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Rotating hydraulics in deep-water channel flow

By LARS RYDBERG, *Department of Oceanography, University of Gothenburg, P.O. Box 4038, S-40040 Gothenburg, Sweden*

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ABSTRACT

Rotating deep-water channel flow is considered. The concept of critical flow is redefined for "wide" channels (with depth variations across the channel) where strong variations of the interface height occur along the flow. Numerical solutions are obtained for the flow structure across the channel, and a criterion for the interior interface height upstream the channel as a function of prescribed flow is presented. The solutions are compared to hydrographic observations from the Bornholm Strait (within the Baltic) and the Faroe Bank Channel with encouraging results. The present theory allows for frictional effects within the channel, and the results differ qualitatively from earlier contributions to this subject.

1. Introduction

Since the thermal wind equation was first derived, a great many different boundary conditions have been used to determine the velocity distribution from density observations. Even in the very simple two-layer case with a deep and quiescent upper layer and a given volume flow in the lower layer, the interface topography is not given by geostrophy alone, but we must add some other condition to determine this topography. In this paper, which deals with geostrophically balanced deep-water channel flow between oceanic basins, we shall assume this flow to be locally critical, a condition which thus "freezes" the interface height relative to an arbitrary bottom profile. The condition provides a perfect blocking of downstream "information", implying that the paper in this respect is a contribution to the subject rotating hydraulics.

This subject, as it may be recovered in deep-water flow between oceanic basins connected by a channel, has only recently been theoretically treated in the literature. One interesting contribution with empasis on laboratory experiments was presented by Whitehead, Leetma and Knox (WLK) (1974). They introduced a predictive theory for the interface height in the upstream basin, h_u (two-layer fluid with a density difference $\Delta\rho$) as a function of prescribed flow, q_0 rotation rate, f reduced gravity $g' = g(\Delta\rho/\rho)$ and form of the channel (a short rectangular channel of width b).

For increasing rotation rate, the theory predicts the upstream height ot increase also, until it finally becomes independent of the width, i.e. $h_u = h_u(q_0, f, g')$. Theory and experiments agree well, although the independence of width does not show up in their experiments, possibly due to frictional effects.

As WLK assumed the upstream basin to be infinitely deep, the form of the channel became less important. The assumption was made to obtain zero velocity in the upstream basin. Later, Sambuco and Whitehead (1976) in another laboratory experiment invoked a "form factor", l, for the channel (sill), which depended on its length and height above the channel floor. For sufficiently low rotation, the upstream height, h_u is again found to be a function of q_0, f, g' and l.

In a recent paper by Gill (1977) a more sophisticated theory is presented allowing for the upstream surface height (Gill uses a one-layer model) to vary locally. Gill thus includes the form of the upstream basin as well as the form of the channel—although it is still rectangular—and this implies boundary currents in the upstream basin in addition to an upstream height. Gill also shows how the position of an hydraulic control may be shifted due to variations in flow rate.

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The theoretical framework by WLK is based on cross-stream geostrophic balance and conservation of potential vorticity (frictionless flow). To determine the cross-stream variations of the interface height, $h(x)$ in the channel (i.e. at the sill), WLK uses a flow maximation principle known from non-rotating hydraulics. The principle results in velocities at the sill which increase from the right, rotation-lagging part of the flow (where the interface height, h is great) towards left (where the interface height becomes small), looking downstream. This feature, which appears also in the paper by Gill—who uses the same assumptions—does not seem very realistic (compare Fig 2). Unfortunately, no velocity observations were reported in the paper by WLK, but from field observations (e.g. Walin and Petrén, 1976; Rydberg, 1978) we easily conclude that high velocities coincide with a thicker flowing layer. The frictionless theory should break down when h becomes small, and a bottom layer dominated by friction will presumably develop.

A theory which, on the other hand, includes frictional effects in a typical overflow situation, is the one by Smith (1975). His model considers the effects of entrainment (and friction) downstream from a presumably critical section, and does not explicitly discuss parameters such as upstream height. We believe, however, that his model could be used also with the upstream height as an "initial value" instead of a given initial deep-water flow (compare e.g. Taylor, Turner and Morton (1956) on buoyant plumes where they may give as an initial value either an initial byouancy flow (c.f. upstream height) or the sum of byouancy and momentum flow in a later stage (c.f. Smith's model, with given geostrophic flow and density difference)).

Another frictional theory for rotating two-layer channel flow is presented in a recent report by Lundberg (1977). He uses the length-wise pressure gradient and a parameterization of the friction to obtain estimates of the cross-stream interface slope and the flow through the channel. His numerical model—like the one by Smith—thus falls within the regime where the internal Froude no, $F_i = (v/c_i)^2$ is generally less than one.

In this paper, however, we shall confine ourselves to two-layer channel flow which is hydraulically controlled, and where the flow is thus independent of downstream conditions. The hypothesis of locally critical flow, $v(x) = c_i(x)$ is applied to a section (or a region), where the depth, $h_0(x)$ varies locally. The hypothesis compensates for the maximation principle used by WLK and Gill, but implies—at least if the interface in the upstream basin is not strongly tilted—that friction must be locally important upstream from the critical section where $F_i = 1$. We need and shall not bother in detail on how this friction takes place, and shall not worry about its eventual effects on the density distribution upstream from this section. We just note that an evaluation of the upstream interface height, $h_u(x, y)$ implies that we need an equation, different from the frictionless, potential vorticity equation used by WLK and Gill. This link will be discussed at length in the theoretical part, Section 2. Numerical solutions are obtained for velocity and interface height within the channel, and for the maximum upstream height, which are compared with observations from the deep-water flow to the Baltic and the flow through the Faroe Bank Channel (Section 3). Section 4, finally, will be devoted to some general comments on rotating channel flow and its relation to the upstream conditions. (The problem treated here is much like non-rotating critical flow, where the depth varies in the cross-stream direction, the great difference being that in such a case, friction is not needed to adjust the flow upstream from the critical section. Such flow was recently touched upon in a paper by Anati et al. (1977), who observed velocities in a short triangular channel in laboratory, and found $F_i = 1$ to hold locally.)

2. Theory

Two large and deep basins on an f-plane are connected by a channel (Figs. 1a–c). The height of this channel, h_0, varies locally but the channel is assumed to be shallow compared to the surrounding basins. Deep water is formed within—or brought into—the upstream basin filling its deeper parts until it eventually flows through the channel at a rate, q_0 equal to the production of deep water. In the downstream basin, the deep water is either mixed up (and heated) with surface water, which recirculate to the upstream basin, or flows on to another basin further downstream. The density of the surface water in the upstream basin is ρ_0, that of the deep water $\rho_0 + \Delta\rho$, the flowing system assumed to be two-layered with a step-like interface. The depth of the deepwater within the

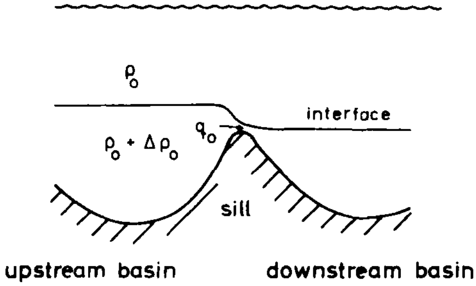
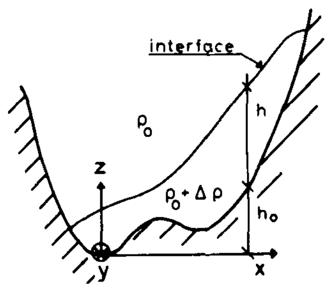
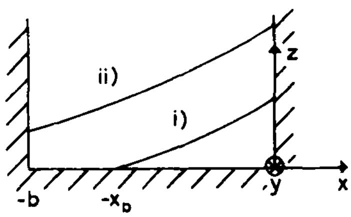


Fig 1a



(looking downstream)

Fig.1b



(looking downstream)

Fig. 1c

Fig. 1a–c. Idealized two-layer channel flow, showing the various quantities introduced in Section 2, Fig. 1b, with arbitrary bottom profile in the cross-section, Fig. 1c, with a rectangular cross-section.

channel, h, is assumed to be small compared to the surface water depth. The interface level downstream from the channel is situated below the sill depth (where the sill depth presumably coincides with the shallowest part of the channel, but not necessarily). The deep interface in the downstream basin may be referred to as "over-mixing", but it may also be due to other factors, e.g. the connections with basins further downstream. The channel is assumed to be relatively short, a condition which is usually not crucial.

We began this section by assuming large and deep basins without quantifying this scaling. In general, however, we think of a basin where the width below the interface is many times the internal Rossby radius of deformation (Ro_i), and the depth away from the boundaries (where the interface hits the bottom) is reasonably larger than the expected local variations in interface height, which may be caused by the flow through the basin. This implies low velocities in the interior compared with those in the channel, but also implies that we can accept locally high velocities along the boundaries (width $\approx Ro_i$) of the basin, where the interface height above the bottom is not large. A discussion of the flow in the upstream basin will appear in section 4.

The question now is whether it is possible to establish a relation between the upstream conditions—presumably an interior upstream height, $h_u = h_u(q_0, f, \Delta\rho, h_0)$—and the flow of deepwater through the channel, which is independent of downstream conditions. For this reason, let us assume that a critical section (or region) exists within the channel, which effectively prevents downstream information from progressing upstream. At least if the channel (i.e. the width of the flowing layer) is wide, and the thickness of the flowing layer, h, is moderate or small, it seems reasonable to assume locally critical conditions at the critical section, implying that the velocity $v(x)$, is everywhere equal to the internal phase speed, $c_i(x) = \sqrt{[g(\Delta\rho/\rho)\cdot h]}$. We shall furthermore assume the flow to be in geostrophic balance, and the equations governing the motion at the critical section are thus,

(i) geostrophic balance

$$fv = g' \frac{\partial}{\partial x}(h + h_0) \tag{1}$$

where $g' = g(\Delta\rho/\rho)$, thus assuming constant density upstream the critical section. This implies that we neglect entrainment caused by friction at the interface, although we expect friction to occur.

(ii) locally critical flow

$$v^2 = g' h \tag{2}$$

(iii) the flow of deepwater, q_0 given by

$$q_0 = \int_x vh\, dx \tag{3}$$

Explicit solutions for v and h are easily found for the case of a rectangular cross-section, whereas an arbitrary form for the critical section leads to a non-linear equation, which may be solved numerically. For comparison with earlier theories let us study the rectangular section first.

2.1. *Rectangular cross-section* ($h_0 = 0$, width, b)

Equation (1) may now be written as

$$fv = g' \frac{\partial h}{\partial x} \tag{1'}$$

which, combined with eq. (2) integrates to

$$\left.\begin{aligned} v &= \frac{fx}{2} + C \\ h &= \frac{1}{g'}\left(\frac{fx}{2} + C\right)^2 \end{aligned}\right\} \tag{4}$$

The integration constant, C, may be determined from eq. (3), but we see from Fig 1c, that there are two possible cases, one where the interface hits the bottom, and the other when it reaches the left side of the channel. The cases may be seen as corresponding to wide and narrow channels, respectively. The simplest one is found when the interface intersects the bottom at $x = -x_b$, and where $h, v = 0$. In this case $C = \frac{fx_b}{2}$ and the eq. (4) may be rewritten as

$$\left.\begin{aligned} v &= \frac{f}{2}(x + x_b) \\ h &= \frac{f^2}{4g'}(x + x_b)^2 \end{aligned}\right\} \tag{5}$$

These expressions may be inserted in eq. (3), which is integrated according to

$$q_0 = \int_{-x_b}^{0} \frac{f^3}{8g'}(x + x_b)^3\, dx = \frac{f^3 x_b^4}{32g'}$$

and the value of x_b is thus found as

$$x_b = 2\left(\frac{2g' q_0}{f^3}\right)^{1/4} \tag{6}$$

The results for v, h and x_b are immediately comparable with those obtained by WLK. As we see from Fig. 2, there is a great qualitative difference between the results. A comparative discussion will appear in Section 4. WLK calculated the results also for a narrow channel, i.e. in our case for a channel where $b < 2(2g'q_0/f^3)^{1/4}$. Determination of v and h for this case is straightforward, but involves the solution of a third-degree equation, thus giving rather complicated expressions. As the real interest of this paper is directed towards studies of constrictions with variable depths, these expressions are not written here.

2.2. *Arbitrary bottom profile* (see Fig. 1b)

From eqs. (1) and (2) we find

$$fv = \frac{\partial}{\partial x} v^2 + g' \frac{\partial h_0}{\partial x} \tag{7}$$

This equation may be solved numerically by a Runge-Kutta method. A solution is shown in Fig 3 for a cross-section where $h_0 \propto x^4$. Different start points $x = -x_b$, v, $h = 0$ have been chosen and q_0 is calculated from eq. (3) afterwards. We note that the flow is very sensitive to the choice of starting point and that this point must be situated so that the slope $\partial h_0/\partial x$ is less than zero. The observations in the Bornholm Strait (Walin and Petrén, 1976) show in general also a strong tendency for the halocline to hit the bottom on the left side in a very concentrated zone (Fig. 7a–c).

We further realize that if we choose our start point to far left, the inclination, $\partial h_0/\partial x$, at the right side of the channel will not be large enough and the interface will never hit the bottom. In such a case the outflow (and the upstream height) becomes dominated by frictional effects. The examples from nature given in Section 3 will hopefully clarify the effects of variable stratification and various bottom profiles.

2.3. *The upstream height*

Two problems remain unsolved so far, first, how to find where in the channel the critical section is situated. This is simple if a narrow and steep-sloped (compared to the interface slope) cross-section exists within the channel, otherwise it will be necessary to evaluate eq. (7) for different cross-sections.

The other problem concerns the upstream height, h_u, above the deepest point at the critical section. In non-rotating hydraulics there is a simple link between the channel flow and the upstream height through the Bernoulli equation and the upstream height is approximately uniform. In rotating hydraulics, on the other hand, we expect the upstream height, h_u, to be a function of position in the upstream basin. When the basin is deep and large, however, i.e. the flow within the basin is weak, it seems reasonable to assume that the upstream height does not vary strong locally, at least not in the interior, and that a spatial mean value of h_u in the interior of the basin is a useful parameter, which may be directly related to the outflow of deep water. In the introduction, we mentioned that friction must be locally important upstream from the constriction, and at this end it is probably worthwhile to recall the assumptions made by WLK and Gill (1977). WLK prescribes a uniform upstream height with a deep lower layer to ensure zero velocity and potential vorticity in the upstream basin. The cross-stream geostrophic balance at the constriction, in combination with the potential vorticity equation $((\partial v/\partial x) + f = 0)$ give rise to high velocities where the interface height is small and vice versa (compare Fig. 2) whereas the Bernoulli equation (which is not independent of the potential vorticity equation) may be seen as the link between the channel flow and the uniform upstream height. Also in the theory by Gill, although he prescribes a finite upstream height and non-zero potential vorticity, the same characteristic features of the flow normally—but not necessarily—occur at the constriction. With a uniform upstream interface (WLK) it is obvious that a general frictionless link between constriction and upstream basin is not useful for the case presented here, and this

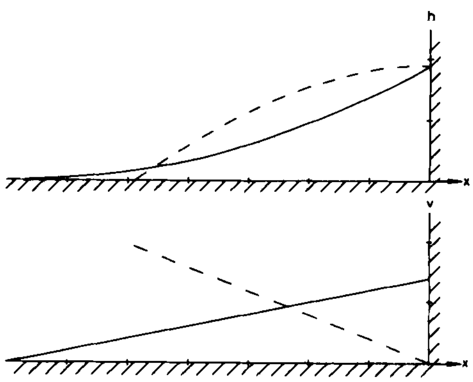


Fig. 2. Relative interface height, $h(x)$ and velocity, $v(x)$ in a wide rectangular channel. Comparison between results obtained with the present theory (full curves) and the frictionless theory (dashed curves) by Whitehead et al. (1974), with equal choice of flow, rotation rate and stratification. (Whitehead, $\partial v/\partial x = -f$; present theory, $\partial v/\partial x = f/2$.)

is normally true even if the interface in the upstream basin is moderately tilted and the velocites there are moderately small (Gill) compared to those at the constriction. We thus conclude that friction must be important upstream from the constriction, and that the Bernoulli link cannot hold in general.

This friction may be generated at the bottom and/or at the interface but we expect the friction to be more important when the thickness of the flowing layer becomes small and/or the velocity becomes high. A combination of a deep flowing layer with moderate or low velocities should, on the other hand, give rise to minimum frictional influence. On these grounds we may assume that one single particle path determines the upstream height through the condition

$$h_u = g'^{-1}\left|\frac{v^2}{2} + g'(h_0 + h)\right|_{\max} \tag{8}$$

which means that the particle path (stream-line) passing through the critical section at the point x, where $((v^2/2) + g'(h_0 + h))$ is maximum, is thus frictionless and obeys the Bernoulli equation. Along other particle paths there is more or less loss of potential energy due to friction upstream of the constriction. Note, however, that for these particle paths the upstream height may of course be lower.

The link between the locally critical flow at the constriction and the conditions in the upstream basin is undoubtedly a debatable point. This paper does not give the whole solution of this object. The assumption above just implies that the stream-line passing the constriction with the greatest "head" needs the upstream height, h_u (compare also Section 4), and—as pointed out by an excellent reviewer—this is the price we pay for simple approximate and hopefully useful theory. At best, it thus offers us the possibility of calculating the channel flow from one suitably chosen point in the upstream basin, where h_u is observed once the channel topography and the stratification are known.

For the rectangular cross-section, the upstream height is then determined by the stream-line on the right-hand side of the channel, where both v and h are maximized. At this wall (compare Fig 1c), using eqs. (5) and (6) we thus find the interface height

$$h(0) = h_r = \left(\frac{2f}{g'}q_0\right)^{1/2} \tag{9}$$

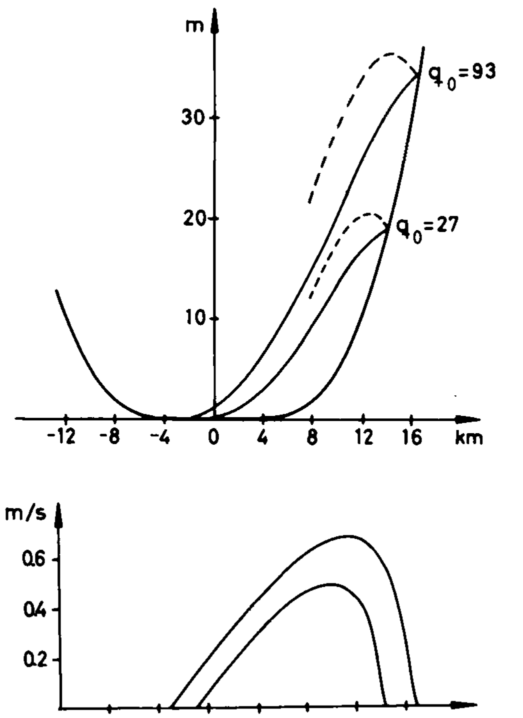


Fig. 3. Solution of eqs. (3) and (7) for interface height (upper figure) and velocity (lower figure) in a channel where $h_0 = 5 \cdot 10^{-16} \times x^4$, $(\Delta\rho/\rho) = 3 \cdot 10^{-3}$ and $f = 1.2 \cdot 10^{-4}\ \mathrm{s}^{-1}$. The flow, q_0 is given in $10^3\ \mathrm{m}^3\ \mathrm{s}^{-1}$. Dashed curves correspond to the solution of eq. (8), their maximum indicating the upstream height.

and from eqs. (2) and (8) we find the upstream height

$$h_u = \left(\frac{9}{2}\frac{f}{g'}q_0\right)^{1/2} \tag{10}$$

which, in terms of h_r, may be expressed as $h_u = \frac{3}{2}h_r$. This result is well known from similar problems of estuarine circulation in wide and deep fjords with short constrictions. As mentioned before, a rectangular cross-section is not realistic in nature. In any real channel the velocity will become zero where the interface hits the bottom on the right-hand side, which implies $h_r \approx h_u$—approximately, because in a critical section with variable depth, $h_0(x)$, eq. (8) may be fulfilled by a stream-line away from the point where the interface intersects the bottom (this may be the case also in laboratory experiments with rectangular channels, as the velocity near the right wall will be greatly reduced due to friction, and h_u is then given by a stream-line away from the right wall).

In Fig. 3 we have drawn the curve $((v^2/2) + g'(h + h_0))$. We see that the upstream height differs only little from the interface height on the right-hand side, and this is normally the case also for natural channels. Thus, in natural conditions and with the certain restrictions concerning the channel form discussed above, the relation between upstream height and deep-water flow is given implicitly by the eqs. (3), (7) and (8).

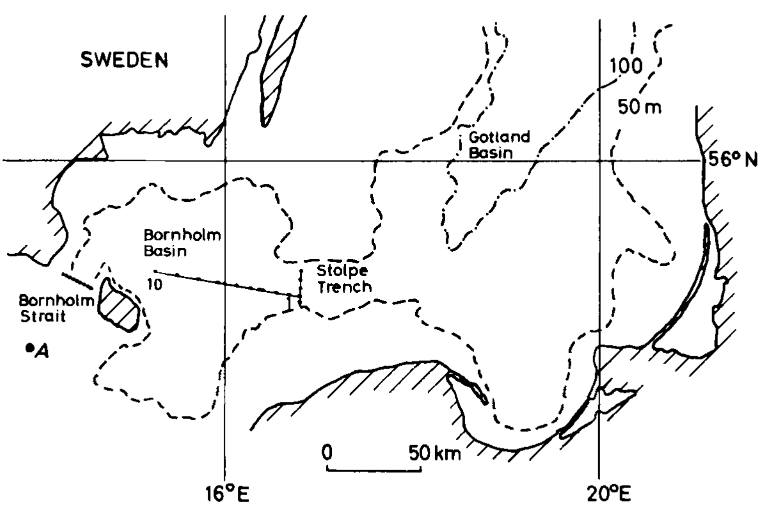


Fig. 4. Map over the Southern Baltic, indicating cross-sections at the Bornholm Strait and the Stolpe Channel.

3. Observational evidence

Observational evidence of hydraulically controlled flow are found within the deep-water flow to the Baltic (see Fig. 5). This deep water flows, after the passage of the Danish sounds, through a series of basins and narrow channels, the Arkona Basin, the Bornholm Strait, the Bornholm Basin, the Stolpe Trench and the Gotland Basin. A map of the Baltic is shown in Fig 4. Simultaneous observations of velocity and salinity with narrow spacing have been performed by our institute at two cross-sections, one in the Bornholm Strait and one in the Stolpe Trench. Some of these observations and estimates of mean flow have been reported by Walin and Petrén (1976) and by Rydberg (1978). Many of the individual observations performed reveal features typical for critical flow.

These hydrographic observations make it possible to compare observed velocities and interface heights at the cross-sections with calculations of these quantities based on eqs. (3) and (7). Numerical solutions of these equations, obtained for various stratifications and flows typical for the Bornholm Strait, are first shown in Figs. 6a, b. Further, comparisons between observed and calculated velocities and interface heights, respectively, are shown for some different situations (Fig. 7a–c). The calculations are based on the observed flow, $q_0 = 21{,}500\ \mathrm{m^3\ s^{-1}}$, $13{,}000\ \mathrm{m^3\ s^{-1}}$ and $6000\ \mathrm{m^3\ s^{-1}}$, respectively, and an observed stratification $(\Delta\rho/\rho) = 4 \cdot 10^{-3}$. Figs. 7a–c show that the calculated interface height is found below the observed mid-pycnocline depth ($S = 12‰$) and that the difference is more pronounced for lower flow rates. Presumably, this discrepancy is due to frictional boundary layers near the bottom and the halocline, which may have a notable influence, more important for thinner flowing layers. (According to observations by Lundberg (1977) the thickness of the bottom boundary layer is of the order of 2 m, thus violating the present theory for weak flows.) Accordingly, the velocities calculated are higher than those observed, but not far above, and it is easily seen that high velocities coincide with deep flowing layers, as prescribed by the theory. Remarkable flows are rarely found in the left channel, which is possibly due to upstream depth conditions.

We shall further make use of the observed mean flow and the mean stratification within the channel to calculate the mean upstream height from eqs. (3), (7) and (8). The upstream height may then be compared to data based on salinity observations performed at stn A (Fig. 4) performed by The Fishery Board of Sweden (Hydrographic data

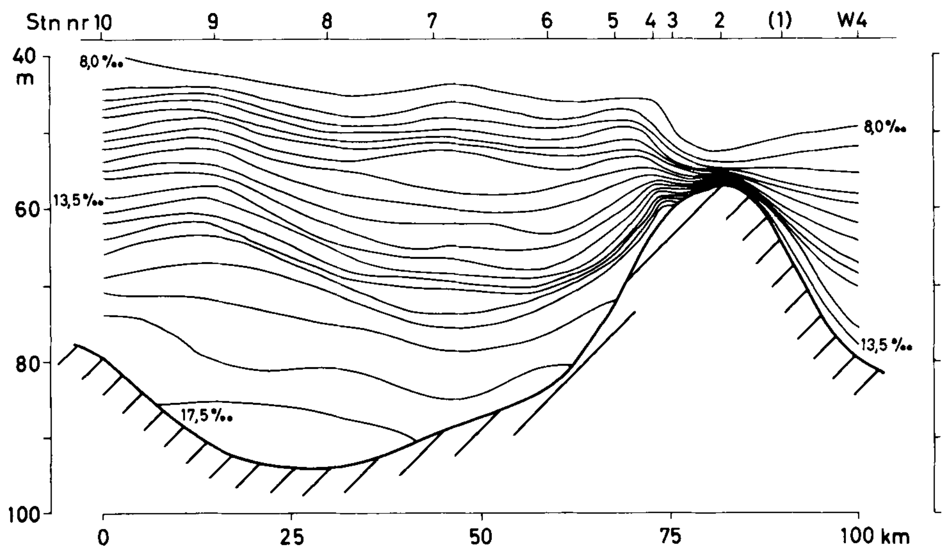


Fig. 5. A length-section mapping salinity within the Bornholm Basin (compare Fig. 4). Indications of hydraulically controlled flow are strong (from Lundberg and Rydberg, 1979).

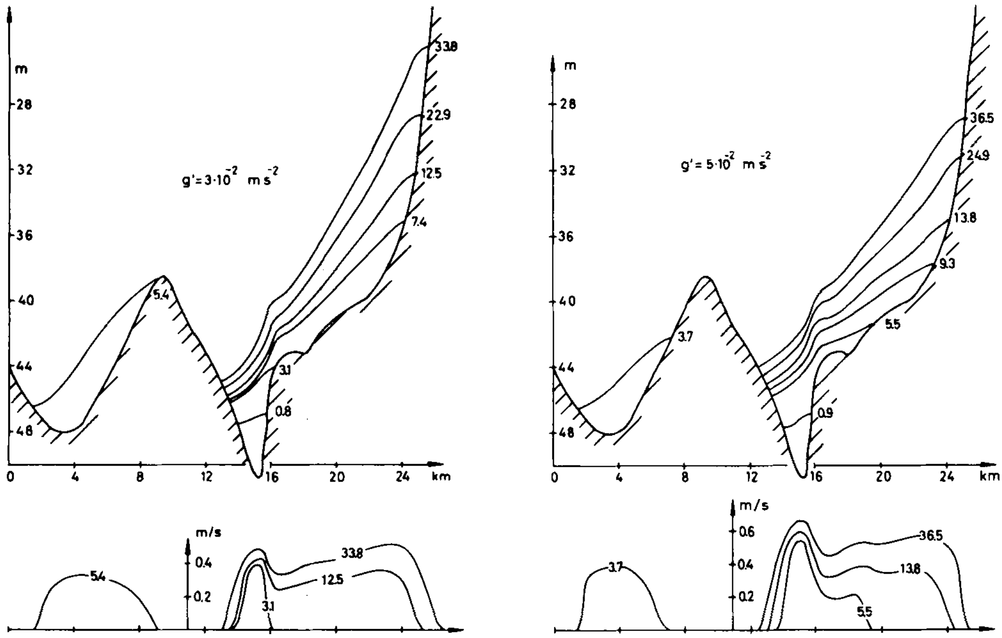


Fig. 6a, b. Solution of eqs. (3) and (7) for the cross-section in the Bornholm Strait, showing interface height and velocity. The solutions are evaluated for different flow rates (expressed in 10^3 m^3 s^{-1}) and variable stratification.

reports (1963–76)). These observations—made approximately four times a year—can be used to calculate mean depth of isohalines, which gives us a reasonable estimate of the upstream height. For the Bornholm Strait the observed mean flow (1973–75) below the 12‰-isohaline is $q_0 = 9000$ m^3 s^{-1} and the mean stratification roughly $(\Delta\rho/\rho) = 4 \cdot 10^{-3}$. The calculated *depth* of the interface where it hits the bottom of the right-hand side becomes $d_r = 35$ m, approximately equal to the upstream depth according to eq. (8). The mean depth (1963–76) of the 12‰-isohaline at stn A is found to be equal to 37 m, whereas the mean depth (1973–75) of the same isohaline where it hits the bottom at the cross-section in the Bornholm Strait is equal to 35 m. The flow in the Bornholm Strait, however, varies strongly—sometimes it may be negative—and there are several occasions when the flow seems to be frictionally balanced, probably due to high downstream levels (compare Lundberg, 1977). These facts make it of course dubious to compare mean quantities. Simultaneous observations of flow and upstream height would have been preferable, and such observations will be performed.

The Faroe Bank Channel—linking the Atlantic with the Norwegian Sea (NS)—through which there is a continuous outflow of cold NS water to the Atlantic, possesses another region where we expect hydraulically controlled flow to occur. In this channel, frictionally controlled boundary layers are presumably less important, as the flows involved are orders of magnitude greater ($q_0 \sim 10^6$ m^3 s^{-1}) than in the Bornholm Strait and the thickness of the flowing layer of the order 100 m. Some cross-sections within and upstream of the channel mapping salinity and temperature are available from Tait (1967), Fig. 8. We may thus, through eqs. (3), (7) and (8), calculate the instantaneous flow, q_0, through the channel by using observations from either section IV (within the channel) or section I (upstream).

It is of course a challenging task to compare the results. If we assume that the cross-section number IV represents the critical section and estimate the density difference between surface and deep water as $(\Delta\rho/\rho) = 5 \cdot 10^{-4}$ (NS water, $S = 34.95$‰, $T = 0°$; Atlantic water $S = 35.3$‰, $T = 8°$) we may easily calculate interface height and velocity at this cross-section for different flows, q_0. The results are shown in Fig. 9. The velocities are of order $v \approx 0.8$ m s^{-1}, the deep-water thickness $h \approx 100$–150 m. Neither the velocity nor the deep-water thickness is

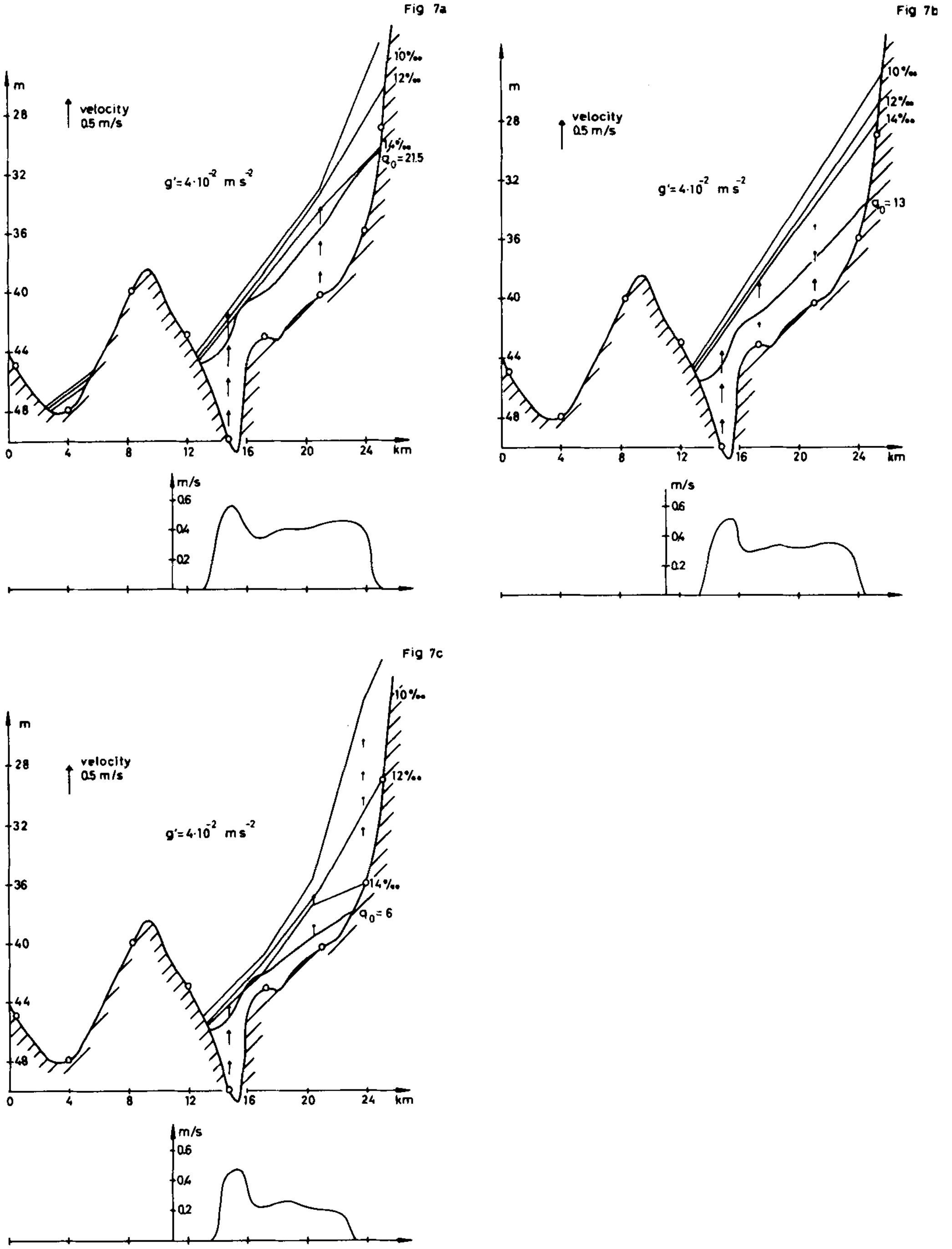


Fig. 7a–c. Calculated interface height (thick curve, upper figure) and velocity (lower figure) compared with observed salinity (isohalines, upper figure) and velocity (arrows, upper figure) for the Bornholm Strait. The observations are performed in the verticals above (O). Calculations according to eqs. (3) and (7) are based on the observed stratification, $(\Delta\rho/\rho) = 4 \cdot 10^{-3}$, and the total flow estimated from the current measurements. Three occasions are shown with different flow, $q_0 = 21{,}500$ m^3 s^{-1}, 13,000 m^3 s^{-1} and 6000 m^3 s^{-1}.

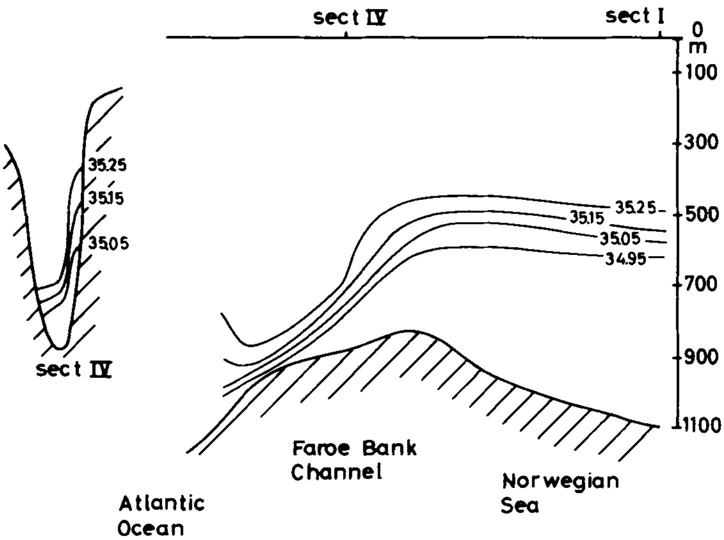


Fig. 8. An example of salinity structure along and across the Faroe Bank Channel according to Tait (1967). The situations of cross-sections I and IV are indicated in the figure.

very sensitive to variations in the flow, but the depth of the interface, on the other hand, is very sensitive—both for variations in flow and in stratification.

We may further calculate the mean depth of the mid-pycnocline ($S = 35.12‰$), from the three observations at section IV presented by Tait, as a reasonable estimate of the mean interface depth of a two-layer model. This interface (see Fig. 10) intersects the bottom on the right side of the channel at $d_r = 520$ m, which according to Fig. 9 implies a deep-water flow of $q_0 = 1.2 \cdot 10^6$ m^3 s^{-1}. The theoretically calculated interface corresponding to that flow is drawn in Fig. 10. We see that the coincidence is almost perfect on the right side of the channel. On the left side there is a reasonable difference. Two reasons may be concerned, (i) friction is important near the bottom and (ii) the control section may be situated further upstream where the depth is 50 m less. No observations were made in that cross-section, however.

Concerning the upstream level, we find, according to eq. (8), that a volume flow of $1.2 \cdot 10^6$ m^3 s^{-1} implies an upstream interface depth, $d_u = 510$ m—insignificantly above d_r (compare the dashed-dotted curve on Fig. 10). The mean depth of the uppermost part of the mid-pycnocline ($S = 35.12‰$)—corresponding to an estimate of an interior d_u—may be calculated from the observations at cross-section I reported by Tait. This is found to be approximately 500 m, i.e. almost exactly equal to d_u. A spatial (and time) mean value of the depth of the mid-pycnocline at section I, on the other hand, is equal to 535 m, thus somewhat below the calculated value of $d_u = 510$ m.

The above calculations are performed without it being possible to compare them to the actual flow, and indirect velocity calculations on the basis of the thermal wind equation will, of course, not be of any help.

Direct current observations within the channel are sparse, but Crease (1973) has reported some measurements with a Swallow float near section IV (see (⊕), Fig. 10). He found a maximum velocity of 109 cm s^{-1} at a depth of 760 m. The float drifted downstream gradually loosing its high velocity. It was left in the left part of the channel with a speed of 26 cm s^{-1}. This indicates that friction downstream and on the left bank, implying weaker density gradients and smaller interface slopes.

The examples given above show that the very simple assumptions made are capable of describing the channel flow and its cross-sectional variations. The relation between the interior upstream height and the flow through the channel, given through a combination of eqs. (3), (7) and (8) is at least not contradicted by observations, but concerning this part of the study, we suffer from adequate

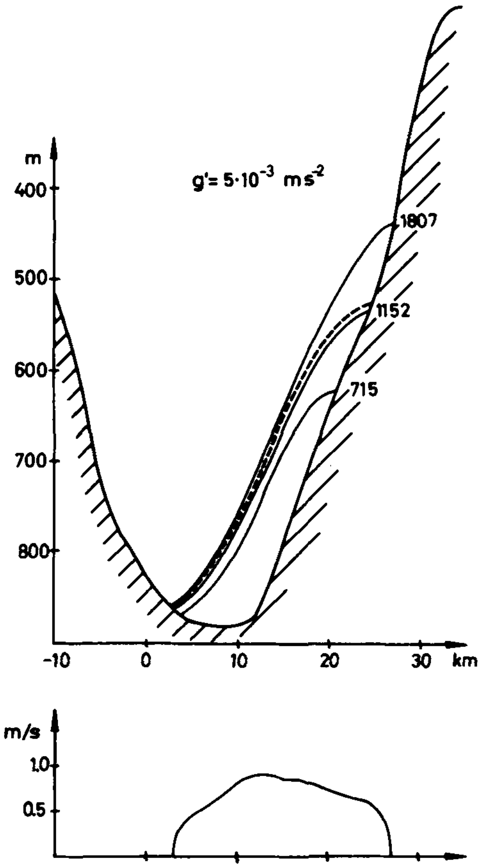


Fig. 9. Interface depth at cross-section IV in the Faroe Bank Channel for different flows, q_0 (expressed in 10^3 m^3 s^{-1}), where $(\Delta\rho/\rho) = 5 \cdot 10^{-4}$. The dashed curve indicates the mean flow, $q_0 = 1.2 \cdot 10^6$ m^3 s^{-1}, calculated from eqs. (3) and (7) on the basis of three hydrographic sections (salinity and temperature observations) by Tait (1967). The lower curve indicates the velocity for the flow, $q_0 = 1.2 \cdot 10^6$ m^3 s^{-1}.

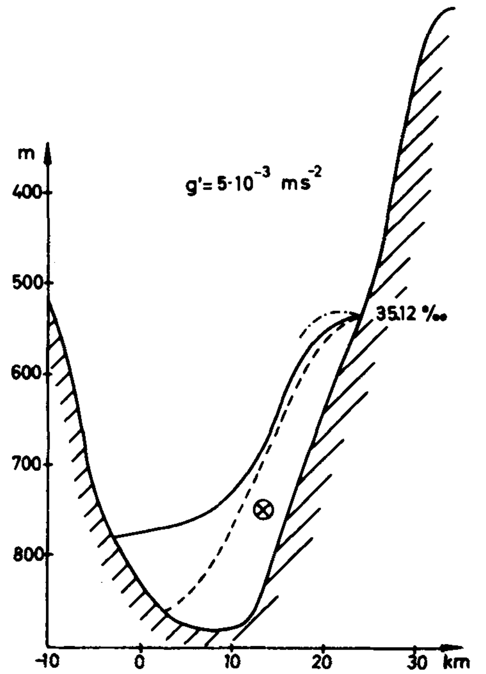


Fig. 10. A comparison between mean depth of the isohaline $S = 35.12‰$ (approximate mid-pycnocline) from the observations by Tait (1967), and the calculated interface depth for $q_0 = 1.2 \cdot 10^6$ m^3 s^{-1}. Maximum upstream height is found from the dashed-dotted curve—according to eq. (8).

simultaneous observations. For both parts, however, continuous salinity and temperature profiles have been transformed for use in a two-layer description. This is not satisfactory. A natural development should aim at a description where we make use of depth of isopycnals as upstream condition, and calculate the flow q_0 as a function of density (or $\Delta\rho$) on the basis of a Richardson number analogy within the channel. At least for the deep-water flow to the Baltic, data for $q_0(\rho)$ are available, presented on the basis of a theory by Walin (1977).

4. Discussion

We now ask ourselves, where and when does locally critical flow occur? In the beginning of Section 2 we tried to introduce the general features of a fluid system where criticial flow is expected. The most powerful reason for assuming the flow to be locally critical was that this assumption was the only one which could satisfy the demand that the downstream conditions should not directly influence the flow through the channel, but that this flow was related to the upstream conditions only. Of course, this is not generally true. A coupling between upstream and downstream conditions must exist, but it may take place mainly through the surface waters. In some way, both flow and density structure are given by the mixing intensity, e.g. the flow of deep water to the Baltic is determined by the overall mixing within the Baltic (and freshwater supply). Locally, however, this mixing may be looked upon as incidental and the flow (or the upstream conditions) may be taken as prescribed. A critical flow in a channel is thus expected to coincide with steep interface slopes along the flow axis—steep compared to those observed in the interior of the basin—and of the same order as those perpendicular to the flow. If, on the other hand, the interface slopes are small, the flow is probably dominated by friction, thereby allowing for downstream influence.

Potential energy losses due to friction upstream from the critical section must be considerable, but taking the whole upstream basin into consideration, it seems quite reasonable that adjustment for the channel flow can occur. This means that we do not expect the upstream basin to obtain the upstream height in general, but rather that we allow for a circulation in the upstream basin, presumably clockwise and bounded around its edges with a width of order the internal Rossby radius of deformation (if driven by a deep water source in the center of the basin). The clockwise flow, quasi-geostrophic in nature (with little friction), will prevent the spill of deep water over the main part (left side, looking downstream) of a wide-crested sill, while the interface in the interior may be found on a level well above the sill depth. The observations upstream from the Faroe Bank Channel indicate this situation. The circulation in the Bornholm Basin is at present being studied (Lundberg and Rydberg, 1979) on this subject.

An important question at this point is, of course, how well does the present theory cope with observations? The qualitative difference between the results obtained here and earlier results—mainly those by WLK—has been referred to before. Expressions for velocity and interface height for equal q_0 and $(\Delta\rho/\rho)$ at the sill for a wide rectangular channel were compared in Fig. 2. The velocity structure is totally different in the two cases, and of course the inclination of the interface becomes also different. As is seen, the WLK theory gives a less widespread flow than does the present theory, but this feature is not easily revealed in nature. The height of the interface on the right wall, on the other hand, is exactly the same for both theories, which is more of a coincidence. This means that a calculation of the flow, q_0, based on the interface height on the right wall, h_r, will give identical results, namely

$$q_0 = \frac{g' h_r^2}{2f}$$

while the upstream height will of course differ with a factor 3/2 (as $h_u = h_r$ in WLK, and $h_u = \frac{3}{2}h_r$ in the present theory). Even for very small deviations from a vertical wall, however, the present theory also gives $h_u \approx h_r$ (compare eq. (8)), and it will be problematic to compare the validity of the theories on the basis of upstream levels. Velocity observations remain however, and from those obtained in the Bornholm Strait we may at least say that they do not contradict the present theory. The highest velocities coincide with the thickest flowing layer and vice versa. But for the results and the discussion above we may note once more that the bottom profile within the channel plays a dominant role in this theory, and especially the right wall inclination. If this inclination is weak, again: friction must dominate the outflow.

Observations of currents and salinities within the Bornholm Basin, the Bornholm Strait and the Stolpe Trench have been a progressive work at our institute since 1973. The main purpose concerns ecological modelling of the Baltic (see Walin, 1977). Upstream conditions, circulation in the upstream basin and critical flow are subjects which are continuously being observed.

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ГИДРАВЛИКА ВРАЩАЮЩЕГОСЯ ГЛУБОКОВОДНОГО КАНАЛА

Рассматривается течение воды в глубоком вращающемся канале. Для "широких" каналов (с изменением глубины поперек канала), где вдоль потока встречаются сильные вариации высоты раздела, переопределяется концепция критического потока. Получены численные решения для структуры потока поперек канала и дается критерий для определения внутренней высоты раздела вверх по потоку в канале как функции заданного течения. Решения сравниваются с гидродинамическими наблюдениями в Борнхольмском проливе в Балтийском море и в проливе фэрои Бэнк. Результаты сравнения обнадеживающие. Данная теория учитывает эффекты трения в канале и результаты качественно отличаются от предыдущих исследований по этому вопросу.