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On the solar modulation of the atmospheric lunar tide

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ABSTRACT

The lunar semidiurnal term $L_2(p)$ has been investigated in detail on both a local and planetary basis and was found to be sensitive to the local diffusive character of the lowest layers of the atmosphere. Accordingly it is possible to explain the solar modulation of the M_2 pressure tide represented by the lunisolar diurnal term $L_1(p)$ and the seasonal behaviour of $L_2(p)$ as well as the variation of the phase angle λ_1 with altitude.

1. Introduction

In 1918, Chapman made the first valid determination of the lunar semidiurnal variation in surface pressure at a nontropical station. Since then several similar determinations of the lunar semidiurnal barometric tide have been made so that its global distribution and its annual variation are well described (Haurwitz and Cowley, 1969). Much improved models of the lunar semidiurnal atmospheric tide have also been developed. For instance, Sawada (1956) calculated the lunar semidiurnal atmospheric tide for various height-dependent temperature structures and Geller (1970) explored the effect of annually changing temperature structures on the lunar semidiurnal tide; Hollingsworth (1971) studied the effects of the lunar semidiurnal ocean tide on the atmosphere and Palumbo (1979) provided evidence for such an effect and Lindzen and Hong (1974) calculated the lunar semidiurnal tide in the presence of meridianal temperature gradients and zonal winds.

It is only recently, however, that a daily modulation of the lunar semidiurnal tide has been investigated. Since the response of the atmosphere to the tidal potential might vary according to solar time (due to the dependence of thermal, dynamical and chemical atmospheric conditions on solar time) it seems likely that the semidiurnal term in surface pressure data $L_2(p)$ also varies with solar time. Palumbo (1975) investigated this effect for pressure data for several stations and found a large and significant modulation with the night-time amplitude typically half as great as the daytime amplitude. Moreover, he found that $L_2(p)$ was dependent on local meteorological conditions; specifically, after the elimination of the convexity effect (Chapman and Lindzen, 1970) the amplitude was smaller when the mean 24-hour pressure was greater than 1000 mb (anticyclonic conditions) than when it was lower than 970 mb (cyclonic conditions). Here, the phenomenon is investigated in further detail with particular reference to its dependence on the diffusive character of the atmosphere. A mechanism is proposed that may also account for the observed seasonal modulation in $L_2(p)$.

2. Observation results for Naples

The use of mean atmospheric pressure, as a stability criterion, for the classification of the days represents only a first approximation. A more suitable measure of the diffusive character of the atmosphere is provided by the bulk Richardson number (Pasquill, 1974) which may be deduced from radiosonde observations via the formula:

$$Ri_B = \frac{g}{T}\frac{\Delta\theta z}{u^2(z)}$$

Here, g = acceleration of gravity (ms^{-2}) T = temperature (K) θ = potential temperature (K) z =

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Table 1. *The $L_2(p)$ tide and the Richardson number from Naples data divided according to day and night, season and mean pressure $\bar{p}$*

Season	Mean pressure (mb)	$l_2(p)$ (μb)	Richardson number
All day	1010.7	24.5 ± 3.1	0.2
All night	1010.8	16.5 ± 3.1	0.8
D months (Nov to Feb)	1011.2	20.6 ± 3.5	0.6
J months (May to Aug)	1010.5	27.6 ± 3.2	0.3
All	<970	26.4 ± 5.5	0.3
All	970–1000	20.2 ± 9.6	0.5
All	>1000	15.1 ± 4.7	0.7

altitude (m), taken as 2500 m. u = velocity of the wind (ms^{-1}). The value of Ri_B increases with atmospheric stability.

Naples is conveniently close to the mean position of four radiosonde stations in Italy (Rome, Brindisi, Cagliari and Messina) therefore a mean value of Ri_B deduced from these stations represents a mean local stability parameter for comparison with $L_2(p)$ data for Naples. As one could expect the largest variation of Ri_B with synoptic meteorological conditions has been found in the lowest layers. The investigation was therefore confined to the first 2500 meters roughly corresponding to the depth of the boundary layer during marked daytime convective conditions in Naples. Table 1 gives mean values of Ri_B for this layer for midday and midnight (GMT). The data were then subdivided into approximately equal groups according to Lloyd's three seasons: (D = Nov, Dec, Jan, Feb; E = Mar, Apr, Sept, Oct; J = May, June, July, Aug) and moreover, into three approximately equal groups according to the daily mean pressure $\bar{p}$:

(i) $\bar{p} < 970$ mb (prevailing cyclonic conditions)
(ii) $\bar{p} > 1000$ mb (prevailing anticyclonic conditions)
(iii) intermediate pressure (associated to no clear weather patterns).

Bi-hourly values of surface atmospheric pressure recorded at Naples (40° 50′N, 14° 15′E) during the same period as the radiosonde data (1950 to 1970) have been divided similarly into groups according to an objective subdivision of Ri_B. Each group of data was analysed by the Chapman–Miller method (Malin and Chapman, 1970) in order to determine the amplitude $l_2(p)$ of the semidiurnal component and its probable vector error, p.e. The results are presented in Table 1 together with corresponding values for Ri_B. As expected, the night-time values for Ri_B are much greater than the daytime values. This implies that the atmosphere is almost always more stable during the night than during the day regardless of prevailing meteorological conditions.

The last three lines of Table 1 show that Ri_B increases with $\bar{p}$, so that the lowest 2.5 km of the atmosphere is more stable in a bulk sense in prevailing anticyclonic conditions. This implies that the mean pressure $\bar{p}$ may be considered a rough stability criterion for classification purposes.

3. Observation results on planetary base

Since the global distribution of bulk Richardson number Ri_B is unavailable, it is impossible to correlate Δl_2 (the difference between $l_2(p)$ observed and computed by models) to Ri_B. However, we may correlate Δl_2 with $\bar{p}$, based on the approximate correspondence we have demonstrated, using World Weather Records (1959). Haurwitz and Cowley (1969), in order to obtain a simple numerical measure of how well their planetary model fits the observed $L_2(p)$, have computed an "error" of the harmonic representation as follows: let a_2 and b_2 be, respectively, $l_2 \sin \lambda_2$ and $l_2 \cos \lambda_2$ as deduced from a network of 264 planetary grid points based on determinations of $L_2(p)$, while a_2^* and b_2^* are the corresponding values computed using the main wave ($s = 2$) with Hough and/or Legendre functions. Then they defined the error "E" by

$$E = \{1/264[(a_2 - a_2^*)^2 + (b_2 - b_2^*)^2]\}^{1/2}$$

The 264 values of a_2, b_2, a_2^* and b_2^* are given in Fig. 2 and Fig. 4 of Haurwitz and Cowley (1969).

Table 2. *Mean pressure $\bar{p}$ in mb, $\Delta l_2(p)$ in μb, error E in μb for mean latitudinal belts in degrees for both hemispheres*

Latitudinal belts (deg)	Mean pressure (mb)	$\Delta l_2(p)$ (μb)	E (μb)	$\frac{E}{l_2(p)}$
0°–10°	1011.4	4.5	13.1	0.56
10°–20°	1012.0	5.6	15.5	0.36
20°–30°	1013.2	4.2	11.2	0.40
30°–40°	1014.6	0.3	9.6	0.26
40°–50°	1014.5	2.6	6.2	0.29
50°–60°	1013.6	3.6	7.6	0.14

The mean values of $\bar{p}$, Δl_2 and E are reported in Table 2 for 10° mean latitudinal belts, for both hemispheres. The Haurwitz and Cowley values have been analysed here in further detail. Their computed data appear to be underestimated since almost all individual observed values exceed the computed ones. Moreover, it can be seen from Table 2 that the ratio of E to $l_2(p)$ computed decreases as the latitude increases. This cannot be attributed to the quality of the observed data since, on the contrary, in the lowest latitude $L_2(p)$ is determined more accurately and there is a very satisfactory network of stations all over the world. These results imply that there is a global effect on $L_2(p)$ which is not represented within the model.

To search for a possible effect of the mean pressure $\bar{p}$ and thus of stability on $l_2(p)$ the correlation between the individual Δl_2 and $\bar{p}$ was computed and found to be −0.85, which coincides with the correlation between the E and $\bar{p}$. Both the correlation coefficients between Δl_2 and $\bar{p}$ and E and $\bar{p}$ of Table 2 are found to be close to −0.82.

The mean value of E in all seasons was found to be about 10 μb (i.e. the same order of the contribution of stability to $l_2(p)$) as evaluated from the data of Naples (Table 1) and other individual stations (Table 3). The data of Table 2 indicate that the difference $l_2$obs − $l_2$comp decreases as $\bar{p}$ increases or that $l_2$obs is higher than $l_2$comp in prevailing unstable conditions. This would support the hypothesis of a planetary effect of $\bar{p}$ on $l_2(p)$ as found for Naples.

If the influence of $\bar{p}$ on $L_2(p)$ can be definitively proved, it would allow corrections to the observed data to be made, leading to a better fit between planetary models and the observed global distribution of $L_2(p)$.

4. Observation for other individual stations

The results found by Palumbo (1975) on a semidiurnal pressure tide $L_2(p)$ for Zürich, Naples and Catania indicate a greater amplitude for $L_2(p)$ during daytime than during night-time. Here, we present another method of analysis demonstrating the same result by analysing data on the diurnal lunisolar variation in surface pressure data $L_1(p)$. In order to see the relationship between $L_1(p)$ and the day–night variation in $L_2(p)$, we shall consider

Table 3. *Lunisolar pressure tide amplitude in μb, phase angle in degrees*

Station	Altitude (m)	l_1 (μb)	λ_1 (degrees)	Method
Hong Kong	33	13.2 ± 3.5	280°	Chapman–Miller
Naples	40	11.7 ± 1.1	274°	Chapman–Miller
Catania	60	23.3 ± 5.3	255°	day–night
Zürich	410	9.1 ± 3.9	265°	day–night
Addis Ababa	2443	10.5 ± 3.6	100°	Chapman–Miller
Mt Fuji (Summer)	3773	25.9 ± 11.9	43°	Chapman–Miller

a hypothetical tidal variation where only L_1 and L_2 are present, i.e.

$$P = l_1 \sin (t + \lambda_1 - 2\nu) + l_2 \sin (2t + \lambda_2 - 2\nu)$$

where t is the solar time and ν is the lunar phase angle, both measured in degrees. ν changes by no more than about 6° both during the day and during the night and it can thus be regarded as approximately constant during any half day. The mean daytime P_D and night-time P_N value of P may be derived as follows:

$$P_D = \frac{1}{\pi} \int_{\pi/2}^{3/2\,\pi} P dt = -\frac{1}{\pi} l_1 \sin (\lambda_1 - 2\nu)$$

and

$$P_N = \frac{1}{\pi} \int_{-\pi/2}^{\pi/2} P dt = \frac{2}{\pi} l_1 \sin(\lambda_1 - 2\nu)$$

implying that:

$$l_1 \sin (\lambda_1 - 2\nu) = \frac{\pi}{4} (P_N - P_D)$$

so that $P_N - P_D$ which can be estimated more accurately than the absolute value of either P_D or P_N provides a measure of the L_1 term.

$L_1(p)$ may thus be determined by finding separate day and night values of $L_2(p)$ or by the classical Chapman–Miller analysis. Although the latter is a much more refined method of analysis and provides the significance level of determination, the former method can be used for stations where the series of data is not sufficiently long to utilize the latter. Palumbo and Mazzarella (1979) have recently improved the results of the Chapman–Miller method by introducing filtering techniques.

Table 3 gives values of $L_1(p)$ determined for several stations. The first method of analysis was used for the Zürich and Catania data and the Chapman–Miller method was used for Hong Kong, Addis Ababa, Naples and for Mt Fuji.

P_N and P_D are derived from half the entire data set. The vector probable errors for P_N and P_D are therefore assumed to be $\sqrt{2}$ times the p.e. of $L_2(p)$, derived from the whole data series using the Chapman–Miller method.

Although this method does not rigorously determine the p.e.'s, it has been checked against such a rigorous determination using the Naples data set when the p.e.'s were virtually identical.

Note that all the $L_1(p)$ determinations appear to be statistically significant in the sense that in all cases p.e. $< l_1/3$. Another indication of the reality of these determinations is the relative constancy of the phase angles of $L_1(p)$ for the first four stations. There is an indication that λ_1 decreases rapidly with the altitude of the station.

Putting the mean phase angle $\lambda_1 = 269°$ for the first four lower stations of Table 3 into the lunisolar diurnal term, we find that the enhancement of the diurnal curve of

$$L = L_1 + L_2 = l_1 \sin (t + 269° - 2\nu) + l_2 \sin (2t + 74° - 2\nu)$$

takes place during daylight hours (Fig. 2, Palumbo 1975).

The rapid decrease in λ_1 with increasing station altitude, as shown in Table 3, might be related to the opposite sense of the variation of Ri_B from night to day at these mountain stations compared with that at the other low-level stations.

5. Explanation of seasonal effect

It is well known that the diffusive character of the lowest layers of the atmosphere is generally higher in summer time. Thus for Naples Ri_B is about twice as large in winter as in summer. The seasonal variation of $l_2(p)$ at Naples is opposite to that of Ri_B (see Table 1). The difference between $l_2(p)$ in summer and in winter is 7 μb. The same holds for the seasonal planetary values obtained by Haurwitz and Cowley (1969) where the difference between the summer and winter values of the harmonic constants for the main wave ($s = 2$) of $L_2(p)$ was found to be 10 μb (for the normalized associated Legendre function) and 11.6 μb (for Hough functions). Thus planetary data for the Northern Hemisphere provide the same results as found from Naples and allow the seasonal variation of $l_2(p)$ to be related to the annual cycle of the diffusive character of the lowest layers of the atmosphere. The same does not appear to hold for the Southern Hemisphere since the majority of the stations are coastal where the surface layers are more frequently neutrally stratified, and the seasonal cycle of Ri_B is likely to be very small.

6. Discussion

It appears that $l_2(p)$ is sensitive to variations in the diffusive character of the lowest layers of the atmosphere and therefore to local climatic conditions. The local contribution to $l_2(p)$ (significantly evaluated on both a local and planetary basis) is represented by the $L_1(p)$ term which is the solar modulation of $L_2(p)$. It is surprising to note the great attention paid to L_2 and the lack paid to L_1 since there is no very large difference in their amplitudes. The close relationship between L_1 and the variable local weather conditions makes its determination very difficult and this is the major reason for its relative neglect. On a worldwide basis these difficulties are exaggerated because of different climatic conditions among the stations. A world-wide pattern of this solar modulation on L_2 could only be detected from a spherical harmonic analysis of L_1, but these terms are unfortunately not available for stations other than those few examined here. The refinements of the methods of analysis provided by Schlapp (1978), Palumbo and Mazzarella (1979), which enable the evaluation of lunar terms from series of data much shorter than required in earlier studies, will allow the filling of this gap in lunar studies. L_1 might be responsible for the wide scattering in individual determinations of l_2 because of the sensitivity of $L_2(p)$ to local bulk stability. Because of this it appears that L_2 does not sufficiently reflect the entire lunar oscillation to provide a useful means for its determination.

Finally, it is suggested that an investigation of L_1 should be made prior to any analysis on L_2 of the effect of the ocean because of the latter's well-known effect on local microclimate. As mentioned previously, there are no existing theoretical studies relevant to the modulation of L_2 by either solar time or by local tropospheric conditions. Geller (1970) has looked at changes in L_2 resulting from global changes in the static stability profile with altitude using a one-dimensional model. Tarpley and Basley (1972) have previously suggested a solar modulation of L_2 to explain their observations on lunar daily variations in electron drift within the E-Region at Jicamarca. Geller discounted this suggestion of Tarpley and Basley. Lindzen and Hong (1974) have modelled the lunar semidiurnal tide using a two-dimensional model in which the zonal wind and temperature vary with both altitude and latitude. If one wished to develop a theory which models the observations presented here, it would have to be very complex and of a new generation.

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О СОЛНЕЧНОЙ МОДУЛЯЦИИ АТМОС ФЕРНОГО ЛУННОГО ПРИЛИВА

Детально локально и в планетарном масштабе исследуется лунный полусуточный прилив $L_2(p)$. Найдено, что он чувствителен к локальному диффузионному характеру нижележащих атмосферных слоев. В соответствии с этим результатом объясняются солнечная модуляция прилива M_2 по давлению, представляемая лунносолнечным членом $L_1(p)$ и сезонное поведение $L_2(p)$, также как и вариация с высотой фазового угла λ_1.