Tellus (1980), *32*, 164–175

Influence of the Norwegian Trench on the wind-driven circulation of the North Sea

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(Manuscript received June 17; in final form October 2, 1979)

ABSTRACT

A simple three-dimensional numerical model of the North Sea basin is used to investigate the effect of the Norwegian Trench on wind-induced circulation in that basin. It is shown that the presence of the Trench can cause a main flow to pass through it, driven principally by sea-surface gradient set up by the wind. This flow affects the circulation of the entire northern part of the North Sea. A possible driving mechanism for the Norwegian Coastal Current may be inferred.

1. Introduction

During March and April 1976, as part of the JONSDAP 76 (Joint North Sea Data Acquisition Program, 1976) Oceanographical Exercise, current measurements were taken in the Norwegian Trench along latitude 59°20′. On average, north-going currents were found throughout the entire depth of the Trench (Furnes and Saelen, 1977). A three-dimensional numerical model of the wind-induced circulation of the North Sea for the period of measurement (Davies, 1979) essentially has reproduced this observed current structure. Furthermore, the northwards transport through the Trench derived from the model agrees in order of magnitude with that estimated from the observations. The current measurements taken overall in the North Sea during JONSDAP 76, and also the model results, show the Trench flow to be the main residual outflow of the North Sea.

It appears, therefore, at least on present evidence, that there are circumstances in which the wind-driven circulation of the North Sea is able to account for an important part of the observed residual flow out of the North Sea through the Norwegian Trench. But, accepting this, why should the main outflow be located in the Trench? The present paper investigates this question by considering a simple rectangular model of the North Sea and adjoining northerly shelf, with and without a deep cut representing the Norwegian Trench. Three-dimensional motions induced by uniform steady wind fields are determined using a numerical modelling approach of earlier proven design (Heaps, 1974). The results demonstrate that the effect of including the Trench is indeed to channel the wind-driven transport along its length, thereby profoundly affecting the residual circulation pattern of the entire North Sea. The combined but distinctive influences of wind stress and sea-surface gradient are held to be mainly responsible for such behaviour.

2. Model basins

Two model basins are considered, illustrated in Fig. 1. The first consists of a rectangular North Sea of depth 65 m, communicating with a rectangular area of adjoining shelf of depth 260 m. The basin is bounded on its northern side by open ocean and on the other three sides by land.

A second basin consists of the one described above modified by the inclusion of a rectangular trench of depth 260 m running down the eastern side of the North Sea area from the shelf region. Adjacent and parallel to it is a slope region, rectangular in planform, in which the depth

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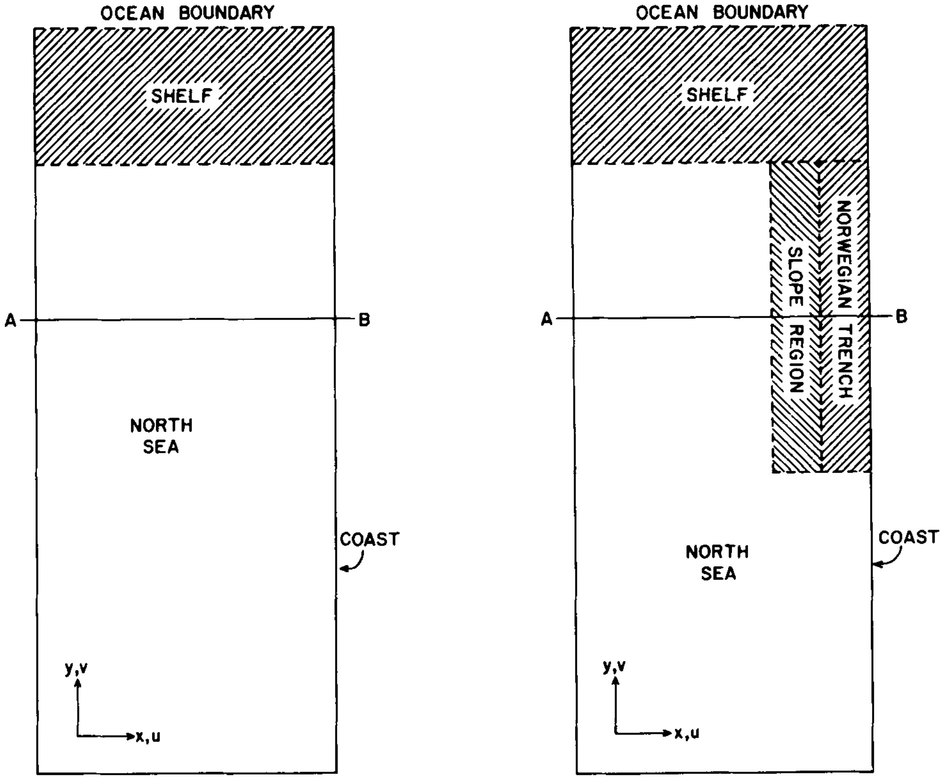

Fig. 1. Rectangular North Sea models with simplified depth distributions, without and with representation of the Norwegian Trench. Areas and associated depths are: □ = 65 m, ■ = 260 m, ■ = 65–260 m, linear west to east variation.

decreases linearly from the 260 m of the trench to the 65 m of the North Sea on passing from east to west. The rectangular trench and the slope region together give a simple representation of the Norwegian Trench.

Patterns of three-dimensional steady circulation maintained in each basin by uniform north and west winds, acting over the North Sea rectangle, have been computed. Before discussing these results, a brief description of the mathematical method used is given.

3. Basic equations and their solution

The equations governing the motion of the sea under wind stress are taken in the form (Heaps, 1974):

$$\frac{\partial \zeta}{\partial t} + \frac{\partial}{\partial x}\int_0^h u\, dz + \frac{\partial}{\partial y}\int_0^h v\, dz = 0 \qquad (1)$$

$$\frac{\partial u}{\partial t} - \gamma v = -g\frac{\partial \zeta}{\partial x} + \frac{\partial}{\partial z}\left(N\frac{\partial u}{\partial z}\right) \qquad (2)$$

$$\frac{\partial v}{\partial t} + \gamma u = -g\frac{\partial \zeta}{\partial y} + \frac{\partial}{\partial z}\left(N\frac{\partial v}{\partial z}\right) \qquad (3)$$

Here t denotes time and x, y, z are Cartesian coordinates forming a left-handed set in which x, y are measured in the horizontal plane of the undisturbed sea surface and z is the depth below that surface. Also h is the undisturbed depth of water, ζ the elevation of the sea surface and u, v the components of current at depth z in the directions of increasing x, y. Additionally, γ is the geostrophic coefficient and g the acceleration of the Earth's gravity, both constants, while N is a coefficient of vertical eddy viscosity.

At the sea surface:

$$-\rho(N\, \partial u/\partial z)_0 = F_s, \quad -\rho(N\, \partial v/\partial z)_0 = G_s \qquad (4)$$

where F_s, G_s are components of wind stress in the x, y directions and ρ is the density of the sea, assumed to be a constant. At the sea bottom, adopting a linear law of bed friction with coefficient k:

$$-\rho(N\,\partial u/\partial z)_h = k\rho u_h,$$
$$-\rho(N\,\partial v/\partial z)_h = k\rho v_h \quad (5)$$

Here, suffix 0 denotes evaluation at $z = 0$ and suffix h evaluation at $z = h$.

With k and N/h taken as constant values, subject to conditions (4) and (5) the dynamical equations (1), (2), (3) may be transformed by vertical integration (Heaps, 1972) to yield

$$\frac{\partial \zeta}{\partial t} = -\sum_{r=1}^{M}\left\{\frac{\partial}{\partial x}(ha_r\phi_r u_r) + \frac{\partial}{\partial y}(ha_r\phi_r u_r)\right\} \quad (6)$$

$$\frac{\partial u_r}{\partial t} = -\lambda_r u_r + \gamma v_r - ga_r\frac{\partial \zeta}{\partial x} + \frac{F_s}{\rho h} \quad (7)$$

$$\frac{\partial v_r}{\partial t} = -\lambda_r u_r - \gamma u_r - ga_r\frac{\partial \zeta}{\partial y} + \frac{G_s}{\rho h} \quad (8)$$

for $r = 1, 2, \ldots, M$ where

$$u_r = \frac{1}{h}\int_0^h u\cos(\alpha_r\xi)\,dz,$$
$$v_r = \frac{1}{h}\int_0^h v\cos(\alpha_r\xi)\,dz \quad (9)$$

Here, M is an integer and

$$\xi = z/h \quad (10)$$

$$\lambda_r = N\alpha_r^2/h^2 \quad (11)$$

$$a_r = \sin\alpha_r/\alpha_r \quad (12)$$

$$\phi_r = 2/(1 + a_r\cos\alpha_r) \quad (13)$$

where α_r is the rth positive root, in ascending order, of

$$\alpha\tan\alpha = c\ (c = kh/N) \quad (14)$$

Finite-difference solutions of eqs. (6), (7), (8) determine the spatial fields of ζ, u_r, v_r ($r = 1, 2, \ldots, M$) through time. A forward time-stepping procedure is employed. Then the current components u, v at any depth z may be found from an inverse transformation:

$$u = \sum_{r=1}^{M}\phi_r u_r\cos(\alpha_r\xi) + (2hF_s/\rho N\pi^2)\,R_M(\xi) \quad (15)$$

$$v = \sum_{r=1}^{M}\phi_r u_r\cos(\alpha_r\xi) + (2hG_s/\rho N\pi^2)\,R_M(\xi) \quad (16)$$

where

$$R_M(\xi) = (3\xi^2 - 6\xi + 2)\,\pi^2/12 - \sum_{r=1}^{M-1}\cos(r\pi\xi)/r^2 \quad (17)$$

Depth-mean currents are determinable from

$$\bar{u} = \sum_{r=1}^{M}\phi_r u_r a_r, \quad \bar{v} = \sum_{r=1}^{M}\phi_r v_r a_r \quad (18)$$

According to (15) and (16), currents are expressed in terms of a series of cosine modes through the vertical, corresponding respectively to $r = 1, 2, \ldots, M$ in ascending order. In practice, M has to be chosen sufficiently large for the convergence of the series.

Our application of the preceding equations to the evaluation of wind-induced flow in the sea basins of Fig. 1 is now described. With the x and y axes taken parallel to the bounding sides as shown in the figure, numerical solutions of (6), (7), (8) were developed through time representing motions generated from an initial state of rest:

$$\zeta = u_r = v_r = 0 \quad \text{at} \quad t = 0 \quad (19)$$

under the action of either a northerly wind stress field:

$$F_s = 0, \quad G_s = -1.5\ \text{N/m}^2 \quad (20)$$

or a westerly stress field:

$$F_s = 1.5\ \text{N/m}^2, \quad G_s = 0 \quad (21)$$

These fields were applied uniformly over the water surface of the North Sea rectangle at time $t = 0$ and then kept constant. Employing a stress of magnitude 1.5 N/m^2 follows the procedure of an earlier investigation which used three-dimensional modelling to study wind-driven motion in a rectangular basin (Heaps, 1972). After 100 h of real time it was found that the wind currents had settled down into, essentially, a steady-state circulation pattern. Results from this stage are presented here.

The solutions were developed satisfying zero normal flow on the land boundaries:

$$u_r\cos\psi + v_r\sin\psi = 0 \quad (r = 1, 2, \ldots, M) \quad (22)$$

and outward radiation, like a simple progressive wave, on the open sea boundary in the north:

$$\left.\begin{aligned}\phi_1 a_1 v_1 &= (gh)^{1/2}\,\xi/h\\ v_r &= 0, \quad r \geqslant 2\end{aligned}\right\} \tag{23}$$

where ψ denotes inclination of a boundary normal to the x-axis, directed out of the model area. Current modes higher than the first are set to zero in (23) thereby prescribing a simplified vertical current structure along the open boundary.

An earlier model omitted the shelf area and imposed the radiation condition along the northern edge of the North Sea rectangle. However, allowing only the first current mode along this edge suppressed the development of differential flow through the depth along it. This had the effect of inhibiting the proper establishment of a three-dimensional wind-driven circulation within the North Sea area. With the shelf area included, the restricting influence of the radiation condition is removed to the north, where the currents are weak and unidirectional through the depth, and is remote from the North Sea area and the wind action. Three-dimensional motion in the North Sea, driven by wind over it, is then sensibly unaffected by conditions on the open boundary.

The finite-difference scheme used in the numerical solution of (6), (7), (8) was the same as that employed by Heaps (1972). The computations were based on a rectangular grid with construction lines parallel to the x, y axes subdividing the model area uniformly. Mesh lengths $\Delta x = 400/27$ km and $\Delta y = 800/51$ km were taken in the x and y directions respectively. A time step of 1 min was used in the calculations.

Horizontal dimensions in Fig. 1 may be conveniently expressed in units of Δx and Δy. Thus, the North Sea rectangle is of width $27\Delta x$ and length $51\Delta y$, while the total northwards extension of the model, incorporating the shelf area, is $64\Delta y$. The rectangular trench and the slope region have widths $5\Delta x$ and $4\Delta x$ respectively, both being of length $25\Delta y$ extending down half the length of the North Sea rectangle.

Constant parameters employed in the work, used previously (Heaps, 1972), are:

$$\begin{aligned} N/h &= 0.001 \text{ m/s}, & \gamma &= 0.1222 \times 10^{-3}\text{ s}^{-1},\\ M &= 10, & g &= 9.81 \text{ m/s}^2\\ P &= 1025 \text{ kg/m}^3, & c &= 2\\ k &= 0.002 \text{ m/s}, \end{aligned}$$

Here, the geostrophic coefficient corresponds to a latitude $\phi = 56°56'$ representative of the North Sea. Thus, with $\omega = 7.292 \times 10^{-5}\text{ s}^{-1}$ the angular speed of the Earth's rotation,

$$\begin{aligned}\gamma &= 2\omega \sin \phi\\ &= 2 \times 7.292 \times 10^{-5} \times \sin \phi \text{ s}^{-1}\\ &= 0.1222 \times 10^{-3}\text{ s}^{-1}\end{aligned}$$

The friction model used here is based on the assumptions:

$$k \propto U_1, \quad N \propto U_1 h$$

where U_1 is some representative velocity (Bowden, 1953). Therefore

$$kh/N = c \tag{24}$$

where c is a constant. On the basis of previous dynamical computations for both the Irish Sea and the North Sea (Heaps, 1974; Davies, 1979) we are led to believe that

$$c = 2, \quad k = 0.002 \text{ m/s}$$

are generally representative values under these assumptions so that

$$N/h = 0.001 \text{ m/s}$$

In the shallow part of our rectangular model, with $h = 65$ m, this gives $N = 0.0650\text{ m}^2/\text{s}\ \phi$ and in the deeper part, with $h = 260$ m, it gives $N = 0.2600\text{ m}^2/\text{s}$. Clearly, there is an unrealistic jump in the value of N from the shallow to the deeper region but this is an inevitable consequence of the assumption that N is directly proportional to the depth h. It seems correct that a jump in h should indeed be accompanied by a jump in N since eddy sizes through the depth are likely to change correspondingly.

We do not expect the nature of the frictional assumptions to significantly affect the essential nature of the circulation patterns derived here. More realistic numerical models give similar results (Davies, 1979).

4. Wind-induced circulations

Consider first the computed circulations in the model basins of Fig. 1 associated with a *northerly* wind stress field of 1.5 N/m² acting over the North

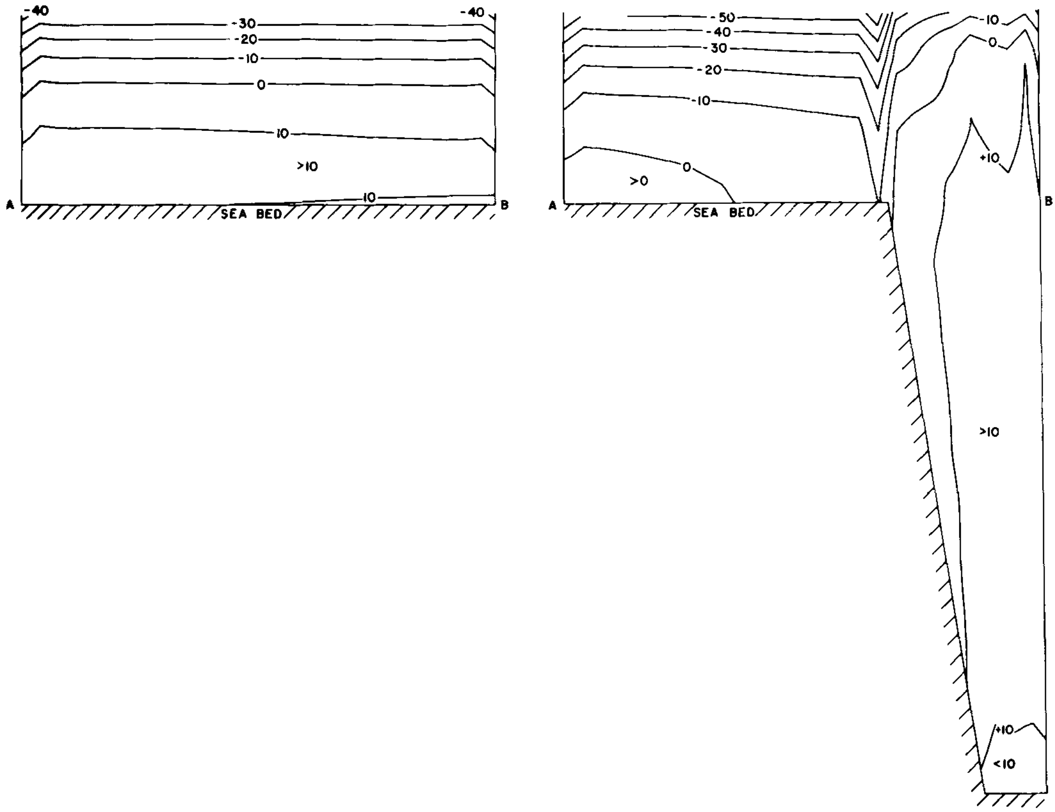

Fig. 2. Currents acting normally across the vertical section AB, showing v-contours in cm/s; positive values indicate flow to the north and negative values flow to the south. The results correspond to a uniform northerly wind field of 1.5 N/m^2 over the North Sea rectangle, without and with a Norwegian Trench.

Sea rectangle. Patterns of wind-induced flow with and without the Norwegian Trench are compared. Perhaps the most striking demonstration of the effect of the Trench on the distribution of currents is shown in Fig. 2 where the north-going v-currents over a vertical west–east section AB (located halfway along the length of the Trench, Fig. 1) are contoured. Without the Trench there is a flow into the basin near the surface and an outward return flow at depth; this vertical structure is sensibly uniform across the entire width of the section. With the Trench included, however, there is inflow at nearly all depths of the shallow North Sea (reaching a maximum near the edge of the slope region) and outflow through most of the deep Trench. Fig. 3 shows the change in the distribution of northerly transport across the section. Without the Trench transport at any position is near to zero due to the balance of inward and outward flows. With the Trench, the transport is inward across the shallow North Sea and outward, reaching higher values, in the Trench.

Fig. 4 shows the horizontal distributions of transport, surface and bottom current with the Trench included. Without the Trench these distributions are uniform, everywhere reflecting a flow towards the south near the sea surface and a return flow towards the north-east near the sea bed. The

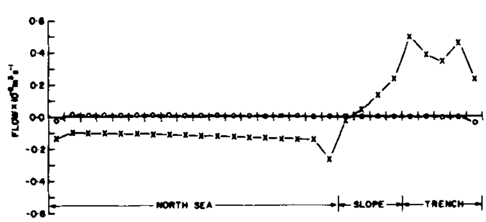

Fig. 3. Variation in horizontal transport (vertically-integrated current per grid length Δx) across section AB, corresponding to the northerly wind stress field; —O—O— without trench, —x—x— with trench.

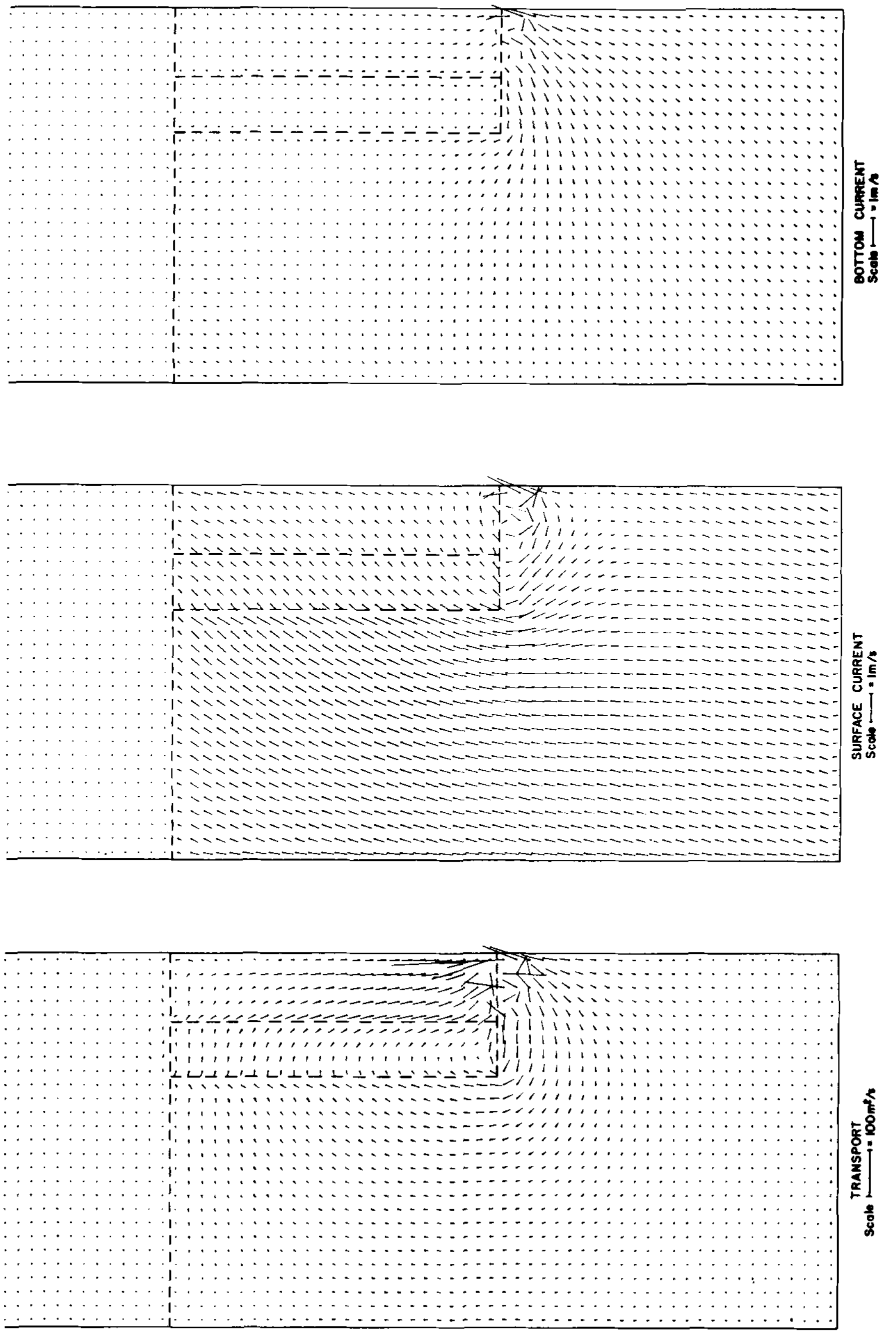

Fig. 4. Transports (m^2/s), surface and bottom currents (m/s) due to northerly wind stress field, with the Norwegian Trench included.

flow over the whole of the North Sea is then similar to that shown in Fig. 4 for the southern part of the model area. Modifications to the currents, brought about by the presence of the Trench, are therefore easily discernible in Fig. 4. Examining the latter in detail, the pattern of transports clearly shows how a south-going flow in the shallow North Sea turns eastwards into the southern end of the Trench where there are two distinct local gyres. Flow northwards in the Trench is evident, turning to the west in the slope region to complete an overall anti-clockwise circulation. The surface currents, being dominated by wind action, are large and mainly directed between south and south-west. Even so, there is a turning eastwards at the southern end of the Trench. Manifestly, the surface currents in the Trench are smaller than those in the adjacent North Sea area. The pattern of bottom currents shows how the general return flow at depth converges towards the southern end of the Trench, being continued by a northwards flow within the Trench itself.

Fig. 5 indicates that the inclusion of the Trench brings about a radical alteration in the distribution of sea-surface elevation, ζ. With a uniform North Sea, the northerly wind maintains a regular gradient of ζ from north to south producing a total rise in ζ of approximately 200 cm. The effect of the Trench is to reduce elevation to near-zero in its own area and in the neighbouring slope region. As a consequence, elevations everywhere are diminished and a skewness is introduced into the pattern of sea-surface rise.

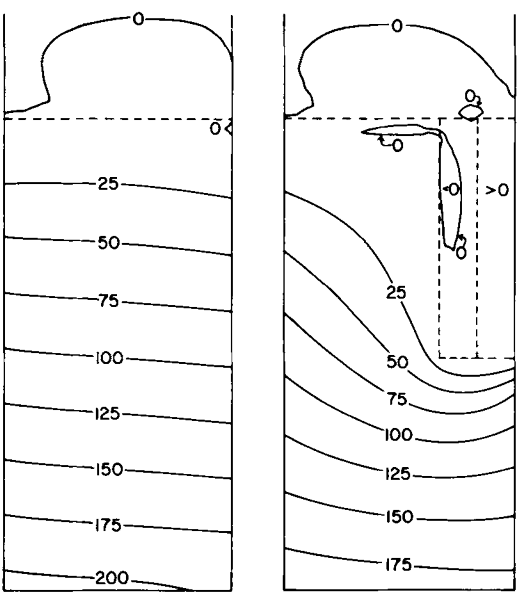

Fig. 5. Contours of sea-surface elevation (cm) due to the northerly wind stress field, without and with a Norwegian Trench.

An explanation of the changes in circulation described above depends on recognizing the separate actions of wind and surface gradient. The flows produced by both of these forcing agencies are, of course, modified by the Earth's rotation and requirements of continuity. The main role of gradient is in driving water northwards through the Trench where the wind effect is weak and largely confined to the near-surface waters. To satisfy continuity, water is drawn into the Trench across its southern end, causing the surrounding North Sea currents to be deflected to the east. Water from the shallow North Sea must sink as it enters the Trench. The introduction of the Trench weakens the north–south gradients over the northern part of the shallow North Sea and the south-going wind-driven flow in this region therefore intensifies. This is particularly noticeable near the slope region where the gradient is weakest. Further to the south, however, the gradients become steeper, especially so immediately south of the Trench. Here they are effective in opposing the wind influence, reducing the magnitudes of the south-going currents and allowing development of the easterly flow into the Trench.

Consider next the computed circulations associated with a *westerly* wind stress of 1.5 N/m^2 acting over the North Sea rectangle. The v-currents over section AB are contoured in Fig. 6. The influence of the Trench is again to concentrate north-going flow along its length, driven by surface gradient. South-going flow in the shallow North Sea, mainly Ekman-type, is strengthened by the requirement to compensate for the outward Trench flow. The redistribution of transport across section AB is illustrated in Fig. 7. The effect of the Trench in altering the balance of flow is even more pronounced than in the case of the north wind (Fig. 3) with considerably higher transports in the channel.

Fig. 8 shows how the Trench influences the distribution of surface currents. Their magnitudes are reduced in the area of the channel but increased

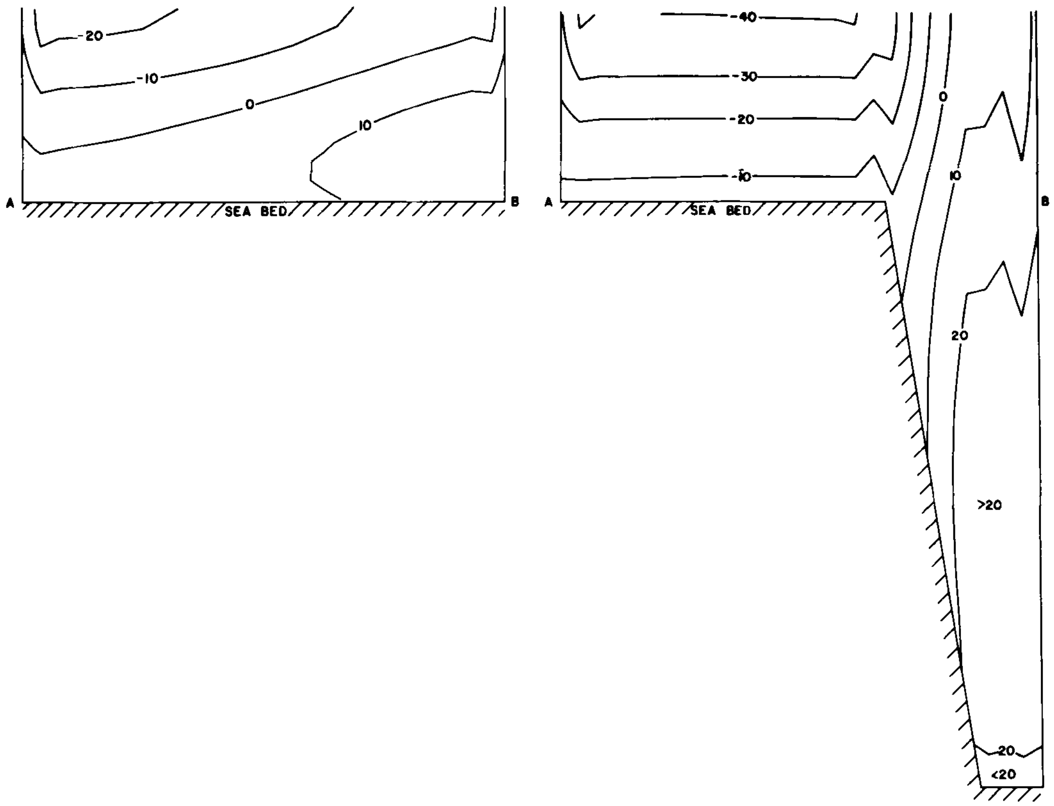

Fig. 6. Currents acting normally across the vertical section AB due to a westerly wind stress field of 1.5 N/m^2 over the North Sea rectangle, without and with a Norwegian Trench. Notation as in Fig. 2.

and deflected to the south in the neighbouring North Sea. Highest values are located near the southern end of the Trench where there are local gyres. Northward flow through the Trench at lower levels continues the surface circulation.

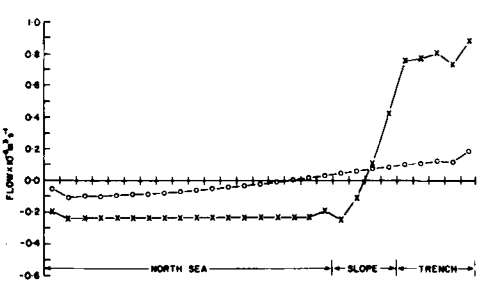

Fig. 7. Variation in horizontal transport across section AB without and with a Norwegian Trench, corresponding to the westerly wind stress field. Notation as in Fig. 3.

Fig. 9 shows distributions of bottom current. Without the Trench, the currents are small and directed to the north-west over most of the North Sea rectangle. There is a stronger set of north-going currents in the north-eastern region, part of a vortex centred around the northern edge of the rectangle. With the Trench included, the currents are channelled along its length and this flow is continued by a weak anticlockwise circulation in the shallow North Sea.

Fig. 10 demonstrates again, this time in the case of the west wind, that the presence of the Norwegian Trench importantly alters the pattern of sea-surface elevation. Positive elevations are reduced to negative and near-zero values in the Trench area, preventing the regular build-up of sea level under the wind's influence.

Combining linearly the fields of motion due to the northerly and westerly wind stress fields has

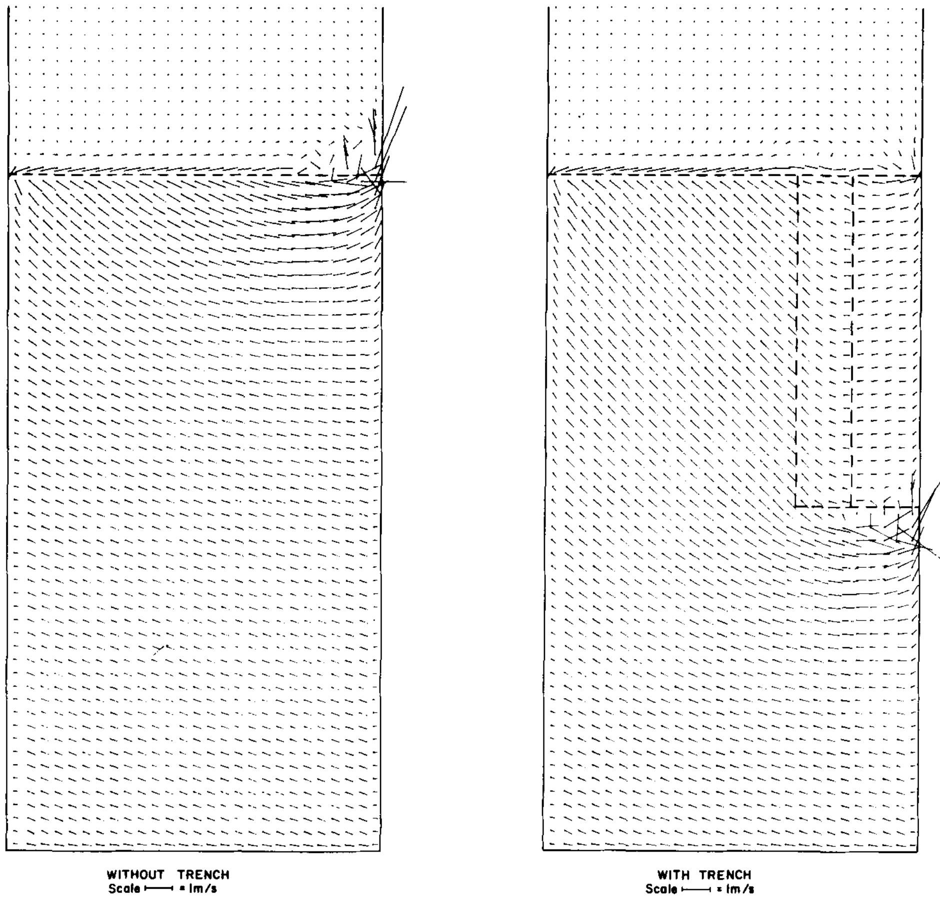

Fig. 8. Surface currents due to the westerly wind stress field, without and with a Norwegian Trench.

yielded the total cross-sectional flow through the Trench and slope region for a stress field of 1.5 N/m^2 acting in various directions. This flow, counted positive to the north and negative to the south, is plotted against wind angle in Fig. 11. It is apparent that winds from south-west through west to north-east (the strongest and most common) produce north-going trench flows—evidenced in reality by the Norwegian Coastal Current. From Fig. 11, a south-west wind stress of 1.5 N/m^2 yields a northwards Trench flow of 1.61×10^6 m^3/s. Scaling down this stress to one of 0.3 N/m^2, it then becomes a mean for the JONSDAP period: mid-March to mid-April 1976, therefore leading to a Trench flow of $(1.61/5) \times 10^6 = 0.32 \times 10^6$ m^3/s. This is smaller than the average value of 10^6 m^3/s deduced from the JONSDAP observations by Furnes and Saelen (1977).

The foregoing underestimation of the flow through the Norwegian Trench by the present model may be largely explained by the failure of the model to take account of a major inflow to the North Sea between the Orkneys and Shetlands. Davies (1979) has shown that the inclusion of this flow, in a more realistic nonlinear modelling of the average wind-induced circulation of the North Sea for the JONSDAP period, does certainly produce a flow through the Trench of approximately the same value as that derived from the observations. The simplicity of the present model, particularly with respect to its bathymetry, coastal configuration and frictional properties, means that it cannot be used

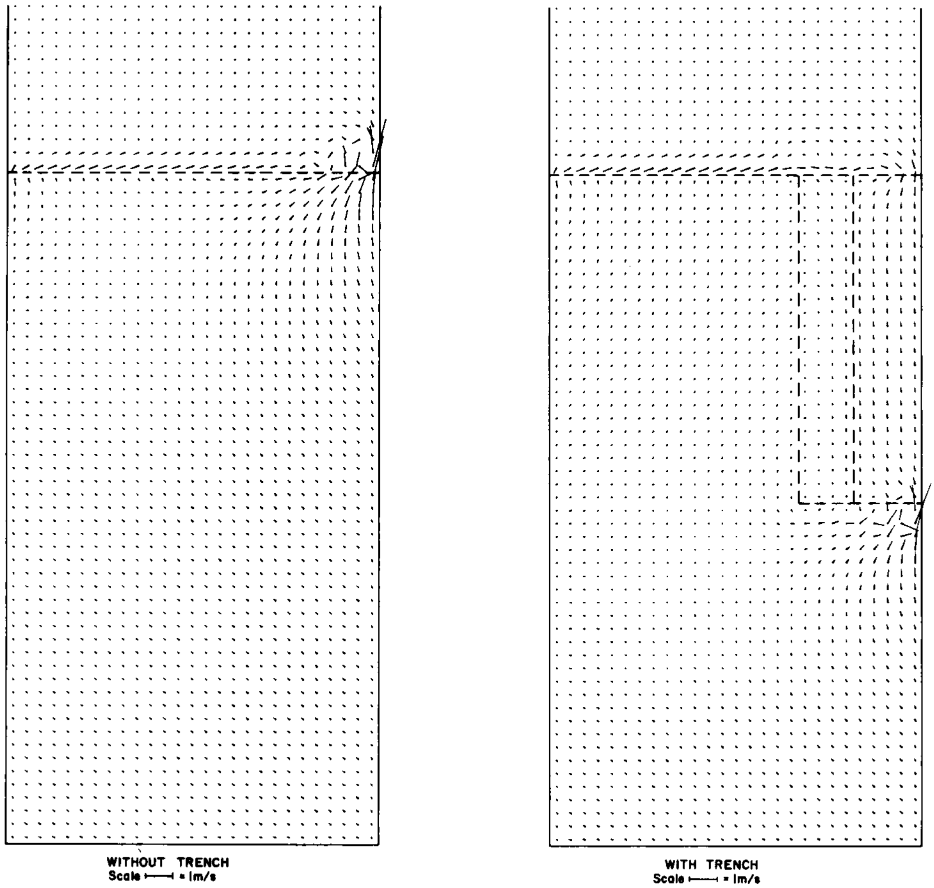

Fig. 9. Bottom currents due to the westerly wind stress field, without and with a Norwegian Trench.

to reproduce real conditions. Heaps (1978) has already indicated that a linearization of friction in the northern North Sea is probably unjustified. But the main point of the model is that it identifies the Norwegian Trench as a major influence determining the circulation characteristics of the northern North Sea, especially so in the formation of the Norwegian Coastal Current.

5. Concluding remarks

(i) An idealized model of the wind-driven three-dimensional circulation of the North Sea has indicated that the Norwegian Trench channels flow along its length and thereby has an important influence on the system of currents in the whole northern area of the North Sea basin.

(ii) A north-going flow through the Trench driven by sea-surface gradient, compensated by a south-going flow in the neighbouring North Sea area where wind is the main driving agent, are features demonstrated by the model (with north and west winds) which appear to have counterparts in reality, namely: the Norwegian Coastal Current and the inflow current along the western edge of the Trench (Dooley, 1974).

(iii) The model predicts local gyres at the southern end of the Trench which may reflect real circulations in the Skagerrak and Kattegat (Davies, 1979).

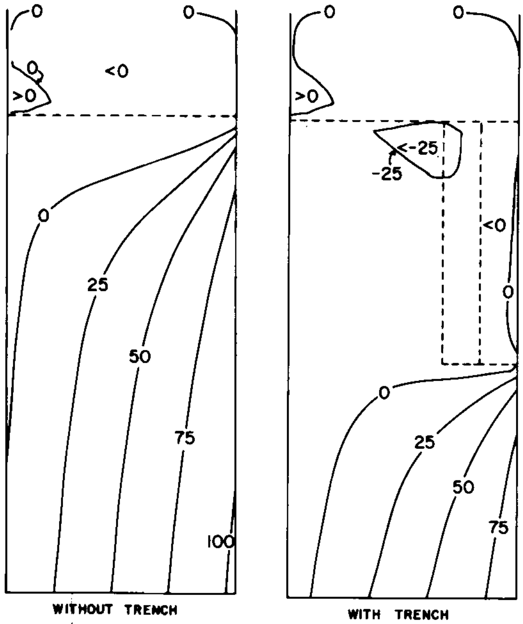

Fig. 10. Contours of sea surface elevation (cm) due to the westerly wind stress field, without and with a Norwegian Trench.

(iv) The model is too simple to reproduce all the features of North Sea wind-driven circulation. This is not its purpose. Rather, the approach is more of an analytic one which seeks basic reasons for specific features of the circulation.

(v) Furnes (1979), using a simple analytic model of wind-induced circulation in the North Sea, has shown that uniform north–south and west–east gradients of depth influence the flow along the Norwegian coast in the same sense as in Fig. 11 of the present paper.

6. Acknowledgements

The authors are grateful to Mr R. A. Smith for help in preparing the diagrams.

The work described in this paper was funded by a Consortium consisting of the Natural Environment Research Council, the Ministry of Agriculture, Fisheries and Food, and the Departments of Industry and Energy.

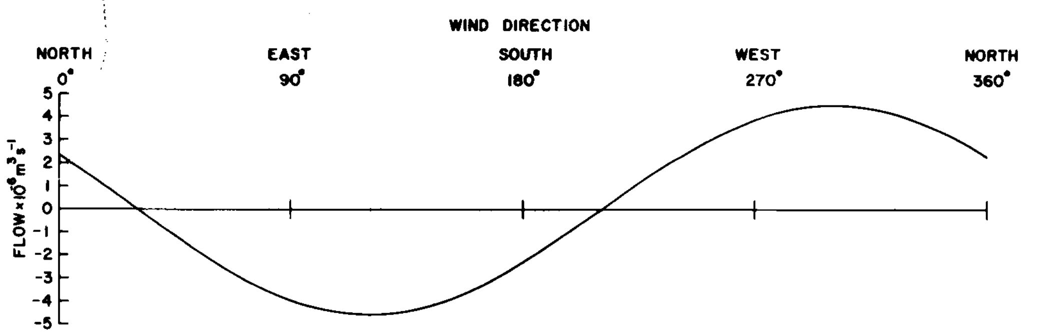

Fig. 11. Flow through the Norwegian Trench (including the slope region), positive to the north and negative to the south, as it varies with wind direction in degrees clockwise from the north. The results correspond to a wind stress of 1.5 N/m^2 over the North Sea rectangle.

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ВЛИЯНИЕ НОРВЕЖСКОГО ЖЕЛОБА НА ВЕТРОВУЮ ЦИРКУЛЯЦИЮ В СЕВЕРНОМ МОРЕ

Для исследования влияния Норвежского желоба на ветровую циркуляцию в Северном море используется простая трехмерная численная модель этого бассейна. Показано, что присутствие желоба может приводить к тому, что основной поток проходит через него, причем он вызывается градиентом на поверхности моря, вызываемом ветром. Этот поток влияет на циркуляцию всей северной части Северного моря. Отсюда также можно получить и вероятный механизм генерации Норвежского прибрежного течения.