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Some consequences of non-proportionality between fluxes and reservoir contents in natural systems

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ABSTRACT

We study the effect on simple reservoir systems of non-proportionality between fluxes and reservoir contents. In particular, we assume that the flux F out of a reservoir is related to the reservoir content M by $F/F_0 = (M/M_0)^\alpha$, where α may be greater than, equal to or less than one. It is demonstrated that adjustment rates and equilibrium distributions are critically dependent upon the value of the parameter α.

A simple model of the global carbon cycle is used as an illustration. We show among other things that, because of non-proportionalities in the fluxes between the ocean surface layer, the atmosphere and the terrestrial system, the decay time of a man-made carbon emission into the atmosphere is much longer than the turn-over time with respect to exchange with the deep ocean layers.

1. Introduction

In the study of man's impact on the environmental cycles one frequently meets the question of how rapidly a man-made excess of a compound in a natural reservoir would decay. Similarly it may be of interest to know how the amount of material undergoing a certain natural process would be affected if the amount of available material were altered. A customary approach here is to assume direct proportionality between fluxes and amounts. If the amount of material available to undergo the process increases, the amount processed is assumed to grow in the same proportion. For a wide class of natural processes, this is a very well-motivated approach. Radioactive decay, many forms of chemical decomposition and advective transport are a few examples of processes that increase in a rate proportional to the number of molecules available.

However, it is also obvious that not all processes follow this simple rule. In many cases, the increase will be smaller than proportional, or perhaps even none at all. For example, carbon is necessary to the photosynthesis in the sea, but the life processes are much more limited by nitrogen and phosphorus. It is therefore not likely that increased concentrations of inorganic carbon in the marine environment would enhance the production of organic matter. On the other hand, there also exist processes in nature where the corresponding flux would respond to an increased pool of available substance with a relatively much larger increase. The evaporation of CO_2 from the ocean surface is a function of the concentration of inorganic carbon in the water. For chemical reasons, the amount of inorganic carbon in the water need only increase by 1% in order for the evaporation to increase by nearly 10%.

The assumption of proportionality may be a tempting first approximation whenever there are no arguments either in favour or against. However, when this simplification is done, certain fundamental properties of the cycle may be quite erroneously described. In the present paper, we shall discuss two aspects of environmental cycles and man's impact on them that are sometimes overlooked, since they depend critically on the occurrence of non-proportional fluxes in the cycle.

(i) *Adjustment time.* We shall define a time-scale tentatively called adjustment time, which is a measurement of the time required for the decay of a perturbation of the reservoir content. It will be

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shown that the adjustment time is equal to the turn-over time when proportionality is assumed between contents and sinks, but in a general case it may be quite different from the turn-over time of the reservoir. For a discussion about the definition of turn-over time and other related time scales, reference is made to a paper by Bolin and Rodhe (1973).

(ii) *Equilibrium distribution.* As long as all fluxes between the reservoirs of a cycle increase in proportion to the contents, it can be shown that an excess will eventually distribute between the reservoirs in the same proportions as their respective initial contents. When non-proportional fluxes occur the shift to a new equilibrium can imply a large change in the relative sizes of the reservoirs.

In the following section we consider the mathematical background for (i) and (ii) in a fairly general way. Using a simplified model of the carbon cycle as an illustration, we then demonstrate that serious errors can arise from neglecting these two effects.

2. Theory

Consider a compound in a well-mixed open reservoir. In order to illustrate the ideas that were mentioned qualitatively in the introduction, we first want to formulate a simple model for how the flux F of the compound out of the reservoir is related to the content M of the reservoir. In the most general case, the outflow is not only a function of the total mass in the reservoir. It can also depend on the age distribution of this material or on external influences. A study of the flow of compounds through reservoirs in geophysical systems under such conditions has recently been presented by Lewis and Nir (1978). In the present paper, however, we assume that the formulation

$$F = F(M)$$

is always valid.

For a small (infinitesimal) change in M we have

$$dF = F'(M)dM \tag{1}$$

where it has been assumed that $F(M)$ is differentiable. In a small neighbourhood of $(M_0, F_0 \equiv F(M_0))$ we may write

$$F - F_0 \approx F'(M_0)(M - M_0) \tag{2}$$

This relation implies a proportionality between M and F only in the special case when $F_0 = F'(M_0) \cdot M_0$. In order to derive from eq. (1) an alternative simplified non-proportional expression for F we introduce the function $\alpha(M) \equiv F'(M) \cdot M/F$ so that

$$\frac{dF}{F} = \alpha(M)\frac{dM}{M} \tag{3}$$

In some applications we have $\alpha(M) \approx \alpha(M_0)$ in the interval of interest. Then

$$\frac{F}{F_0} \approx \left(\frac{M}{M_0}\right)^{\alpha(M_0)} \tag{4}$$

In the following calculations, which aim at giving some simple illustrations of the implications of non-proportionality between fluxes and reservoir content, we have chosen to work with eq. (4) rather than the linearized eq. (2). The assumption about a constant α excludes the possibility of describing certain fundamental types of reservoir behaviour. For example, as M increases the sink process may shift character into another functional form. For large values of M there may then be no further growth of F and α would thus approach zero.

Most of our qualitative conclusions could have been derived equally well by considering eq. (2). Even if there may be reasons, based on physical or chemical arguments, to claim that eq. (4) in some situations is more realistic than eq. (2), our choice of eq. (4) is somewhat arbitrary. The difference from an assumption about proportionality is, however, substantial. In Fig. 1, which shows a hypothetical relation between mass and flux, we have tried to illustrate the nature of the different assumptions discussed so far. Note that the functions given by eqs. (2) and (4) both have the same slope as the real curve in the equilibrium point (M_0, F_0).

2.1. *Adjustment time for a disturbed reservoir*

Let us study a single reservoir with content M connected with another reservoir with infinite content (Fig. 2). The flux F_0 into the reservoir is constant whereas the flux out of the reservoir is related to the mass inside the reservoir by relation (4). In the equilibrium state M has a value M_0 such that $F(M_0) = F_0$. We now consider a situation when there is an initial imbalance between M and F. Such an imbalance may be the result of a more or less sudden change in M or, alternatively, a similar

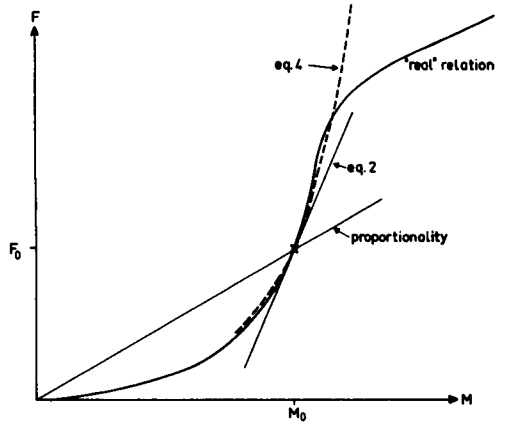

Fig. 1. Graphical representation of a hypothetical relation between the flux F out of a reservoir and its content M. The thin lines and the dashed line show three model relations discussed in the text.

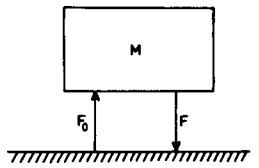

Fig. 2. Reservoir with content M connected with a reservoir of "infinite" content. Fluxes defined in text.

change in F. Thus, suppose that M is initially given a value M_* which is different from the equilibrium value M_0. We want to investigate how fast M readjusts to M_0 if the flux into the reservoir remains constant and equal to F_0.

The differential equation describing the adjustment process is given by

$$\frac{dM}{dt} = F_0\left(1 - \left(\frac{M}{M_0}\right)^{\alpha}\right); \quad M(t=0) = M_* \tag{5}$$

We non-dimensionalize the equation with the aid of the equilibrium turn-over time of the system ($\tau_0 = M_0/F_0$),

$m \equiv M/M_0$

$\tau \equiv t/\tau_0$

and get

$$\frac{dm}{d\tau} = 1 - m^{\alpha}; \quad m(\tau = 0) = m_* \tag{6}$$

Instead of attempting to solve this equation explicitly we rest content with studying the implied adjustment rate. As a measure of this rate we take the time scale T defined by

$$T \equiv \frac{M - M_0}{F - F_0} \tag{7}$$

where $F = F_0(M/M_0)^{\alpha}$ is the flux out of the reservoir at time t.

Jacquez (1972, pp. 69–71) formulated a more general non-linear reservoir system and studied its behaviour for small perturbations. The time scale defined in (7) corresponds to the adjustment rate coefficient in his equation (5-9) only in the special case of small perturbations.

We wish to illustrate that the adjustment time may change substantially in the course of the adjustment process and to stress the difference between adjustment time and turn-over time. We therefore rewrite (7) and obtain

$$T = \tau_0 \frac{m - 1}{m^{\alpha} - 1} \equiv \tau_0\, \gamma \tag{8}$$

It is readily seen that

$$T \gtreqless \tau_0 \quad \text{if and only if } \alpha \gtreqless 1 \tag{9}$$

The numerical values of the factor $\gamma = (m - 1)/(m^{\alpha} - 1)$ for different values of m and α are given in Table 1.

Table 1. *Values of the ratio between the adjustment time scale T and the equilibrium turn-over time τ_0 for different values of the deviation mass m and the exponent α*

m \ α	8	4	2	1	1/2	1/4	1/8
8	4.0×10^{-7}	1.7×10^{-3}	0.11	1	3.8	10.3	23.6
4	4.6×10^{-5}	1.2×10^{-2}	0.20	1	3.0	7.2	15.9
2	3.9×10^{-3}	6.7×10^{-2}	0.33	1	2.4	5.3	11.0
1	(0.125)	(0.25)	(0.5)	(1)	(2)	(4)	(8)
1/2	0.50	0.53	0.67	1	1.7	3.1	6.0
1/4	0.75	0.75	0.80	1	1.5	2.6	4.7
1/8	0.88	0.88	0.89	1	1.4	2.2	3.8

It is clearly seen from Table 1 that as the adjustment proceeds, i.e. m approaches 1, the adjustment rate changes. For $m > 1$ and $\alpha > 1$, as well as for $m < 1$ and $\alpha < 1$, the adjustment rate decreases (i.e. γ increases) as the adjustment proceeds and the excess declines. Such a situation is illustrated in Fig. 3. The opposite is true for the situations when $m > 1$ and $\alpha < 1$ or $m < 1$ and $\alpha > 1$. It is only in the simple exponential case, $\alpha = 1$, that the adjustment rate remains constant (and equal to τ_0).

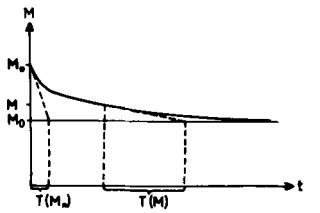

Fig. 3. Illustration of the definition given in eq. (7) of an adjustment time scale T. Note that in the general case ($\alpha \neq 1$) T is not a constant but varies as the adjustment proceeds. The situation illustrated in this figure corresponds to a α-value > 1.

The time for the mass to reach a specified fraction of the equilibrium value is, in the general case, not a unique function of T but depends also on the value of α. When the disturbance is small (i.e. $m \approx 1$) we may expand the factor γ according to

$$\gamma = \frac{m-1}{m^{\alpha}-1} \approx \frac{1}{\alpha} \tag{10}$$

which yields

$$T \approx \frac{\tau_0}{\alpha} \tag{11}$$

This relation could also be derived from eq. (5-9) in Jacquez (1972). Table 1 indicates how well eq. (11) approximates eq. (8). It is seen that, at least for $\alpha \lesssim 2$, eq. (11) may be an acceptable approximation provided

$$\tfrac{1}{2} \lesssim m \lesssim 2$$

We may conclude this section with the following summary.

(i) For a reservoir with proportional (first-order) sink process ($\alpha = 1$) the adjustment is exponential and the adjustment time equals the equilibrium turn-over time. This is a normal situation which applies well to a number of geophysical and other systems (cf. the introduction).

(ii) If $\alpha > 1$, i.e. the sink process is more strongly dependent upon the mass of the reservoir, the adjustment is faster than the turn-over time of the undisturbed system. For large positive disturbances ($m \gg 1$) the adjustment rate is initially very large, i.e. the adjustment time is very small. The adjustment time increases very significantly as the adjustment proceeds and approaches τ_0/α as m approaches 1. For large negative disturbances ($m \ll 1$) the adjustment time is initially close to the equilibrium turn-over time τ_0. During the adjustment process the adjustment time decreases and approaches τ_0/α. As an example of a reservoir with $\alpha > 1$ we may take a lake where the geometry of the outlet is such that even a small increase in the lake level—and amount of water in the lake—gives rise to a large outflow. In hydrological terms this situation may be expressed as the condition that the exponent of the outflow rating curve is larger than the exponent of the volume–depth curve (Jeng and Yevdjevich, 1966). The adjustment time of the lake is the time scale describing the return of the lake level to its normal value after a period of rain. It is not uncommon with lakes where this time scale is several times shorter than the turn-over time of water in the lake.

(iii) If $\alpha < 1$, i.e. the sink process is less dependent upon the mass of the reservoir, the adjustment time is larger than the equilibrium turn-over time. In this case the adjustment time shows less drastic deviation from τ_0/α. For positive deviations ($m > 1$) the adjustment time decreases as the adjustment proceeds, while for negative deviation ($m < 1$) the opposite is true. As an example of this situation, one may imagine a lake with gently sloping shorelines but with a canyon-formed outlet. In such a lake even a substantial addition of water will give rise to only a small increase of the outflow.

2.2. *Equilibrium distributions*

The series of transfers, by which one particular atom is moved between the reservoirs in a cycle, constitutes a random walk. We cannot predict the way an individual atom will be transferred between the various reservoirs in the future. However, the random walk is governed by certain characteristics.

To every reservoir there is a transit time distribution function describing the probability for an atom to remain in the reservoir for a given length of time. The reservoir to which the atom is subsequently transferred is also in some cases random, and can be described by a probability distribution. The parameters describing these distributions are closely related to the bulk properties of the cycle at steady state, such as the inventories and the fluxes between the reservoirs. For example, the relative sizes of the reservoirs are the same as the fractions of the total time that an average atom spends in them, and the relative sizes of the fluxes out of a reservoir determine the probability for an atom to leave the reservoir via either of these fluxes. Another observation of interest in this context is the fact that the average transit time for an atom in a reservoir, i.e. the first moment of the transit time distribution function, is equal to the ratio of the reservoir content to the sum of all the sinks, the turn-over time (Eriksson, 1971; Bolin and Rodhe, 1973).

In order to understand the character of man's impact on a cycle, it is necessary to clarify whether or not it will affect these characteristics of the motion. If the human impact has no effects of this kind, it follows that a finite man-made input will eventually distribute between the reservoirs in the same proportions as the matter that was originally in the system. When the new equilibrium is reached, all reservoirs and fluxes will in this case have increased by the same fraction. Thus we have a situation where proportional relations apply between reservoir contents and effluxes. Conversely it follows that when non-proportional fluxes occur in a cycle, man-made inputs will exert an effect on the characteristic parameters of the random walk of the atoms. For example, the fraction of the total time that an average atom spends in a given reservoir may then change. From a macroscopic viewpoint this means that the relative sizes of the reservoirs will be altered. Also, the turn-over time for a reservoir may have changed when the new equilibrium is established. If this occurs the average length of a stay of an atom in the reservoir is not the same as before.

With these considerations as a background, we shall formally investigate two simple questions.

(i) Consider the situation of Fig. 2. Suppose that the flux F_0 is changed to a new constant value by the addition of a (man-made) flux F'. How will the mass of the reservoir and its turn-over time have changed when the new equilibrium is established?

(ii) Two reservoirs with finite masses, which are connected by non-linear fluxes, receive an addition of a specified amount of mass. How will the mass be distributed between the reservoirs when equilibrium has been attained and how will the fluxes have been affected?

In situation (i) let F' denote the constant additional flux into the reservoir and M_E the new equilibrium value. It follows from the assumptions made in Section 2.1 that

$$F_0\left(\frac{M_E}{M_0}\right)^{\alpha} = F_0 + F' \tag{12}$$

If $F' \ll F_0$ we may write

$$\frac{M_E - M_0}{M_0} \approx \frac{F'}{\alpha F_0} \tag{13}$$

It is readily seen that if $\alpha < 1$ (>1) the relative increase of M is larger than (smaller than) the relative increase of F. It is a straightforward matter to generalize this model to include more than one box, cf. Section 3.2.

The turn-over time of the reservoir in the new equilibrium condition, τ_e, is given by

$$\tau_e = \frac{M_E}{F_0 + F'} = \tau_0\left(1 + \frac{F'}{F_0}\right)^{1/\alpha - 1} \tag{14}$$

This expression shows how the turn-over time, i.e. the average residence time is affected by the additional flux F'. It is only in the case $\alpha = 1$ that the turn-over time remains unchanged. When $\alpha > 1$, the turn-over time will have decreased ($\tau_e < \tau_0$). The reverse is true for $\alpha < 1$.

For the second question we consider two boxes, as in Fig. 4, and assume that the system receives an additional mass, which distributes such that fluxes in both directions are equal. If the two increases

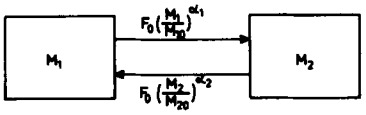

Fig. 4. Two reservoirs connected by non-proportional fluxes.

ΔM_1, and ΔM_2 are small, they can be shown to satisfy the relation

$$\frac{\Delta M_1}{M_{10}} \approx \frac{\alpha_2}{\alpha_1} \frac{\Delta M_2}{M_{20}} \tag{15}$$

It follows that, if α_1 is different from α_2, an addition to the system will be partitioned between the two reservoirs in a ratio different from the ratio of the initial masses.

3. Applications to a simple model of the global carbon cycle

In order to illustrate some of the ideas we put forward in the previous sections, we will apply them to a simple model of the global carbon cycle. Our aim is neither to "explain the carbon cycle" nor to suggest a new and improved model of this cycle. Several of the already existing models take into account the various physical and chemical processes in a more realistic way than is done here. The purpose is rather to demonstrate, qualitatively, how the behaviour of models of this type depends critically upon some basic characteristics of the model.

Specifically, we address ourselves to the problem of how models of the carbon cycle can be employed to provide tentative answers to questions such as the following.

(i) What will be the adjustment time of atmospheric CO_2 following a drastic reduction of man-made emissions? In particular, why do some computer models show a time scale for decline of excess CO_2 considerably longer than the turn-over time of carbon in the atmosphere?

(ii) How will the man-made addition of CO_2 eventually be distributed between the various carbon reservoirs?

3.1. *Adjustment time*

Let us consider the four-box model of the carbon cycle shown in Fig. 5. The terrestrial reservoir includes the carbon in the living organisms as well as in the humus layer. The surface layer of the ocean corresponds to the mixed layer and extends to a depth of some 100 m. In this first section we shall assume that the deep ocean reservoir responds very slowly to changes in the other reservoirs and that it can be treated approximately as an infinite reservoir. In view of the relatively large

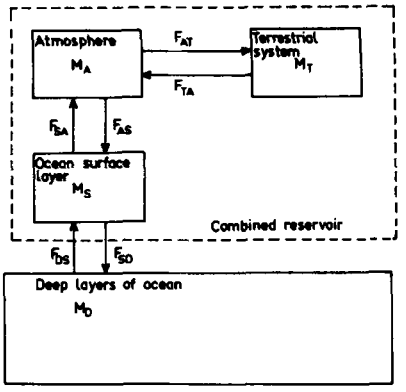

Fig. 5. Simplified model of the carbon cycle.

amount of carbon in the deep oceans—about ten times as much as in the other three reservoirs together—and the correspondingly long turn-over time, this may be an acceptable approximation when considering time scales of up to a few hundred years. By treating the three other reservoirs as a "combined reservoir" we then have a situation identical with that shown in Fig. 2 and we may apply the equations derived in Section 2.1. In order to do so we need to derive an effective α-value for the "combined reservoir" with respect to its exchange with the deep ocean reservoir. This requires that we specify the character of the fluxes between the various reservoirs, even those occurring inside the "combined reservoir".

The following assumptions are made concerning the relations between the fluxes and the reservoir contents. These assumptions, although not quite arbitrary, could certainly have been formulated differently.

$$\left.\begin{aligned} F_{DS} &= F_0 \quad (= \text{constant}) \\ F_{SD} &= F_0 \frac{M_S}{M_{SO}} \\ F_{SA} &= F_1 \left(\frac{M_S}{M_{SO}}\right)^{\beta_S} \\ F_{AS} &= F_1 \frac{M_A}{M_{A0}} \\ F_{AT} &= F_2 \left(\frac{M_A}{M_{A0}}\right)^{\beta_A} \\ F_{T0} &= F_2 \frac{M_T}{M_{T0}} \end{aligned}\right\} \tag{16}$$

where M_{S0}, M_{A0} and M_{T0} are the equilibrium values of the mass in the different reservoirs.

The coefficient β_S (>1) in the expression for F_{SA} arises from the distribution of the carbon in the surface layer reservoir between dissolved CO_2, H_2CO_3, HCO_3^- and CO_3^{--}, and the chemical equilibrium between these compounds (the buffer factor). The numerical value of β_S has been estimated to be near 9 (Keeling, 1973).

Similarly β_A (<1) is due to the fact that CO_2 generally is not the limiting factor for vegetation growth. This means that even a substantial increase in M_A may not produce a very large increase in F_{AT}.

We use the following rough estimates of the various fluxes and reservoir contents to give a quantitative illustration of these and the following relations. $M_{S0} = 1000$, $M_{A0} = 700$, $M_{T0} = 3000$ (unit: 10^{15} g) and $F_0 = 45$, $F_1 = 100$, $F_2 = 60$ (unit: 10^{15} g/yr). These figures are in broad agreement with those given by Bolin et al. (1979) but should not be taken to be very exact and well known.

The equilibrium turn-over times in the various reservoirs are given by

$$\left.\begin{aligned} \tau_{0S} &\equiv \frac{M_{S0}}{F_0 + F_1} \approx 7 \text{ yr} \\ \tau_{0A} &\equiv \frac{M_{A0}}{F_1 + F_2} \approx 4 \text{ yr} \\ \tau_{0T} &\equiv \frac{M_{T0}}{F_2} \approx 50 \text{ yr} \end{aligned}\right\} \tag{17}$$

Let us now consider the "combined reservoir" consisting of the ocean surface layer, the atmosphere and the terrestrial system (cf. dashed box in Fig. 5). The total mass of this reservoir is $M = M_S + M_A + M_T$ and its equilibrium turn-over time with respect to exchange with the deep ocean

$$\tau_0 = \frac{M_0}{F_0} = \frac{4700}{45} \approx 100 \text{ yr} \tag{18}$$

In order to apply eq. (6) to this system we must determine an effective α for the "combined reservoir" (cf. Section 2.1). Thus we write the flux from it to the deep layer of the ocean as

$$F = F_0 \left(\frac{M}{M_0}\right)^{\alpha} \tag{19}$$

Since this flux is identical with F_{SD} we have

$$F_0 \frac{M_S}{M_{S0}} = F_0 \left(\frac{M}{M_0}\right)^{\alpha} \tag{20}$$

If we consider time scales long enough for internal readjustment to take place inside the "combined reservoir" we have in addition

$$\left.\begin{aligned} F_1 \left(\frac{M_S}{M_{S0}}\right)^{\beta_S} &\approx F_1 \left(\frac{M_A}{M_{A0}}\right) \\ F_2 \left(\frac{M_A}{M_{A0}}\right)^{\beta_A} &\approx F_2 \left(\frac{M_T}{M_{T0}}\right) \end{aligned}\right\} \tag{21}$$

After eliminating M_A and M_T from eq. (20) with the aid of eq. (21) we obtain

$$\left.\begin{aligned} &\left(\frac{M_S}{M_{S0}}\right)^{1/\alpha} \left(1 + \frac{M_{A0}}{M_{S0}} + \frac{M_{T0}}{M_{S0}}\right) \\ &= \frac{M_S}{M_{S0}} + \frac{M_{A0}}{M_{S0}} \left(\frac{M_S}{M_{S0}}\right)^{\beta_S} \\ &+ \frac{M_{T0}}{M_{S0}} \left(\frac{M_S}{M_{S0}}\right)^{\beta_S \beta_A} \end{aligned}\right\} \tag{22}$$

If $M_S \approx M_{S0}$ (this seems to be a fair approximation even if $M_A > M_{A0}$) it follows that

$$\alpha \approx \frac{M_{S0} + M_{A0} + M_{T0}}{M_{S0} + \beta_S M_{A0} + \beta_S \beta_A M_{T0}} \tag{23}$$

Using the values 1000, 700 and 3000 for M_{S0}, M_{A0} and M_{T0} respectively, 10 for β_S and 0.2 for β_A we get $\alpha = 0.28$. Applying this result to eq. (11) we get a value of around 400 years for the adjustment time of our system for reasonably small disturbances.

We recall at this point that the time scale estimated here corresponds to the rate of adjustment at a moment when equilibrium has been established *within* the "combined reservoir", but when equilibration between this reservoir and the deep sea is taking place. For the purpose of illustrating the whole transient process, more elaborate numerical models should be preferred. Investigations of this question have been made by Keeling and Bacastow (1977), Revelle and Munk (1977) and Siegenthaler and Oeschger (1978). Although the adjustment rates implied by their models are generally of the same order of magnitude as those

we have derived, differences in model structure cause them to deviate considerably both from our estimate and from one another. The results by Siegenthaler and Oeschger deserve special comment in this context. Studying the response to a pulse input into the atmosphere, on the one hand they explicitly distinguish between a very rapid time scale of about 20 years for the equilibration between the atmosphere and the mixed layer and, on the other hand, a much longer time scale of several hundred years for the process involving the deep sea. This is a nice example of a situation when the adjustment rate in the initial stage of a decline may be a poor indicator of the time required to reach a specified fraction of the new equilibrium state.

In conclusion, it is seen that because of the existence of non-proportional fluxes inside the "combined" reservoir, its adjustment time is several times larger than its equilibrium turn-over time.

The physical explanation for this situation is the following. Because of the large value of β_S, the increase of M_S need not be very large in order to balance the exchange at the sea surface even if the atmospheric content is substantially increased. This tends to limit the rate of exchange of the surface layer, and thereby also of the "combined reservoir" with the deep sea. The effect of the other non-proportionality, that of the flux from the atmosphere to the terrestrial system, is similar. A large value of β_A would have implied a large increase in the terrestrial carbon pool, thereby decreasing the rate of adjustment with respect to the deep sea. On the other hand a small value of β_A, as in the present model, tends to limit the amount of excess carbon stored in the terrestrial reservoir, thereby acting in the opposite way. Evidently, in the present model the effect of the large β_S-value dominates the opposing effect of the small β_A-value.

3.2. *Equilibrium distributions*

Let us look at the second situation described in Section 2.1, that of a closed system having received an extra addition of mass during an unspecified period of time. When applying it to the CO_2 problem we use the same model as the one described in Fig. 5. The only difference, compared to the previous set of relations (eq. (16)), is that we now consider the deep layers of the ocean as an explicit, finite, reservoir with a linear flux into the surface layers equal to

$$F_0 \cdot M_D / M_{D0}$$

The governing system of equations for the equilibrium distribution reads

$$\left.\begin{aligned} \frac{M_D}{M_{D0}} &= \frac{M_S}{M_{S0}} \\ \left(\frac{M_S}{M_{S0}}\right)^{\beta_S} &= \frac{M_A}{M_{A0}} \\ \left(\frac{M_A}{M_{A0}}\right)^{\beta_A} &= \frac{M_T}{M_{T0}} \end{aligned}\right\} \qquad (24)$$

with the constraint $M \equiv M_D + M_S + M_A + M_T = M_0 + \Delta M$.

With the numerical values of M_{S0}, M_{A0}, M_{T0}, β_S and β_A, as given above, and with $M_{D0} = 35{,}000$ and $\Delta M = 6000$ the new equilibrium distribution would be as in Table 2. A total man-made contribution of 6000 units (10^{15} gC) is a large but possible value. It corresponds roughly to the now known accessible reserves of fossil carbon (Keeling and Bacastow, 1977). It is seen that in this model the major part of the fossil carbon will end up in the deep layers of the ocean. Nevertheless the atmospheric content will remain at about 2.5 times its present value. If the assumption had been made that all fluxes were linear the increase in the atmospheric content would have been only about 15%.

As described in a general way in Section 2.2, the occurrence of non-proportional fluxes implies that as the amount of carbon in the natural cycle increases, the characteristic pattern of motion of the atoms is affected. In eq. (14) an example was given to show how the turn-over time of a reservoir could change. Other examples are shown in Table 2. The altered circulation naturally affects all carbon, and it is of interest to see how the distribution of radiocarbon (^{14}C) is changed. In reality, the amount of radiocarbon in the system has been dramatically enhanced by nuclear weapon tests in the atmosphere. In columns 3, 4 and 5 of Table 2 we have computed the equilibrium distribution of radiocarbon as it would have been after the input of 6000 units of ^{14}C-free fossil carbon if no bomb tests had taken place. We see that

Table 2. *Carbon contents, fluxes, turn-over times and radiocarbon contents for the four-reservoir model of Fig. 5, before and after an input of 6000·10¹⁵ g fossil carbon. Radiocarbon contents were computed neglecting fractionation. The mass unit for radiocarbon was chosen to give N_A (before) = 700*

	Carbon content, 10^{15} g		Radiocarbon content, relative units		Relative change in radiocarbon content	Turn-over times, years		Fluxes, 10^{15} g/yr		
	Before	After	Before	After		Before	After		Before	After
Atmosphere	700	1,880	700	1,610	+130%	4.4	5.5	F_{AT}	60	73
Terrestrial system	3,000	3,660	2,980	3,110	+4.4%	50	50	F_{AS}	100	270
Surface layer	1,000	1,120	960	940	−2.1%	6.9	3.5	F_{SD}	45	50
Deep ocean	35,000	39,100	30,800	29,800	−3.2%	780	780			

because of the changing carbon circulation, about 3% of the radiocarbon content of the ocean is removed to the atmosphere. Because of the less-than-proportional increase in the photosynthesis, only 10% of this is transferred into organic compounds.

We see from Table 2 that although the carbon content of the atmosphere increases nearly 2.5 times, there is only about a 15% reduction in the ratio $^{14}C/C$. It should be remembered that the increase of sea-water carbon content of about 10%, as illustrated in Table 2, would reduce the degree of calcium carbonate saturation. We cannot exclude the possibility that this reduction would initiate carbonate dissolution over extensive ocean regions. Parallel to the adjustment illustrated in Table 2 there would then be successive lowering of the buffer factor β_S and the long-term equilibrium could exhibit a much smaller air-borne fraction than indicated in Table 2.

On the other hand, if no carbonate dissolution takes place, the assumption of a constant buffer factor is certainly an overestimate of the ocean's capacity for carbon uptake. It can be shown that, with an increased amount of carbon in the surface layer, the buffer factor would increase. We may, however, neglect this when our purpose is to illustrate the effect of non-proportionality on the equilibrium distribution.

4. Concluding remarks

We have shown by some simple calculations how non-proportionality of fluxes between reservoirs affects adjustment times in and equilibrium distributions between such reservoirs. Our examples show that assumptions about proportionality may lead to very different estimates of the values of adjustment times and equilibrium distributions. The application on a simple model of the global carbon cycle has shown that the behaviour of this model is critically dependent upon non-proportionalities of the fluxes between the ocean surface layers, the atmosphere and the terrestrial reservoir. In fact, although the flux from the ocean surface layers to the deep sea was assumed to be proportional to the carbon content in the ocean surface layer, it was shown that the same flux is far from proportional to the content of the "combined reservoir" consisting of the ocean surface layers, the atmosphere and the terrestrial system together. One important lesson to be learned from this is that one ought to look very carefully for non-proportional exchange processes not only between the reservoirs considered but also *inside* them.

We have used the carbon cycle to illustrate some of the basic concepts introduced in this paper. Naturally, we do not claim that such very simplified models of the carbon cycle, which we have studied, contain the final answer to the very complex question of how nature will distribute the man-made CO_2 emissions between the major reservoirs. That question should be studied with the aid of much more sophisticated models which take into account more of our knowledge about the physical and chemical processes involved (Keeling and Bacastow, 1977; Revelle and Munk, 1977; Oeschger et al., 1975; Björkström, 1979). However, when using such complicated models it may not always be easy to understand their basic characteristics and to see how the overall behaviour is dependent upon the parameters intro-

duced. We therefore mean that such detailed studies should be guided by rough estimates of the types that we have presented. This will facilitate the creation of a clear picture of the basic structure of the model.

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НЕКОТОРЫЕ СЛЕДСТВИЯ НЕПРОПОРЦИОНАЛЬНОСТИ МЕЖДУ ПОТОКАМИ И КОНЦЕНТРАЦИЯМИ В ЕСТЕСТВЕННЫХ РЕЗЕРВУАРАХ

Мы исследуем эффект непропорциональности между потоками и концентрациями примеси в простых системах резервуаров. В частности, мы предполагаем, что поток F из резервуара связан с содержанием в нем примеси M соотношением $F/F_0=(M/M_0)^\alpha$, где α может больше, равно или меньше единицы. Показывается, что скорости приспособления и равновесные распределения зависят критически от величины параметра α. В качестве иллюстрации используется простая модель цикла углерода. Среди прочих вещей мы показываем, что из-за непропорциональностей в потоках между поверхностными слоями океана, атмосферой и системой континентов время распада углерода, выпущенного в атмосферу в процессе деятельности человека, много больше времени оборота углерода по отношению к обмену с глубокими слоями океана.