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Comments on "Rotating hydraulics in deep-water channel flow" by Lars Rydberg

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The author refers to a rich selection of papers dealing with rotating hydraulics and selects the paper by Whitehead, Leetma and Knox (WLK) (1974) for a detailed comparison with his own results. I am perplexed as to why two theories based on the same laws of mechanics should give such radically different results, particularly in the matter of cross stream velocity distributions. On the strength of Rydberg's experimental evidence, I am prepared to put my money on his version, but I confess to remaining somewhat skeptical and more than a little confused.

The WLK theory predicts the flow in a long, horizontal frictionless channel downstream from a deep quiescent basin. The word long is used in the sense that the predicted flow is steady and uniform in the direction of flow; the downstream control section is distant from the transition zone. The cross-stream structure (velocity, height of interface above the channel floor) is predicted from the cross-stream equation of motion (geostrophic balance) and the conservation of potential vorticity along streamlines. The Bernoulli equation furnishes a relation between the upstream interface height, and the downstream flow. Finally, by analogy with non-rotating hydraulics, the maximum discharge through the canal is sought (optimization in terms of the remaining free parameter). The results reduce to the familiar broad-crested weir solutions for non-rotating hydraulics when f is set to zero. The final step (optimization) is the debatable point in this theory; the structure of the flow and the relation between upstream and channel conditions are consistent with steady, frictionless, flow in a rotating frame.

Rydberg's point of departure is an examination of conditions at the control point in the flow, where conditions are far from uniform in the direction of flow (interface must slope along the channel axis). It is clear that the condition of locally critical flow is sufficient to ensure that the flow is independent of downstream conditions. I know that one can tackle this problem using an initial value approach and the theory of characteristics in the non-rotating case, and the same ought to apply to the rotating case, since the Coriolis terms in the momentum equations do not involve partial derivatives of the dependent variables. The question of where in the channel the flow becomes exactly critical is raised; there is no reason to assume that the curve along which $v^2 = gh$ is perpendicular to the main axis of the channel. That is certainly not the case in the WLK model. On the other hand, it is possible to show that steady frictionless non-rotating flow on a smooth crested weir becomes critical at the summit (Long, 1954) and with a given parameterization of the friction, the singular points of the integral curves of the equation for interface slope along the channel can be located; these are control points for the flow.

For example, the equations of motion and continuity for steady flow in a channel of cross-section $A(z, y)$ may be written

$$u\frac{du}{dy} + g\frac{dz}{dy} = F$$

$$\frac{d}{dy}(Au) = 0$$

where u is the mean flow velocity along the axis, F is the friction force per unit mass, z is the height of the free surface (interface) above level reference surface.

$$\frac{dA}{dy} = \left(\frac{\partial A}{\partial y}\right)_z + \left(\frac{\partial A}{\partial z}\right)_y \frac{dz}{dy}$$

$$= \left(\frac{\partial A}{\partial y}\right)_z + b\frac{dz}{dy}$$

where b is the channel width at level z and $(\partial A/\partial y)_z$ accounts for the changes in channel section with distance, z held constant. These equations may be recast into a single equation for the slope of the interface

$$\frac{dz}{dy} = \frac{\left(\frac{F}{g} + \frac{\left(\frac{\partial A}{\partial y}\right)_z u^2}{gA}\right)}{1 - \frac{u^2}{g(A/b)}} = \frac{\Phi_n}{\Phi_c}$$

$\Phi_n = 0$ defines a "normal" curve where the interface is horizontal—friction effects balance effects of channel slope or changing shape. $\Phi_c = 0$ defines a "critical" curve where the local Froude number is equal to 1. Given the discharge $Au = Q$, the channel topography, and the friction parameterization $F = F(Q, A)$, the normal and critical curves may be drawn (Fig. 1). The curves intersect in a singular point where $dz/dy = 0/0$ is indeterminate. Expansion of the solution about this point reveals whether there is a smooth integral curve of z (y) through the point.

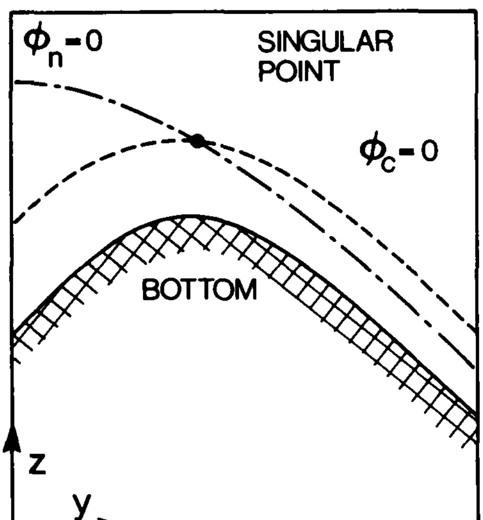

Fig. 1. Location of singular point at the intersection of normal and critical curves.

In the case of flow over a weir without friction, there is always a singular point at the crest of the weir (where $(\partial A/\partial y)_z = 0$). There is only one integral curve which passes through the singular point connecting the upstream subcritical flow with the downstream supercritical flow. The introduction of friction tends to shift the singular point downstream. All of this can be found in Ven Te Chow's text on open channel hydraulics (Ven Te Chow, 1959).

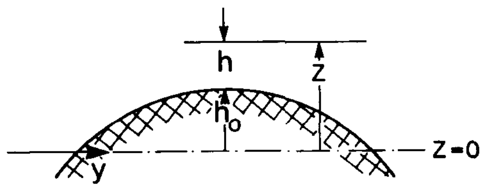

Fig. 2. Definition sketch of crest of a weir.

For a channel of unit width (Fig. 2),

$$F = \frac{-ku^2}{h}, \quad \left(\frac{\partial A}{\partial y}\right)_z = \frac{-dh_0}{dy}, \quad A = h$$

The solution for the singular point is

$$\left(-\frac{ku}{gh} - \frac{dh_0}{dy}\frac{u^2}{gh}\right) = 0; \quad \frac{u^2}{gh} = 1$$

or

$$\frac{dh_0}{dy} = -k$$

For the simple case, the control point is shifted downstream to where the bottom slope is equal to the non-dimensional friction factor or surface drag coefficient.

Provided that the geometry of the channel is equivalent to that of a weir with a well-defined summit and provided that the frictional effects are not too large, Rydberg's assumption of a locally critical flow cross-section at the summit may be acceptable as a working hypothesis.

I am distinctly uneasy however, when Rydberg draws cross-channel profiles of the Bernoulli integral (Fig. 5 in Rydberg's text). If the theory is reasonably well posed this term should not vary greatly across the channel; all the stream tubes must terminate on an essentially level upstream interface. Rydberg sweeps this under the carpet of frictional effects and makes the upstream connection with only one stream tube where the Bernoulli integral is a maximum. The problem is the price to be paid for fixing on the section where the flow is critical.

The assumption that the cross-channel momentum equation reduces to a simple geostrophic balance could be challenged, particularly at the left side of the channel. In the interests of a simple approximate theory, I cannot see a tractable alternative. I am almost certain that the defect in the Bernoulli integral stems from the exclusion of cross-channel momentum and that something along the line of Sambuco and Whitehead's (1976) analysis would have to be introduced to patch things up. Of course you could argue that the stream tube which lies along the right-hand wall contains no cross-flow terms, and thus forms the link with the upstream conditions. The present theory with the mid-channel maximum of the Bernoulli integral makes this a doubtful procedure at the moment. Why is $(V^2/2) + g^1(h_0 + h))$ a maximum in mid-channel? The term for the rectangular channel is maximum at the right-hand side wall. Maybe the problem is with the boundaries. Suppose that the right-hand wall were vertical at the point of intersection with the interface.

We could force the Bernoulli term $(V^2/2) + g^1(h_0 + h)$ to be a maximum on this wall. Setting $V^2 = gh$ or using $fv = g^1(d/dx)(h_0 + h)$, we obtain

$$\frac{d}{dx}\left(\frac{V^2}{2} + g(h_0 + h)\right) = \frac{d}{dx}\left(\tfrac{3}{2}V^2 + h_0\right) = 0$$

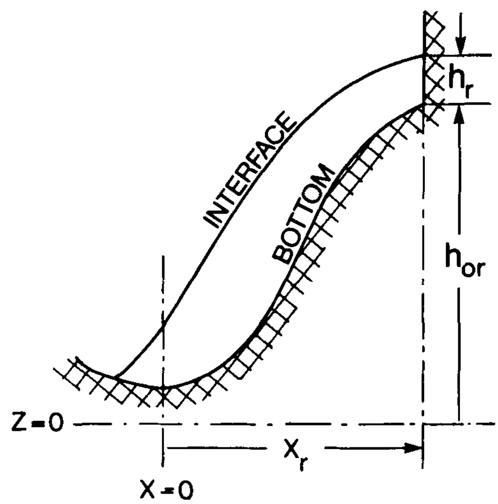

Fig. 3. Definition sketch of flow cross-section with vertical right-hand boundary.

which with the eq. (7) in Rydberg's paper, leads to

$$\left.\begin{aligned} V &= \frac{1}{3}\frac{g^1}{f}\frac{dh_0}{dx} \\ h &= \frac{1}{9}\frac{g^1}{f^2}\left(\frac{dh_0{}^2}{dx}\right) \end{aligned}\right\} \text{at } x = x_r$$

These relationships would now form the starting point for the integration of Rydberg's eq. (7) back towards the origin.

This stratagem might tidy things up in an artificial way but it does not deal with the basic problem. Perhaps it is simply too much to expect to construct a simple model of this flow.

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