{1}{80}$ and Ellison (1956) found $b \approx \frac{1}{12}$. However, in climatological and also dynamical studies, one has commonly used a constant value for the drag coefficient. Krauss (1972) argues that this is justified from the view that the wind interacts primarily with the high frequency part of the wave spectrum which has the same power over a wide range of wind velocities. Krauss (1972, pp. 152 and 164) claims that observations taken under nearly neutral conditions indicate that in the 5 to 16 m/s range the drag coefficient seems to be independent of the wind speed.

In any case, the question of the roughness over sea surface is at present far from being completely clear. Many researchers are inclined to consider that on the sea z_0 does not depend just on the local meteorological conditions, but also on the wind fetch.

In the present study we will present observations which indicate that the drag coefficient in the bulk aerodynamic formulae (1.1) and (1.2) may be considered to be constant for a very wide range of wind speeds. The observations are presented in section 2 and the theory in section 3.

2. Observations from station M (66 °N, 2 °E)

Data from station M are treated in the following manner. After dividing the data for each month into 18 different wind sector groups, the mean value of the differences $(T_s - T_a)$ and $(e_w(T_s) - e_a) = (e_s - e_a)$ within 2 m/s wind speed intervals are plotted against the wind speed (Fig. 1). The data for the two periods 1949–1961 (fully drawn curves) and 1962–1967 (dashed curves) are treated separately, the reason being a change in the observational procedure. By dividing the data into groups as explained, it seems reasonable to assume that we obtain climatological values of the differences $(T_s - T_a)$ and $(e_s - e_a)$ representative for given airmass types and given distributions of the sea surface temperature along the surface trajectories.

It turns out that these differences $(T_s - T_a)$ and $(e_s - e_a)$ are practically independent of the wind speed. In Figs. 1a and b, plots for $(T_s - T_a)$ are shown for the wind direction (310° and 320°) for February and July. This wind sector gives the largest values of the differences. The difference is largest in February, about 5 °C, and smallest in July, slightly less than 2 °C. For the opposite wind

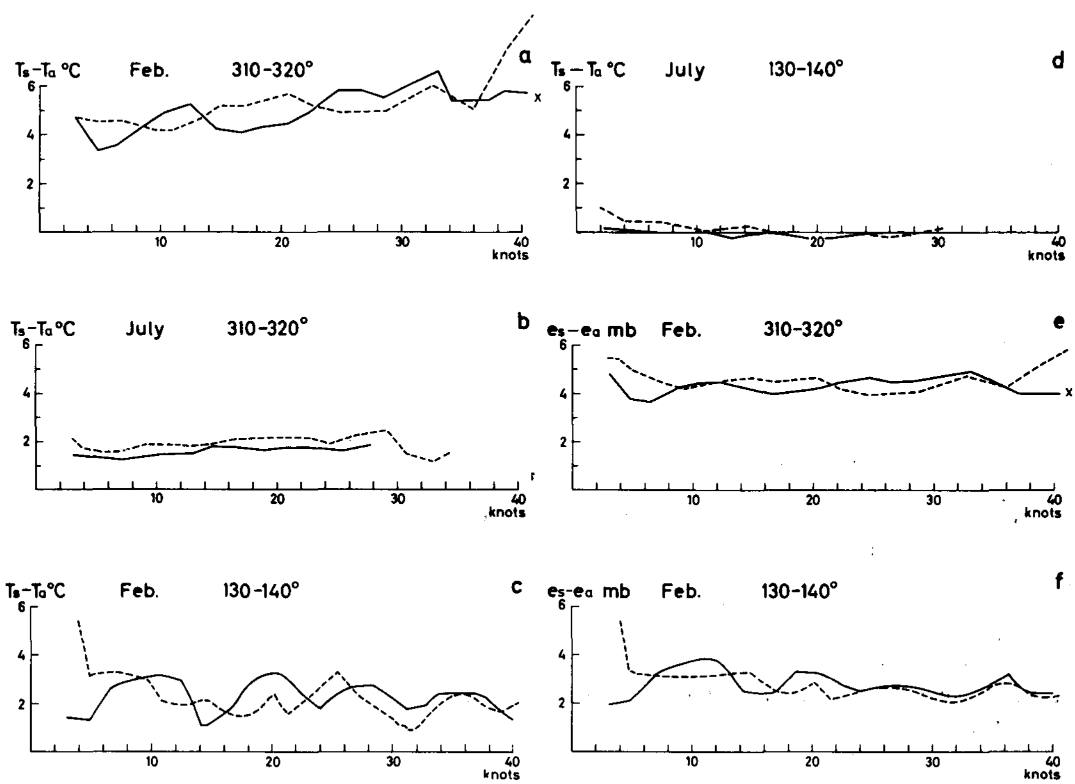

Fig. 1a–f. Mean values of $T_s - T_a$ (°C) and $e_s - e_a$ (mb) plotted against wind speed (knots), at station M for wind directions 310–320° and 130–140°, for the months February and July. Solid curves are for the period 1949–1961 (observations every 3 hours); broken curves for the period 1962–1967 (hourly observations).

directions (130° and 140°) the value of $(T_s - T_a)$ is about 2 °C in February and zero in July (Figs. 1c and d). In February, wind sector (310°–320°), a total of five observations were found having windspeeds exceeding 44 knots (22 m/s). This number of observations was regarded as too small to be climatologically representative. The mean value of $(T_s - T_a)$, 5.4 °C, for these observations is marked with a cross to the right in Fig. 1a.

The mean value of the difference between the saturation water vapor pressure for the sea surface temperature and the water vapor pressure in the surface air $(e_s - e_a)$, is plotted in the same manner and shows the same characteristic feature. In Figs. 1e and f the plots are for February and wind directions (310°–320°) and (130°–140°), respectively. The value of $(e_s - e_a)$ is seen to be about 4 mb for the wind sector 310°–320° and about 3 mb for the wind sector 130°–140°. The mean value of $(e_s - e_a)$, 4.0 mb, for the five observations having wind speed larger than 44 knots is marked with a cross to the right in Fig. 1e.

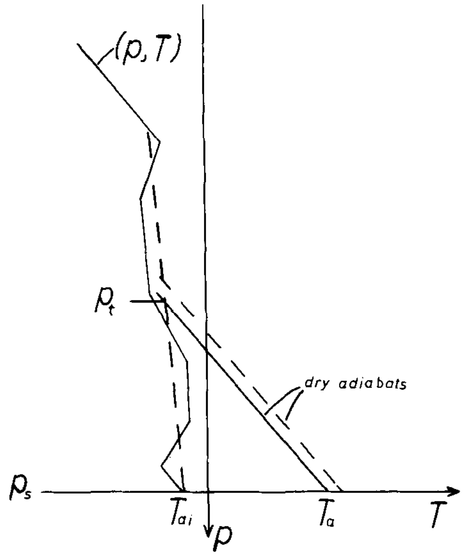

Fig. 2. Initial temperature sounding curve modified by turbulent heat flux. Dashed curve to the left indicates the mean temperature sounding curve defining T_{ai} and $(\overline{\delta T}/\delta p)_i$.

3. Theory

We will assume near neutral equilibrium conditions and thus that the bulk aerodynamic formulae (1.1) and (1.2) are valid. Furthermore we will assume quasi-stationary conditions. The concept of a moving air column that receives the vertical fluxes is also used.

3.1 *Turbulent flux of sensible heat*

The vertical flux density of sensible heat, f_H, is assumed given by eq. (1.1). The air column which receives this heat flux is heated under isobaric conditions. Thus the heat flux goes to increase the enthalpy of the air column. The change in enthalpy per unit mass, h, is given by: $dh/dt = c_p(dT/dt)$. We assume that the turbulent heat flux takes place under neutral conditions. The stratification of the air will then be dry-adiabatic and the temperature change of the mixed layer will everywhere be equal to the increase of the surface air temperature (see Fig. 2). We denote the pressure at the top of the mixed layer p_t. The increase in enthalpy of the air column is then given by

$$\int_{p_t}^{p_s} \frac{dh}{dt}\frac{\delta p}{g} = \frac{c_p}{g}\frac{dT_a}{dt}(p_s - p_t) \tag{3.1}$$

From eq. (1.1) and (3.1) we obtain

$$C_z \rho V_a (T_s - T_a) = \frac{1}{g}\frac{dT_a}{dt}(p_s - p_t) \tag{3.2}$$

One of the basic assumptions in developing formulae for turbulent fluxes is the assumption of stationarity. We therefore assume $dT_a/dt \approx V_a(\partial T_a/\partial x)$ where x measures the distance along the surface trajectory of the air column. When introducing this into eq. (3.2) V_a drops out and we obtain

$$C_z \rho (T_s - T_a) = \frac{1}{g}\frac{\partial T_a}{\partial x}(p_s - p_t) \tag{3.3}$$

From eq. (3.3) we find that the quantity $(T_s - T_a)$ is independent of wind speed if the drag coefficient, C_z (or the heat transfer coefficient) is assumed to be constant.

In order to see which parameters determine the air temperature and thus, assuming T_s to be given, the difference $(T_s - T_a)$ we seek an expression for the height of the mixed layer. This height has been estimated in many sophisticated ways. Here we simply assume that

$$T_t - T_{ai} = -\left(\frac{\overline{\delta T}}{\delta p}\right)_i (p_s - p_t) \tag{3.4}$$

where T_t is the temperature at p_t, T_{ai} is the surface air temperature and $(\overline{\delta T}/\delta p)_i$ a mean value of the temperature change with pressure for the initial temperature sounding curve. We will assume that it is possible to find a representative constant value of $(\delta T/\delta p)_i$ as in the case illustrated in Fig. 2; where the dashed curve to the left indicates the temperature sounding curve corresponding to a given T_{ai} and a constant value of $(\delta T/\delta p)_i$ which fairly well approximates the actual temperature sounding curve, fully drawn. We further simplify the dry-adiabatic temperature gradient to be constant for the column given by $(R/c_p)(T_0/p_0)$, where T_0 and p_0 are appropriate mean values, R is the gas constant per unit mass. Considering the triangle in Fig. 2 formed by the isobar p_s, the dry adiabat with surface temperature T_a, and the temperature sounding curve with constant $(\delta T/\delta p)_i$, the dashed curve to the left in Fig. 2, we find that

$$T_a - T_{ai} = \left[\frac{R}{c_p}\frac{T_0}{p_0} - \left(\frac{\overline{\delta T}}{\delta p}\right)_i\right](p_s - p_t)$$
$$= K(p_s - p_t) \tag{3.5}$$

where K, for a given synoptic situation, with little loss of accuracy may be assumed to be constant. This is not a necessary, but a convenient assumption. Equation (3.5) shows that the depth of the mixed layer depends on the form of the initial temperature sounding curve and the surface temperature which the air column has obtained as a result of the heat flux. Introducing eq.(3.5) into eq.(3.3) gives

$$C_z \rho (T_s - T_a) = \frac{1}{g}\frac{\partial T_a}{\partial x}\frac{T_a - T_{ai}}{K} \tag{3.6}$$

We have assumed K to be different from zero. Assuming the drag coefficient (or the heat transfer coefficient) to be constant, we may introduce a dimensionless independent variable $\zeta = x/L$ where $L = T_0/Kg\rho C_z$. Equation (3.6) may now be written

$$\frac{T_a - T_{ai}}{T_0}\frac{\partial}{\partial \zeta}\frac{T_a - T_{ai}}{T_0} + \frac{T_a - T_{ai}}{T_0}$$
$$= \frac{T_s(\zeta) - T_{ai}}{T_0} \tag{3.7}$$

This equation may be solved numerically for any given distribution of T_s along the surface trajectory. As an example we use the case presented in Figs. 3–5. Figs. 3 and 4 show the surface air temperature at 12 GMT 10 January 1967 and at 00 GMT 11 January 1967, respectively. Fig. 5 gives the sea surface temperature 10–11 January 1967. In Fig. 6 we have plotted the value of T_s for the air column's path between $T_s = 0$ °C and station M. In this case we have from the temperature sounding (Fig. 2) at Jan Mayen (70.8 °N, 8.6 °W): $T_{ai} = -12$ °C and $(\delta T/\delta p)_i = 1$ °C/100 mb. For simplicity we take T_s to be linear; according to eq.(3.7) T_a should then quickly approach a value 6 °C less than T_s. This result corresponds well with the observed value of T_a, which is also plotted in Fig. 6.

3.2 *Turbulent flux of water vapor*

A similar line of reasoning as above may be applied to the turbulent flux density of water vapor, f_v. This flux of water vapor goes to increase the specific humidity of the air column. Assuming the specific humidity within the mixed layer to be constant with pressure we obtain

$$\int_{p_t}^{p_s}\frac{dq}{dt}\frac{\delta p}{g} = \frac{1}{g}\frac{dq_a}{dt}(p_s - p_t) \tag{3.8}$$

Introducing the assumptions

$$q_a \approx \varepsilon\frac{e_a}{p_s} \quad \text{and} \quad q_w(p_s, T_s) \approx \varepsilon\frac{e_w(T_s)}{p_s}$$
$$= \varepsilon\frac{e_s}{p_s} \tag{3.9}$$

we obtain in analogy to eq.(3.3)

$$C_z \rho [e_s - e_a] = \frac{1}{g}\frac{\partial e_a}{\partial x}(p_s - p_t) \tag{3.10}$$

Thus also the quantity $e_s - e_a$ is independent of the wind speed if the drag coefficient (or the water vapor transfer coefficient) is assumed to be constant.

4. Conclusions

By dividing the data into groups as explained in section 2, it seems reasonable to assume that we obtain climatological values of the differences $(T_s - T_a)$ and $(e_s - e_a)$ representative for given airmass types and given distributions of the sea surface

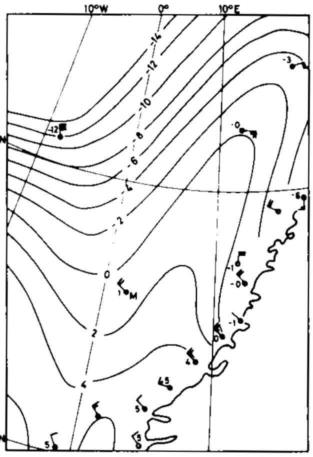

Fig. 3. Surface air temperature (°C), 12 GMT 10 January 1967.

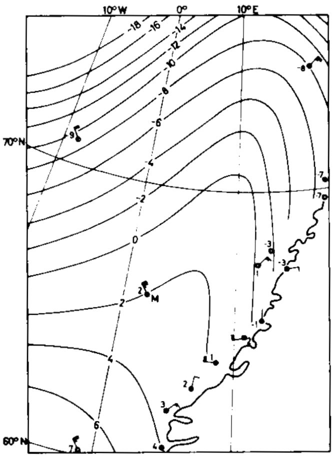

Fig. 4. Surface air temperature (°C), 00 GMT 11 January 1967.

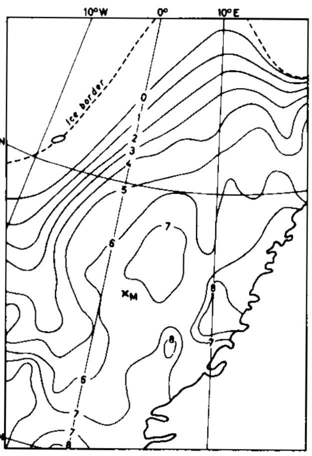

Fig. 5. Sea surface temperature 10–11 January 1967. Ice border is indicated by the broken line.

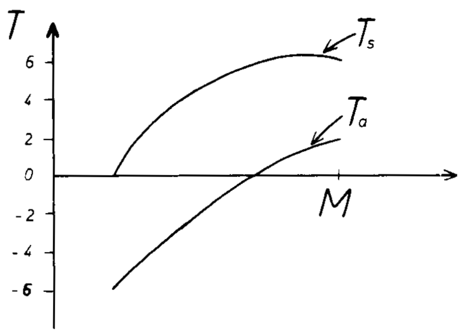

Fig. 6. Sea surface temperature (10–11 January) and surface air temperature (100 GMT, 11 January) along surface trajectory from sea surface isoterm 0 °C to station M.

temperature along the surface trajectories. According to formulae (3.3) and (3.10) these differences should be independent of wind speed if the drag coefficient (or the heat and water vapor transfer coefficients) is independent of wind speed.

All the data from the 19-year period 1947–1967 have been used, and there is no selection of data. It is therefore somewhat surprising that the differences $(T_s - T_a)$ and $(e_s - e_a)$ stay so remarkably constant for all wind speeds. This strongly indicates that the bulk formulae (1.1) and (1.2) are indeed valid using constant values for the transfer coefficients. This is in contrast to the widely used assumption that over the sea the roughness length should be a function of wind speed.

The observations indicate also that the differential eq. (3.7) may be a useful equation for calculating the surface air temperature in a large variety of synoptic cases. On the other hand, when the surface temperature, T_a, is known, eq.(3.7) may be used to test the assumptions involved.

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ТУРБУЛЕНТНЫЕ ПОТОКИ ТЕПЛА И ВОДЯНОГО ПАРА НАД ОКЕАНОМ

В данном исследовании используются результаты наблюдений с корабля погоды на станции М (66° с.ш., 2° в.д.). Для каждого месяца за 19-летний период 1948–1967 гг. данные подразделены на 18 групп в соответствии с направлением ветра. Для каждого интервала скоростей ветра мы вычисляем среднее значение разности между температурой поверхности моря и температурой воздуха у поверхности и разности между давлением насыщенного пара при температуре поверхности моря и давлением водяного пара в воздухе у поверхности. Найдено, что эти разности не зависят от скорости ветра. Показано, что эти наблюдательные факты согласуются с классической теорией турбулентности, которая использует независимость коэффициентов переноса для тепла и водяного пара от скорости ветра.

Выводится также дифференциальное уравнение, с помощью которого можно вычислить температуру у поверхности для воздушного столба, если известны температурный профиль в начальной воздушной массе и температура поверхности моря вдоль траектории воздушного столба у поверхности.