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# Horizontal divergence, acceleration and curvature change, a diagnostic equation with applications

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ABSTRACT

The horizontal divergence of an accelerating flow is discussed in the contexts of meandering currents and the formation of eddy-rings. The purpose is to help discern areas of upwelling and other dynamical features. Two interrelated hypotheses, representing two meso-scale versions of the turning vorticity equation (Chew, 1975), are formulated with the help of the Florida Current data of Schmitz (1969). The first is a generalization of the pioneer work of Bjerknes (1937) on the relation between horizontal divergence and curvature vorticity change. Along a meandering flow in the northern hemisphere, and away from the equator, the hypothesis predicts horizontal convergence where there is a rapid increase in centripetal acceleration, and horizontal divergence where the increase is in centrifugal force. In particular, in a meandering surface flow with a succession of cyclonic and anticyclonic turns, the hypothesis predicts a series of alternating upwelling and downwelling tongues.

The second hypothesis illuminates a study by Fuglister (1972) of the role of vertical motion in the formation and evolution of eddy-rings. Eddies, as cut-off meanders, are initially irregular in shape. According to this hypothesis, an eddy evolving into a ring with circular symmetry is entering a stage of non-divergence and hence a stage of stability.

## 1. Introduction

Horizontal divergence in a surface flow is of general interest because it often brings nutrients to the euphotic zone, and is a key to the dynamics of the flow. The latter is better seen in meteorology where, e.g., horizontal divergence is a central consideration in weather analysis generally, and a building block in the theory of cyclones in particular. In this paper, the divergence intrinsic to an accelerating flow of inviscid fluid is considered. We seek to relate divergence explicitly to the process of acceleration, particularly that related to curvature change when a current turns.

Being a vector, horizontal acceleration has two components. In natural coordinates, they are the downstream acceleration and that related to curvature change, the centripetal acceleration or its equal and opposite the (apparent) centrifugal force. The relationship between these accelerations and the horizontal divergence is contained in the turning vorticity equation (Chew, 1974, 1975). But we will re-derive the relation more directly, calling it the divergence-acceleration equation.

The diagnostic potential of the divergence-acceleration equation is first illustrated using the observational results of Schmitz (1969). This serves two purposes: one, to show how hitherto overlooked information can be extracted from the observations, and two, to assess the relative magnitude of the different terms in the equation. On the basis of this assessment, and the prevalence of meandering flows we next pose two hypotheses. One, a quasi-steady hypothesis, relates the occurrence of divergence to downstream change in centripetal acceleration, or to curvature change since the sign of the centripetal acceleration depends on the sign of the curvature. This allows a ready determination of where divergence is likely to occur, as well as its order of magnitude, as will be illustrated in the Gulf Stream and a much weaker and smaller undulating frontal jet. The other hypothesis, emphasizing the time-dependent elements, relates divergence to curvature vorticity change in

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eddy-rings. This views eddy-ring formation in the North Atlantic and off East Australia as sharing a common dynamics, where the evolution from a first asymmetric stage to a later one of symmetry corresponds to a transition from a first stage of appreciable divergence to a later one of non-divergence.

## 2. The divergence-acceleration equation

We consider a steady, adiabatic flow with horizontal velocity component $\mathbf{V}$ and vertical component $w\mathbf{k}$, where $\mathbf{k}$ is the vertical unit vector, positive upward. We adopt the Bousinesq approximation that the inertia of the fluid is unaffected by its horizontal density variation. And for notation convenience we write

$$d(\ )/dt = \mathbf{v}\cdot\nabla(\ ) + w\,\partial(\ )/\partial z \tag{1}$$

where $\nabla$ is the horizontal del operator, and $z$ the vertical coordinate. Then the equation of motion for the horizontal component of the flow is

$$d\mathbf{V}/dt = -\mathbf{k}\times f\mathbf{V} - (1/\rho_0)\,\nabla p \tag{2}$$

where $f$ is the local planetary vorticity or the Coriolis parameter; $p$ is pressure; and $\rho_0$ is the fluid density such that $\nabla\rho_0 = 0$.

A natural coordinate system uses the downstream and cross-stream directions of the local velocity as horizontal reference. Writing $V$ for the magnitude of $\mathbf{V}$, we denote the downstream direction by the unit vector $\mathbf{s}$, defined by $\mathbf{s} = \mathbf{V}/V$; and the cross-stream direction by the unit vector $\mathbf{n}$, defined by $\mathbf{n} = \mathbf{s}\times\mathbf{k}$, positive to the right of $\mathbf{V}$. Then writing $K$ for the horizontal curvature, we have

$$d\mathbf{V}/dt = \mathbf{s}\,dV/dt - \mathbf{n}KV^2 \tag{3}$$

where $dV/dt$ is the magnitude of the downstream acceleration, and $KV^2$ the magnitude of the cross-stream or centripetal acceleration. Thus the vertical component of the curl of $d\mathbf{V}/dt$ is

$$\begin{aligned}\mathbf{k}\cdot\nabla\times d\mathbf{V}/dt &= \mathbf{k}\cdot\{\mathbf{s}(\partial/\partial s) + \mathbf{n}(\partial/\partial n)\}\\ &\quad\times\{\mathrm{s}\,dV/dt - \mathrm{n}KV^2\}\\ &= +(\partial/\partial s)\,KV^2 + (\partial/\partial n)\,dV/dt\\ &\quad + Kw\,\partial V/\partial z + KVD\end{aligned} \tag{4}$$

Where $D = \nabla\cdot\mathbf{V}$ is the horizontal divergence, and the last two terms a consequence of the changing orientations of the unit vectors $\mathbf{n}$ and $\mathbf{s}$ (Haltiner and Martin, 1957). Since the vertical component of the curl of the right-hand side of (2) is minus $fD + df/dt$, the divergence-acceleration equation is

$$\begin{aligned}(f+KV)\,D &= -(\partial/\partial s)\,KV^2 - (\partial/\partial n)\,dV/dt\\ &\quad - Kw\,\partial V/\partial z - df/dt\end{aligned} \tag{5}$$

The first term on the right-hand side represents the downstream change in centripetal acceleration, the second the cross-stream change in downstream acceleration and has been called the banking term because in surface flows it is related to the lateral tilting of the sea surface, the third a vertical advection modulated by curvature and the fourth the so-called beta effect. Hence, in general, horizontal divergence is dependent on a centripetal effect, a banking effect, a curvature-vertical advection effect, and a beta effect. In contrast, in terms of the relative vorticity $\zeta$, the divergence is given by

$$(f+\zeta)\,D = -d\zeta/dt - \mathbf{k}\cdot\nabla w\times(\partial/\partial z)\,\mathbf{V} - df/dt \tag{6}$$

(Haltiner and Martin, 1957).

## 3. The Schmitz observation and the evaluation of eq. (5)

Between May 24 and June 24, 1965, Schmitz measured the Florida Current along the two sections shown in Fig. 1, repeating them alternately for 12 transects along each section to achieve a representation of the mean flow. His observational results are reproduced in Table 1 for a one-moving-layer approximation. The variables $u$ and $v$ are respectively the eastward and northward velocity components averaged over the layer depth $H$, defined as the thickness of the surface layer with a vertically averaged sigma-$t$ of 25.0. Plots of $v$ and $H$ are given in Fig. 2 where the abrupt shear zones at the extreme edges of the current are assumed to be frictional boundary layers. Of interest is the open region seaward of these boundary layers and sandwiched between the two sections. Regarding this open region Schmitz wrote that "In the anticyclonic (shear) zone, the flow pattern at the southern transect is wider, deeper, and slower than that at the narrower northern transect. However, the current is decelerating slightly in the cyclonic (shear) zone." Clearly, the Bernoulli acceleration is asymmetric, with the right flank accelerating but the left flank decelerating.

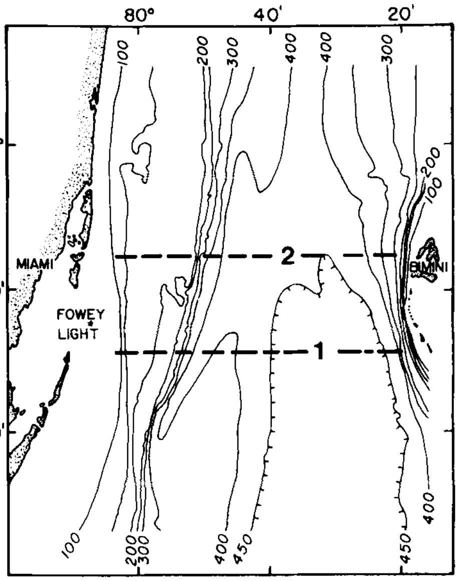


*Fig. 1.* The locations of sections and stations in the Schmitz observation. The north–south distance between the sections is 25 km (depth in fathoms).

There was no measurement of the flow curvature. But while Schmitz assumes the curvature effect to be negligible, we will estimate its importance by using his data to evaluate the other terms in (5), finding the curvature change effect as a residual. In this evaluation, due north ($y$) and east ($x$) are taken, respectively, as the down and cross-stream directions. The small departure is assumed insignificant.

To minimize estimation errors we consider averages for the region. The horizontal divergence $\langle D \rangle$ averaged over the region is obtained by using

$$\langle D \rangle = (B - A)^{-1} \int_A^B D\, dx$$
$$= \{ \int_A^B (\partial v/\partial y)\, dx + (u_B - u_A) \} (B - A)^{-1} \quad (7)$$

where $\partial v/\partial y$ is approximated by the northward speed difference between the two sections divided by 25 km, and $u_A$ and $u_B$ by averaging the corresponding components at $A$ where $x = 10$ km, and at $B$ where $x = 83$ km. This yields

$$\langle D \rangle = 3.6 \times 10^{-6}/\mathrm{s} \quad (8)$$

due principally to the northward acceleration of the flow since

$$(u_B - u_A)/(B - A) = (-1.5\text{–}6.0)\ (\mathrm{cm/s})/(73\ \mathrm{km})$$
$$= -1.0 \times 10^{-6}/\mathrm{s}$$

Table 1. *The Schmitz data for a one-moving-layer approximation. Reproduced from Schmitz (1969)*

| $x$ coord. (km) | $H$ Layer depth (m) | $u$ E/W avg. current comp. (cm/s) | $v$ N/S avg. current comp. (cm/s) | Transport. stream function ($20 \times 10^6$ m³/s) |
|---|---|---|---|---|
| 6 | 20 | 0* | 0* | 0.00* |
| 10 | 53 | 2 | 86 | 0.01 |
| 15 | 77 | 5 | 110 | 0.02 |
| 20 | 100 | 7 | 128 | 0.05 |
| 25 | 121 | 12 | 154 | 0.09 |
| 30 | 147 | 12 | 154 | 0.14 |
| 35 | 173 | 9 | 152 | 0.20 |
| 45 | 208 | 12 | 145 | 0.34 |
| 55 | 246 | 8 | 129 | 0.50 |
| 65 | 273 | 9 | 116 | 0.66 |
| 70 | 282 | 7 | 111 | 0.74 |
| 75 | 288 | 12 | 103 | 0.81 |
| 80 | 295 | 11 | 96 | 0.89 |
| 83 | 299 | 10 | 94 | 0.93 |
| 86 | 20 | 0* | 0* | 0.96 |
| 5 | 20 | 0* | 0* | 0.00* |
| 10 | 77 | 10 | 94 | 0.01 |
| 15 | 92 | 12 | 119 | 0.03 |
| 20 | 126 | 21 | 139 | 0.07 |
| 25 | 146 | 22 | 145 | 0.12 |
| 30 | 173 | 20 | 146 | 0.18 |
| 35 | 200 | 19 | 139 | 0.24 |
| 45 | 233 | 17 | 130 | 0.39 |
| 55 | 265 | 13 | 118 | 0.54 |
| 65 | 286 | 4 | 96 | 0.69 |
| 75 | 305 | −3 | 79 | 0.82 |
| 80 | 304 | −10 | 67 | 0.87 |
| 83 | 310 | −13 | 59 | 0.90 |
| 87 | 318 | −19 | 46 | 0.94 |
| 90 | 20 | 0* | 0* | 0.95 |

* Indicates assumed value at boundary.

Hence for $f = 6.3 \times 10^{-5}$/s,

$$\langle fD \rangle = 2.27 \times 10^{-10}\ \mathrm{s}^{-2} \quad (9)$$

In the one-moving-layer approximation of Schmitz, the steady-state downstream acceleration reduces to $dV/dt = \frac{1}{2}\partial V^2/\partial s$, and corresponds to the change of horizontal kinetic energy along a transport stream function. According to Table 1 the same stream function is located at $x = 10$ km at both sections, while the stream function $18 \times 10^6$ m³/s is located at $x = 83$ km at the southern section but near 81 km at the northern section.

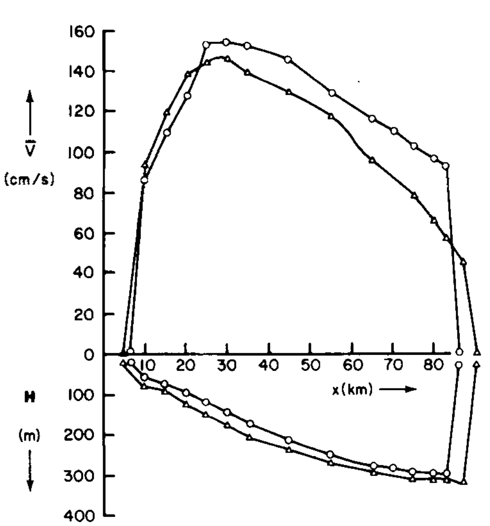


*Fig. 2.* Northward speed ($v$) and surface layer depth or thickness ($H$) vs. distance to the east ($x$). Triangles for the southern section and circles for the northern section as shown in Fig. 1. The narrow zones to the left of $x = 10$ km, and to the right of $x = 83$ and 87 km are assumed to be the frictional boundary layers. Reproduced from Schmitz (1969).

Hence along these stream functions, for $A$ at $x = 10$ km,

$$(\mathrm{d}V/dt)_A = \{\tfrac{1}{2}(\partial/\partial s)(u^2 + v^2)\}_A = -3.1 \times 10^{-4}\ \mathrm{cm/s^2}$$

While for $B$ near $x = 83$ km,

$$(dV/dt)_B = 1.1 \times 10^{-3}\ \mathrm{cm/s^2}$$

Thus we have

$$\langle(\partial/\partial n)\, dV/dt\rangle = \{(dV/dt)_B - (dV/dt)_A\}(B - A)^{-1} = 1.1 \times 10^{-10}\, s^{-2} \tag{10}$$

While $\partial V/\partial z$ is identically zero in the one-moving-layer, it can be as large as 0.0075 $\mathrm{s^{-1}}$, according to Niiler and Richardson (1973, Fig. 6). And if we use (8), at 100 m the vertical speed $w =$ 0.036 cm/s. It follows that for $\langle K\rangle = 1/(150\ \mathrm{km})$, the order of magnitude of the last two terms in (4) are

$$\left.\begin{aligned}\langle KVD\rangle &= \pm\, 0.3 \times 10^{-10}\,\mathrm{s^{-2}}\\ \langle Kw(\partial V/\partial z)\rangle &= \pm\, 0.2 \times 10^{-10}\,\mathrm{s^{-2}}\end{aligned}\right\} \tag{11}$$

The beta term is similarly small at

$$\langle df/dt\rangle = 0.3 \times 10^{-10}\,\mathrm{s^{-2}} \tag{12}$$

for $\langle v\rangle = 121$ cm/s in accordance with Table 1. Hence dropping these small terms we find

$$\langle(\partial/\partial s)\, KV^2\rangle = -\,3 \times 10^{-10}\,\mathrm{s^{-2}} \tag{13}$$

Schmitz gives typical errors of 5–10% of for the $u$ component and 3–5% for the $v$ component. A 15% error for (13) seems reasonable. So summarizing the evaluation we have

$$\left.\begin{aligned}\langle fD\rangle &= 2 \times 10^{-10}\,\mathrm{s^{-2}}\\ \langle(\partial/\partial s)\, KV^2\rangle &= -3 \times 10^{-10}\,\mathrm{s^{-2}}\\ \langle\partial/\partial n)dV/dt\rangle &= 1 \times 10^{-10}\,\mathrm{s^{-2}}\\ \langle df/dt\rangle &= 0.3 \times 10^{-10}\,\mathrm{s^{-2}}\\ \langle KVD\rangle &= \pm 0.3 \times 10^{-10}\,\mathrm{s^{-2}}\\ \langle Kw\ \partial V/\partial z\rangle &= \pm 0.2 \times 10^{-10}\,\mathrm{s^{-2}}\end{aligned}\right\} \tag{14}$$

## 4. A curvature-dominated hypothesis

It is clear from (14) that the leading terms in (5) are the divergence and centripetal terms. So where

$$(\partial/\partial s)\, KV^2 > (\partial/\partial n)\, dV/dt \tag{15}$$

and

$$(\partial/\partial n)\, dV/dt > df/dt \tag{16}$$

we have approximately

$$\left.\begin{aligned}&(f + KV)\, D = -(\partial/\partial s)\, KV^2\\ &\text{or}\\ &D = -(f + KV)^{-1}\,(\partial/\partial s)\, KV^2\end{aligned}\right\} \tag{17}$$

That is, in a curvature-dominated meander where the flow is quasi-steady and $f > KV$, divergence occurs where the centrifugal force $(-KV^2)$ increases downstream.

Where the amplitude of a meandering current is small, $KV^2$ varies from zero to a maximum in a distance approximately equal to a quarter wavelength ($L/4$). We denote this change in $KV^2$ by $\delta KV^2$, and the change in $dV/dt$ across a current of width $N$ by $\delta dV/dt$. Hence from (15) the wavelengths to which (17) is applicable is given by

$$L < 4N(\delta KV^2/\delta dV/dt) \tag{18}$$

barring such idealized regimes as gradient flow where $dV/dt$ is zero but $KV^2$ may vary, accelerating rectilinear flow where $KV^2$ is zero but $dV/dt$ may vary, and geostrophic flow where both $KV^2$ and $dV/dt$ are zero. Where the magnitude of the acceleration ratio in (18) is of the order one, then for $N = 100$ km, the upper limit of applicable wavelengths is of the order

$$L = 0\ (400\ \text{km}) \tag{19}$$

Away from the equator where generally $f > KV$, mass distribution across a meandering flow reflects the approximate balance between the Coriolis and the cross-stream pressure gradient force. In the northern hemisphere and looking downstream this means that along the left flank, colder subsurface water is nearer the sea surface. Therefore, where divergence first succeeds in surfacing cold water, the upwelling will occur on the left side. The rate of upwelling, however, will be uneven in that region, since changes in centrifugal force are generally different for different parts of the flow. This is an important point in the following application of (17).

According to Taft (1971), upwelling north of the Somali Current off the Horn of Africa does not appear to be primarily wind-driven. Rather, it occurs north of where the current turns eastward abruptly. Since the north side is the left side and the abrupt turn corresponds to an abrupt increase in centrifugal force, this is in support of (17).

In the upwelling off Cochin, India, Banse (1968) also denies the winds a primary role. But while he suggest an adjustment process related to the strength of the current, the work of Duing (1970, Fig. 11) shows, interestingly, a meandering current turning anticyclonically off Cochin, again in support of (17).

The flow of the California Current around capes and points of land occurs over distances well within the limit of (19). In following the coastal contours the curvature of the current is positive where it rounds a cape and negative where it turns to resume its southward flow. Hence, according to (17) divergence is expected south of the cape, where the curvature changes from positive to negative, in agreement with the works of Arthur (1965) and Yoshida (1967, p. 63). Similarly, divergence may also be expected in the northbound Davidson Current if it rounds the capes with comparable speed. For the Davidson Current, however, any upwelling will be farther offshore, although it will still be to the south of a cape.

At the sea surface, the cold water from horizontal divergence is sometimes masked by insolation. Nevertheless, upper subsurface isotherms will rise in a divergence and fall in a convergence. Chew (1974) describes two such cases: one in the Loop Current in the Gulf of Mexico, and one in the meandering Florida Current north of Cuba. In both cases upper isotherms ascended as centrifugal force increased, and descended as the force decreased. Two other observed features are the fragments of warm water on the left side, and the change in horizontal temperature gradient. The fragments (Fig. 3) were at locations that could be interpreted as having been separated from the main flow by the upwelling that attends an increase in centrifugal force. More definite is the tendency for the horizontal temperature gradient to decrease with divergence and increase with convergence, a point to remember in the next example.

In his review of primary production in the sea, Yentsch (1974, p. 116) attributes increased productivity in western boundary currents to an imbalance between pressure gradient and Coriolis forces. An example of this imbalance in the Gulf Stream is the situation of February 26, 1975, shown in Fig. 4 from Legeckis (1978). On that day infrared image of sea surface temperature shows the stream to meander between Savannah and Cape Fear with a wavelength of about 250 km between anticyclonic turns. If the stream flow is assumed to be in the general direction parallel to the inshore boundary of the meander, then according to (17), convergence is indicated in the stream segment between Savannah and Long Bay, and divergence in the shorter segment between Long Bay and Cape Fear. Considering first the divergence segment, we estimate the curvatures at both the turns off Long Bay and Cape Fear at about 1/(55 km), positive at the cyclonic and negative at the anticyclonic turn 110 km downstream. Thus, for $V = 100$ cm/s, and $f$ corresponding to 33° N, eq. (17) gives a divergence of the order of

$$D = 4.5 \times 10^{-6}\ \text{s}^{-1} \tag{20}$$

Because of the asymmetry of the meander, the convergence in the upstream segment is smaller at

$$D = -1.3 \times 10^{-6}\ \text{s}^{-1} \tag{21}$$

if we take $-1/(150$ km) for the curvature of the turn off Savannah, 250 km for the downstream distance to the next cyclonic turn, and $V$ and $f$ as before. Downstream of the turn off Cape Fear there is again convergence, according to (17).

This alternating pattern appears reflected in the surface temperature contrast across the inshore boundary. To see this, recall from the preceding example the observed tendency for decreasing horizontal temperature gradient with divergence, and increasing gradient with convergence, a tendency that parallels the meteorological one where divergence promotes frontolysis and convergence frontogenesis. In terms of Fig 4, a weaker temperature gradient means a softer contrast. Therefore, compared to the convergence segments immediately up and downstream, we expect a

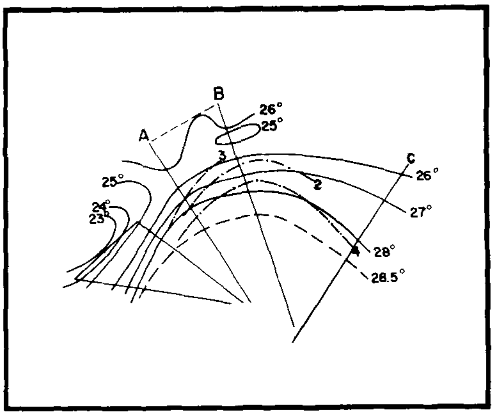


*Fig. 3a.* The 40 m temperature (°C) in the Loop Current in the Gulf of Mexico during a drogue tracking experiment in October, 1970. Water temperature tracked by drogues (dot-dashed curves labelled 2, 3, and 4) ranged from 26 to over 28 °C. The feature of interest lies to the left of the drogues where pools of cool water separated a warm (26 °C) water fragment of unknown dimensions near points marked *A* and *B*, about 85 km apart. Reproduced from Chew (1974).

measure of softness across the inshore boundary of the short divergence segment. This seems to be the case. Moreover, in conjunction with the potential upwelling, there is in the frail appendage that extends south from the anticyclonic turn off Cape Fear, a potential counterpart of the warm water fragments in Fig. 3.

A study by Woods, Wiley, and Briscoe (1977) of vertical motion in a meandering frontal jet both supports and provides evidence that (17) is applicable to flows of wavelengths of order as short as 10 km. They give values of $V = 10$ cm/s, $K = 1/(2$ km), and $L = 8$ km. In a sinusoidal flow this means an amplitude of $K(L/2\pi)^2 = 0.8$ km, and a downstream change in $KV^2$ of about

$$(\partial/\partial s)\, KV^2 = 2.0 \times 10^{-9}\ \mathrm{s}^{-2} \tag{22}$$

This compares to a beta term of $2 \times 10^{-12}\ \mathrm{s}^{-2}$, at most. But to estimate the banking term it is necessary to make some assumptions about the jet. First, for a liberal estimate, we take the components of the downstream acceleration to reinforce one another, so that $dV/dt = (\partial/\partial s)\, V^2 = 1.8 \times 10^{-4}$ cm/s, for a speed of 5 cm/s at one turn and 10 cm/s at the next turn. This immediately give $(\partial/\partial n)\, dV/dt = 1.2 \times 10^{-9}\ \mathrm{s}^{-2}$, for a jet width of 1.5 km. Hence both conditions (15) and (16) are met, and from (17) and (22) a divergence of the order of

$$D = 2 \times 10^{-5}\ \mathrm{s}^{-1} \tag{23}$$

is indicated. A central feature of their work is a

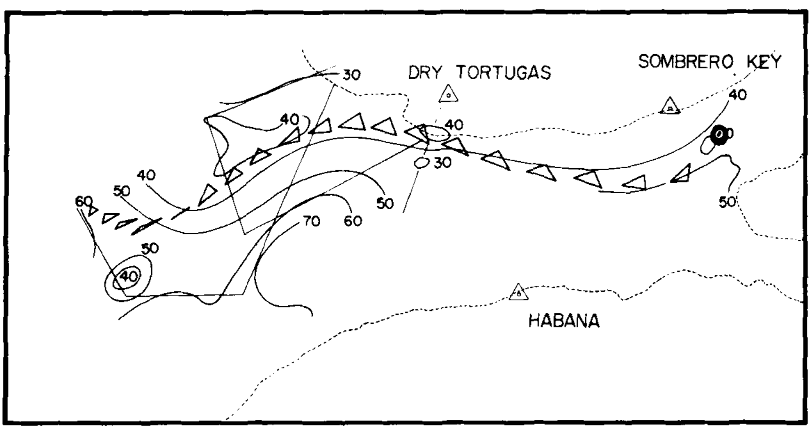


*Fig. 3b.* The depth (m) contour of the 28 °C surface in an upstream segment of the Florida Current during a drogue tracking experiment in August, 1971. The feature of interest is the closed 40 m contour in the left flank of the current as it approached the anticyclonic turn west of Dry Tortugas. Reproduced from Chew (1974).

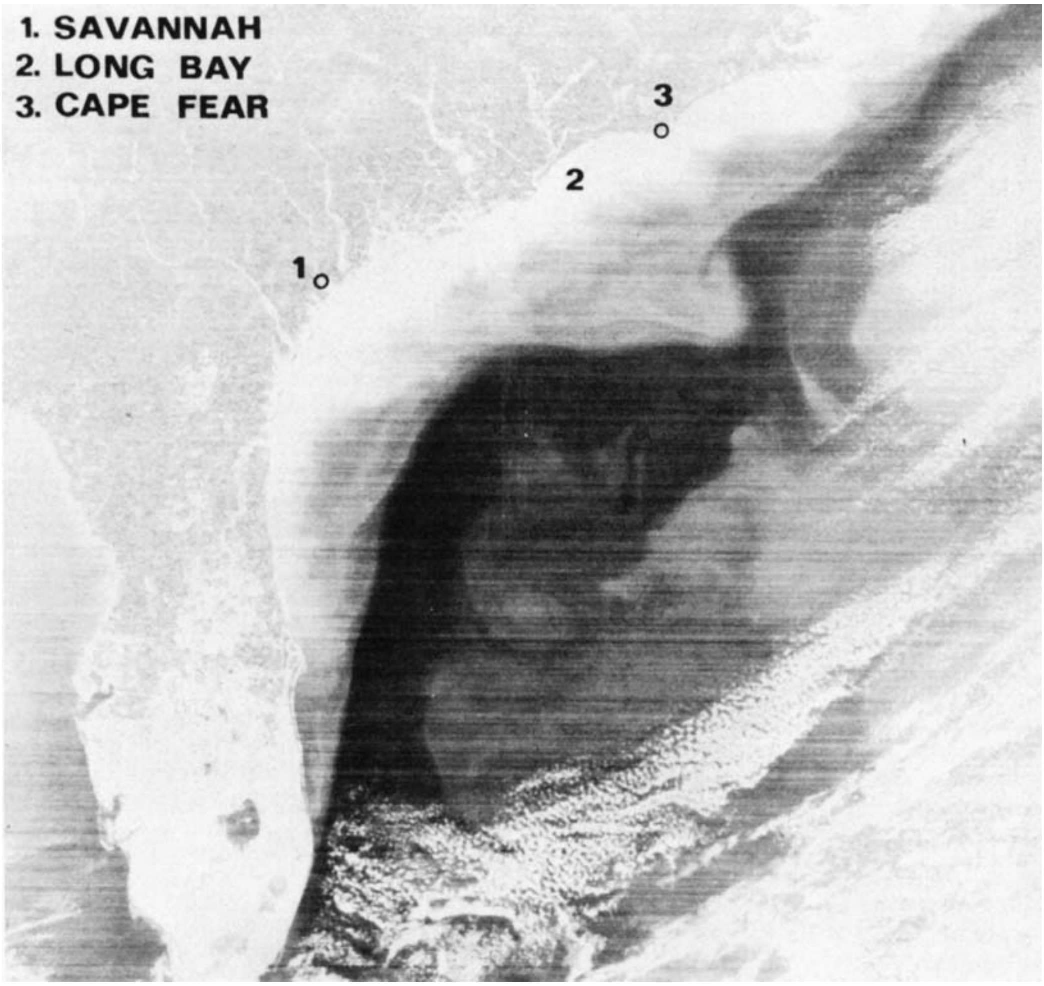


*Fig. 4.* Infrared image for February 26, 1975. Adopted from Legeckis (1978, Fig. 6).

series of alternating upwelling and downwelling tongues, upwellings that could mean upward nutrient flux. For meandering flows, (17) predicts just such a series. Moreover, depending on the sign of $KV$, the factor $(f + KV)$ in (17) may be greater or less than $f$. Thus where a flow is nearly sinusoidal, upwelling tongues tend to be more intense than downwelling ones. Hence, given the prevalence of meandering flow, the mechanism embodied in (17) appears to be a significant process in the general fertility of the sea.

## 5. An eddy-ring hypothesis

In the formation of eddy-rings from amplifying meandering flow, there is a large change in curvature vorticity in both time and space. Hence, for streamline curvature $K_s$, we suppose, in addition to (15) and (16) that

$$(D/Dt)\, K_s\, V > KV\, \partial V/\partial s \tag{24}$$

where $D/Dt = d/dt + \partial/\partial t$ is the individual derivative. Thus the turning vorticity equation (Chew, 1975, eq. 8) reduces to

$$(D/Dt)\, K_s\, V = -(f + K_s\, V)\, D \tag{25a}$$

Or for a layer of thickness $h$,

$$(D/Dt)\{(f_0 + K_s\, V)/h\} = 0 \tag{25b}$$

For the northern hemisphere and for $f > K_s V$, (25) simply states that a layer undergoing convergence (divergence) will gain cyclonic (anticyclonic) curvature vorticity. Three possible features may be inferred from (25). First, whatever the mechanism initiating vertical motion, with cyclonic curvature vorticity increasing with convergence, sufficient convergence will lead to cyclonic cutoffs, and sufficient divergence to anticyclonic cut-offs. This is in accord with the works of Saunders (1971), Fuglister (1972), and Newton (1978), and is consistent with the dynamics of the warm water fragments in Fig. 3 if they were or were becoming anticylonic eddies. Second, where convergence and divergence are of equal magnitude, the factor $(f + K_s V)$ in (25) will favor cyclonic over anticyclonic cut-off. The implied greater population of cyclonic eddy-rings is consistent with the work of Lai and Richardson (1977) and Richardson, Cheney and Worthington (1978) who found three to four cyclonic rings for every anticyclonic eddy in the North Atlantic. But this disparity could also mean a bias toward greater subsidence in the North Atlantic. More important, however, is the longevity of cyclonic rings, bringing us to the last inference that rings are stable configurations. Cut-off eddies are initially irregular in shape. But as an eddy acquires circular symmetry on becoming a ring, $(D/Dt) K_s V$ approaches zero, as would $(\partial/\partial s) KV^2$ in (17) as the flow tends toward a steady state. It follows from (17) or (25) that there is no significant vertical motion in symmetric rings, and remains so as long as no other forces act.

To conclude this section we turn to an observation of eddy formation off East Australia where Nilsson, Andrews and Scully-Power (1977) found upper-layer convergence a cause in the closing off of an anticyclonic warm water eddy. They postulate a baroclinic triggering mechanism, but that aside, the dynamics is in accord with (25). For with $f < 0$ in the sourthern hemisphere, upper layer convergence is precisely what (25) prescribes as necessary to generate the negative curvature vorticity inherent in warm anticyclonic eddies. Moreover, with negative curvature vorticity now reinforcing a negative Coriolis parameter, for the same magnitude of convergence or divergence there is a greater likelihood for more numerous and more intense anticyclonic eddies than for cyclonic eddies. This is consistent with the works of Hamon (1965) and Boland and Hammon (1970).

## 6. Discussion and conclusion

Since the work of Wust (1924) most studies of the Florida Current off Miami have assumed negligible curvature effects. It was not realized that it is not always necessary for the current to have large curvature in order to have correspondingly large curvature effects. Chew (1979) shows examples of important curvature effects in flows with Rossby number, as a ratio of the centrifugal to the Coriolis force, of the order of 0.1 or less. In the light of (5) and (17), the Bernoulli acceleration arising from the channel constriction will, through the divergence effect, bias the curvature of all meander trains passing through. Assessing the full implication of this situation will require a combination of both the Lagrangian and Eulerian approach. And perhaps the absence of such a combination is one reason why progress has also been slow in the study of the upwelling and meandering of the Kuroshio, although Uda (1964) first posed the problem more than a decade ago.

Given the small terms in (14), the predominant divergence effect in (5) must be balanced by an equally large curvature change effect, as posed in (17). Although in (14) the observed field of divergence is tied to a channel constriction, precisely because the linkage between divergence and curvature change is strong even in a channel, we may expect the linkage to be no less in the open sea where flow curvature can be much larger.

Bjerknes (1937) and Bjerknes and Holmboe (1944) are the first to consider the curvature effects on divergence. They treat the longitudinal and transversal components of the horizontal divergence separately. For the longitudinal component they employ a model where the transport is confined to isobaric channels of constant depth, implying an assumption of zero downstream acceleration later made explicit by Palmen and Newton (1969, p. 143). The use of this assumption in (5) by setting the separate component terms in $dV/dt = V\,\partial V/\partial s + w\,\partial V/\partial z$ individually to zero immediately gives

$$\left.\begin{aligned} (f + KV)\,D &= -V^2\,\partial K/\partial s - df/dt \\ \text{or,}\quad & \\ fD &= -V^2\,\partial K/\partial s - df/dt \end{aligned}\right\} \qquad (26)$$

on neglecting $KV$ against $f$. The counterpart in Bjerknes and Holmboe (1944, eq. 5.4) is somewhat different, but may be put in the form,

$$fD = -V(\partial/\partial s)(KV + f) \quad (27)$$

Related, but also different is

$$fD = -(\partial/\partial s)\, KV^2 - df/dt \quad (28)$$

a starting point of numerical study of the so-called "free-inertial jet" (e.g. Godfrey and Robinson, 1971). But besides being inconsistent with the $dV/dt = 0$ assumption, there is no clear evidence that either (27) or (28) is superior to (26) in describing the relation between divergence and curvature change. Thus to the extent that $dV/dt = 0$ holds (26) is a generalization of the Bjerknes approach in that there is no restriction on the depth of isobaric channels and both the longitudinal and tranversal divergence components are included in $D$. But compared to (17) eq. (26) is unnecessarily restrictive and provides no obvious clue, such as, (18), to the range of its validity.

For meso-scale adiabatic flows we conclude that horizontal divergence and change of centripetal acceleration are closely linked and that in the northern hemisphere upwelling is strongly indicated where there is a rapid increase of centrifugal force in the surface flow.

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## ГОРИЗОНТАЛЬНАЯ ДИВЕРГЕНЦИЯ, УСКОРЕНИЕ И ИЗМЕНЕНИЕ КРИВИЗНЫ; ДИАГНОСТИЧЕСКОЕ УРАВНЕНИЕ И ЕГО ПРИЛОЖЕНИЯ

Рассматривается горизонтадьная дивергенция потока с ускорением в связи с меандрирующими течениями и образованием вихрей-рингов. Цель работы-способствовать нахождению областей подъема вод и связанных с ним динамических явлений. Две гипотезы, представляющие две мезомасштабные версии уравнения вихря (Чу, 1975), формулируются с помощью данных Шмитца (1969) по Флоридскому течению. Первый подход является обобщением пионерской работы Бьеркнеса (1937) о связи между горизонтальной дивергенцией и изменением кривизны линий тока. Гипотеза предсказывает, что вдоль меандрирующего течения в северном полушарии и вдали от экватора имеет место горизонтальная конвергенция там, где есть быстрый рост центростремительного ускорения и имеет место горизонтальная дивергенция там, где растет центробежная сила. В частности, в меандрирующем поверхностном течении с последовательностью циклонических и антициклонических круговоротов гипотеза предсказывает последовательность языков подъема и опускания вод.

Вторая гипотеза представляет количественную теорию, основанную на исследовании Фуглистера (1972) роли вертикального движения в образовании и эволюции вихрей-рингов. Вихри, будучи отсеченными меандрами, вначале нерегулярны по форме. Согласно этой гипотезе вихрь, развивающийся в ринг с круговой симметрией, вступает в бездивергентную стадию и, следовательно, в стадию устойчивости.