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# On the effect of $CO_2$ on atmospheric heating rates

By THOMAS P. ACKERMAN, *Australian Numerical Meteorology Research Centre, P.O. Box 5089AA, Melbourne, Australia 3001*

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ABSTRACT

A high resolution longwave radiative transfer model has been developed to investigate the effect of increasing $CO_2$ concentrations on longwave fluxes and heating rates. High spectral resolution has been used in evaluating transmission functions in the 15 $\mu$m $CO_2$ band region with allowance being made for $H_2O$ overlap. Comparison of the results of this model and those of the model of Manabe and Wetherald suggest that the Manabe–Wetherald model is quite accurate and that both models produce similar changes in the heating rate for doubled $CO_2$ concentrations. These changes are quite small ($< 0.1$ K day$^{-1}$) in the troposphere and are of the same order of magnitude or smaller than the estimated accuracy of the models. Larger differences are noted in the stratosphere but model errors are also larger in this region. Through the use of a radiative-convective model it was found that the small changes in the tropospheric heating rate are responsible for more than half of the computed surface temperature change.

## 1. Introduction

The question of the effect of increasing $CO_2$ concentrations on the global climate has engendered a considerable amount of scientific work in the past few years. In a paper comparing the various results Schneider (1975) quotes nine values of the sensitivity of surface temperature to doubled $CO_2$ concentrations. These range in value from 0.7 K to 9.6 K. The most quoted values are probably those of the radiative convective model of Manabe and Wetherald (1967) which produced a figure of 2.4 K and of the general circulation model of Manabe and Wetherald (1975) which produced a figure of 2.9 K. Since both of these models used the same radiation code (Manabe and Strickler, 1964, hereafter referred to as MS), and since this code is based on a band emissivity formulation with a fairly simple approach to the sensitive $CO_2$—$H_2O$ overlap in the 15 $\mu$m $CO_2$ band, we considered it useful to compare the results of this code to those of a high resolution longwave radiation code (hereafter referred to as HR). This comparison should first of all consider the accuracy of the two codes, and secondly should highlight areas of sensitivity in the codes to variable $CO_2$ concentrations.

## 2. The longwave radiative transfer model

The code developed for this study is based on the well-known flux equations written in pressure coordinates:

$$F_r^+(p) = \tau_r(p_0, p)[B_r(g) - B_r(p_0)] + B_r(p) - \int_{p_0}^{p} \tau_r(p', p)\frac{dB_r(p')}{dp'}\,dp' \quad (1a)$$

$$F_r^-(p) = \tau_r(p, p_t)\,B_r(p_t) - B_r(p) - \int_{p}^{p_t} \tau_r(p, p')\frac{dB_r(p')}{dp'}\,dp' \quad (1b)$$

where $F_r^\pm(p)$ = upwards, downwards flux at level $p$ for frequency interval $r$

$\tau_r(p, p')$ = transmission function for the layer bounded by $p$ and $p'$

$B_r(p)$ = blackbody function for the temperature at level $p$

$B_r(g)$ = blackbody function for the temperature of the earth's surface

$p_0$ = surface pressure

and $p_t$ = pressure at the "top" of the atmosphere.

For this model $p_t$ is assumed to be 1 mb.

Since transmission is a complex function of absorber path length, pressure, and temperature, we have used three different expressions for $\tau_r$ in different regions of the spectrum. In the region $1200\ \text{cm}^{-1} < \nu < 2200\ \text{cm}^{-1}$, which is dominated by the 6.3 $\mu$m $H_2O$ absorption band, we have used the formulation of Rodgers and Walshaw (1966). In the window region ($820\ \text{cm}^{-1} < \nu < 1200\ \text{cm}^{-1}$) we have adopted a formulation of $\tau_r$ following the work of Bignell (1970):

$$\tau_r(u, \bar{p}, \bar{e}) = \exp\left[-(k_1 \bar{p} + k_2 \bar{e})\, u\right] \tag{2}$$

where $u$ is the optical path length, $\bar{p}$ the mean pressure along the path, and $\bar{T}$ the mean temperature. $\bar{e}$ is the mean vapour pressure along the path and $k_1$ and $k_2$ are absorption coefficients which are functions of the spectral intervals. Nine spectral intervals were used in the 6.3 $\mu$m based region and four in the window region. It should be noted that calculations in these spectral regions are independent of the $CO_2$ amount and were included in the model for the sake of completeness so that total longwave heating rates could be presented.

In the spectral region $200\ \text{cm}^{-1} < \nu < 820\ \text{cm}^{-1}$, which includes both the 15 $\mu$m $CO_2$ band and the rotation band of water vapour, we have used an empirical formulation devised by Smith (1969). Transmittance data which was taken over a range of optical path lengths, pressures, and temperatures likely to be encountered in atmospheric computations was fitted by a polynomial expansion of the form

$$\tau_r(u, \bar{p}, \bar{T}) = \exp\left[-\exp(W)\right] \tag{3}$$

where

$$W = \sum_{i=0}^{8} c_i\, a_i \tag{4}$$

The $c_i$ are constants derived from the fitting process and are functions of the spectral interval $r$. The $a_i$ are various powers and combinations of $\ln u$, $\ln \bar{p}$ and $\ln \bar{T}$. This fitting process was done over spectral intervals of 5 $\text{cm}^{-1}$ width and was done separately for $CO_2$ and $H_2O$ absorption in the 15 $\mu$m overlap region. The individual transmission functions in the overlap region are then multiplied together to produce the atmospheric transmission function $\tau_r(u, \bar{p}, \bar{T})$ needed to solve eq. (1). This technique is, quite obviously, computationally tedious since the region $200\ \text{cm}^{-1} < \nu < 820\ \text{cm}^{-1}$ divided into 5 $\text{cm}^{-1}$ intervals produces 124 intervals and in 59 of these intervals overlapping absorption between $H_2O$ and $CO_2$ must be considered.

Smith's method has the distinct advantage that it relies on actual transmittances at high spectral resolution measured over values of concentration, pressure, and temperature typically encountered in the atmosphere. Consequently all molecular absorption bands are included without the necessity of specifying different expressions to calculate the transmittance of each (for a discussion of the various bands see, for instance, Ramanathan, 1976). In addition the use of such small frequency intervals produces in general a better agreement between fitted and observed transmission functions than is usually obtained in radiative transfer models.

Two points concerning the accuracy of the model should be mentioned. The first of these is that strictly speaking the overlap between $CO_2$ and $H_2O$ cannot be treated by computing the two independently and then multiplying them together. In order to evaluate the error involved in this procedure a line-by-line model of a portion of the overlap spectrum was run for $CO_2$ and $H_2O$ independently and then in combination for the two gases using a 5 $\text{cm}^{-1}$ slit width. A comparison of the multiplicative transmission function with the combined transmission function showed differences of less than 1% everywhere in the region tested. This suggests that the error introduced by multiplying the two transmission functions is not likely to be of consequence. The second point relates to the absolute accuracy of the method. It is quite difficult to assess the accuracy since errors are introduced into the calculations not only by the original transmission measurements but also by the fitting process. The differences between measured and fitted transmission functions are basically random while the measurement errors may be either random or systematic. In order to evaluate the effect of such errors on the heating rate calculations we have performed two sets of computations for the 15 $\mu$m region of the spectrum ($505\ \text{cm}^{-1} < \nu < 820\ \text{cm}^{-1}$). In the first of these we computed heating rates using transmission functions which were systematically multiplied first by 1.1 and then by 0.9. This should be an upper limit on the errors produced by systematic errors since we expect them to be much less than 10%. For the second set of computations we multiplied the transmission functions by a random number $\alpha_i$ where $i$ refers to the particular spectral interval and $\alpha_i$ is constrained to lie between 0.5 and 1.5. These limits on $\alpha_i$ may seem rather large since Smith claims an r.m.s. error of less than 1% for his fit (Smith, 1969). However examination of his results show that for particular values of pathlength and frequency the error may be as large as 30%. Three different sets of random numbers were used to produce three heating rate profiles.

The results of the two sets of computations are shown in Fig. 1. The profiles produced by the systematic variation in $\tau_r$ (Fig. 1a) are as expected in the lower troposphere with a larger $\tau_r$ increasing the cooling and vice versa. Similarly in the lower stratosphere the larger $\tau_r$ enhances the cooling to space. The reason for the reduced cooling with larger $\tau_r$ evident in the region from 400 mb to 200 mb is not obvious. An examination of the upwards and downwards fluxes shows that they behave systematically, i.e. $F^+$ is larger at all pressures and $F^-$ is smaller when $\tau_r$ is increased, and consequently the net flux also changes in a predictable manner. However, the rate of change in the net flux is not uniform. Because the system is non-linear it is difficult to evaluate the mechanism producing this non-uniform rate of change, but it is apparently related to exchange between layers with similar temperatures above and below the tropopause. The results of the random perturbations (Fig. 1b) are somewhat surprising in that the heating rate profiles for the three perturbed runs, while substantially different from the base case, all tend to resemble each other. Since the average $\alpha_i$ was 0.88 for the first set of random numbers, 1.10 for the second, and 1.03 for the third, we might expect a behaviour similar to that seen in Fig. 1a. An examination of the fluxes again shows that they behave as predicted, i.e. a larger average $\alpha_i$, which implies a larger average $\tau_r$, produces a larger $F^+$ and smaller $F^-$. The reason for the clustering of the three curves is not known. Presumably these problems can be resolved by a detailed dissection of the calculations but this is outside the scope of this study.

The curves presented in Fig. 1 raise several issues apart from the unexplained behaviour discussed above. First of all, it is apparent that the behaviour of the system is non-linear. The heating rates are not linearly related to the transmission function, particularly when random perturbations are applied to $\tau_r$. Secondly, these 10% changes in $\tau_r$ produce some disconcertingly large changes in the heating rates. Since the applied perturbations represent an upper bound on the likely errors, it is sufficient for the purpose of this study to say that our heating rate calculations are accurate to 0.1 K day$^{-1}$, assuming that the actual errors in the transmission functions are not larger than those introduced above. There can be little doubt that this

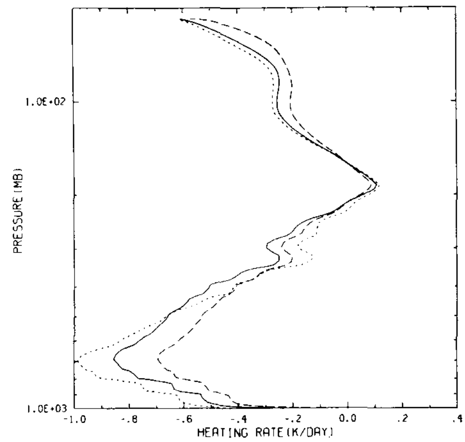


*Fig. 1(a).* Heating rate profiles computed for identical atmospheric conditions but for slightly modified transmission functions for the spectral region 505 cm$^{-1}$ to 820 cm$^{-1}$. Profiles shown are for 1.0 × $\tau_r$ (solid curve), 1.1 × $\tau_r$ (short dash), and 0.9 × $\tau_r$ (long dash).

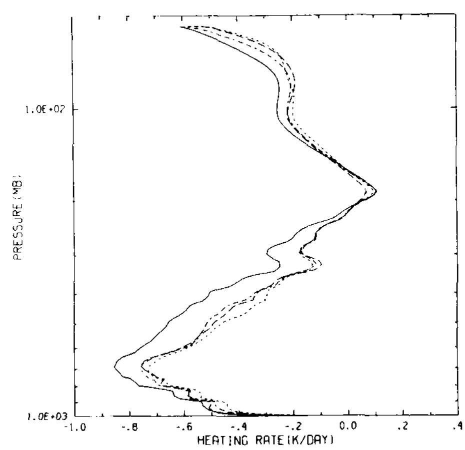


*Fig. 1(b).* Same as (a) except that the series of $\tau_r$ have been multiplied by a sequence of random numbers (see text for further explanation). Profiles shown are for 1.0 × $\tau_r$ (solid curve) and for three different sequences of random numbers (dashed curves).

entire subject of errors in transmission functions and the associated errors in heating rate calculations deserves considerably more study than it has received here.

An example of the transmittance spectra generated by Smith's method is given in Fig. 2a for $CO_2$ alone and in Fig. 2b for $CO_2$ and $H_2O$ overlapped. These spectra were computed assuming a path length of 1 atmosphere with uniformly mixed $CO_2$ at concentrations of 300 p.p.m., 600 p.p.m. and 150 p.p.m. For Fig. 2b the water vapour path length was assumed to be 29.3 kg $m^{-2}$ at a mean pressure of 827 mb and 285 K. These figures also illustrate the importance of treating the overlap region correctly when computing variations due to changing $CO_2$ concentrations. For the case of $CO_2$ alone doubling or halving the concentration changes the average absorption in the spectral region shown by 9%. For the overlap case a similar change in $CO_2$ amount changes the average absorption by only 1.5%. Since the absorption in this region is only part of the total absorption of longwave radiation it is evident that halving or doubling the $CO_2$ concentration changes the total longwave absorption by less than 1%. Relating this back to the errors discussed in the previous paragraph it can be shown that the changes due to $CO_2$ variation are the same order of magnitude or smaller than the possible variations due to errors in computing $\tau_r$. This is not to say that we cannot use this, or other, models to study $CO_2$ variations since presumably errors in the transmission functions are largely random and independent of the $CO_2$ concentration. It is worth noting however that the errors inherent in the model are large and that consequently the quantitative model results must be viewed with a certain reserve.

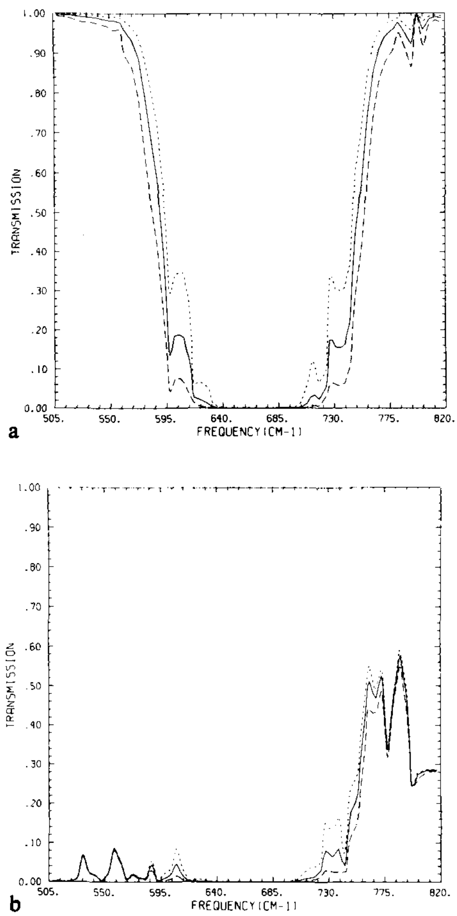


*Fig. 2.* Transmittance curves for $CO_2$ concentrations of 300 p.p.m. (solid curve), 600 p.p.m. (long dash), and 150 p.p.m. (short dash) for (a) $CO_2$ only and (b) $CO_2$ and $H_2O$ overlap. For $CO_2$, $\bar{T} = 255$ K and $\bar{p} = 507$ mb. For $H_2O$, $u = 29.34$ kg $m^{-2}$, $T = 285$ K and $\bar{p} = 827$ mb.

Each of the three expressions which we are using for $\tau_r$ assumes that pressure and temperature are uniform along the path $u$. This is in general not true and we have used the familiar Curtis–Godson approximation to compute average pressures and temperatures. While this is a good approximation for typical atmospheric $CO_2$ and $H_2O$ distributions it can give errors of a few per cent in transmittance calculations in the stratosphere (see Goody, 1964).

Having specified $\tau_r$ the flux equations were evaluated in the following manner. The atmosphere between the surface and 1 mb was divided into layers of equal thickness in pressure. In practice 75 layers were generally used. Values of temperature and water vapour density were specified at each level using a cubic spline under tension to interpolate from given atmospheric profiles. Optical paths between the layers and effective temperatures and pressures for these paths were computed, and the flux equations were evaluated at each layer interface. The integrals in the equations were computed by dividing each of them into two parts, the first consisting of the interval from $p_i$ (the level at which the flux was being evaluated) to $p_{i\pm 1}$, and the second consisting of the interval from $p_{i\pm 1}$ to $p_m$, $p_m$ being either the upper or lower boundary or an intermediate pressure at which the integrand became effectively zero. Each of these integrals was evaluated using a 32-point Gauss quadrature. The fluxes were then multiplied by the appropriate frequency interval width and summed. Heating rates were computed by adding $F^+$ and $F^-$ to give the net flux, fitting this curve with a series of splines, and differentiating the spline fit. Tests of this technique against analytically solvable cases and tests varying the number of layers and quadrature points showed that the fluxes were accurate to one part in a thousand and the heating rates were accurate to 0.01 K as far as the numerical procedures were concerned. This degree of accuracy is certainly sufficient from a numerical point of view since it is about an order of magnitude better than the accuracy that can be expected due to errors in $\tau_r$.

## 3. Comparison of model results

For the purpose of comparison both the HR and the MS codes were used to compute fluxes and heating rates for the longwave spectrum (200 cm$^{-1}$ $< \nu <$ 2200 cm$^{-1}$) excluding the contribution of ozone. Calculations were made using temperature and water vapour profiles representative of five atmospheric regimes: tropical, mid-latitude summer, mid-latitude winter, subarctic summer, and subarctic winter. These profiles were recommended for adoption as international standard models by the IAMAP Radiation Commission Working Group on a Standard Radiation Atmosphere (McClatchey et al., 1976). While using these models in the HR code we found that the temperature profiles must be modified slightly for work involving high resolution in the vertical. Each of them assumes a lapse rate of 6.5 K km$^{-1}$ in the middle troposphere but the temperatures (given at 1 km intervals) were rounded to whole degrees. This resulted in an oscillation in the $dT/dz$ profile which in turn created oscillations in the heating rate profiles. The problem was resolved by carrying an extra digit in the temperatures. Additional computations were made using the profiles given by Stone and Manabe (1968) in order to test the two codes against previous results. Only a representative sample of the calculations will be shown here. The MS code used in this study has the vertical level structure as specified in Manabe and Hunt (1968) rather than that specified by Stone and Manabe (1968).

The heating rate profiles for the standard and doubled $CO_2$ concentration for the mid-latitude summer atmosphere for both models are shown in Fig. 3a. Fig. 3b shows the difference in heating rates (doubled $CO_2$ case minus standard $CO_2$) for the two models. From these comparisons we see that, in spite of the quite different resolution, both spectrally and vertically in the two models, there is rather a good agreement between them in the

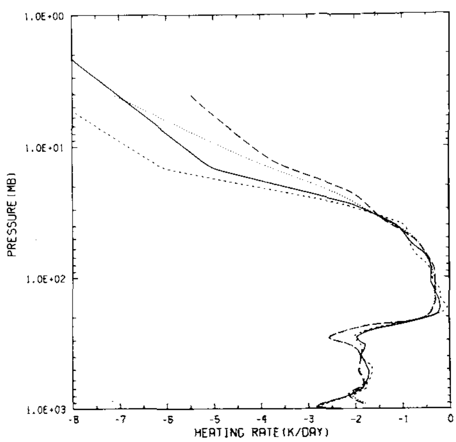


*Fig. 3(a).* Heating rate profiles computed with HR code for $CO_2$ concentrations of 300 p.p.m. (solid curve) and 600 p.p.m. (short dash) and profiles computed with MS code for $CO_2$ concentrations of 300 p.p.m. (long dash) and 600 p.p.m. (dotted) using mid-latitude summer profiles.

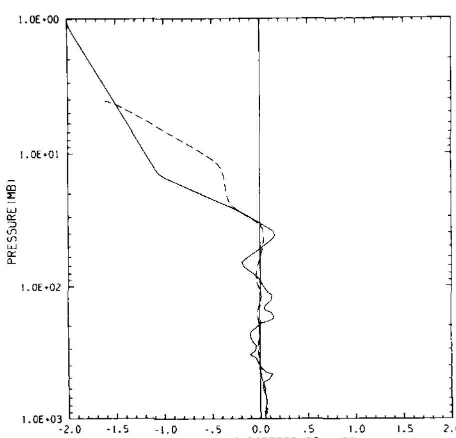


*Fig. 3(b).* Difference between heating rate profiles computed for $CO_2$ concentrations of 300 p.p.m. and 600 p.p.m. for HR code (solid curve) and MS code (long dash).

calculation of heating rates and also in the calculation of heating rate changes due to increased $CO_2$. The most obvious differences in the model results occur in the stratosphere. Due to the fact that both models use the Curtis–Godson approximation, which is considerably less accurate in the stratosphere than in the troposphere, both models have the largest errors in this region and consequently the largest uncertainties. It is interesting to note that the discrepancies in the heating rate differences are considerably less than those in the heating rates. Presumably the errors in the calculation are similar for both $CO_2$ cases which would then give us more confidence in the differences. However, it cannot be proved that this is the case and the discrepancies are well within the error bars of the models. Also the differences between the models are not consistent. The HR model computes larger cooling in the stratosphere than does the MS model for a mid-latitude summer atmosphere (Fig. 3a). On the other hand, for a subarctic winter profile the computed heating rates (Fig. 4) are quite similar to each other with the MS model producing slightly greater stratospheric cooling.

In addition to differences in the stratosphere there is a noticeable difference in heating rates in the lower troposphere in Fig. 3a. This is primarily due to the inclusion of "e-type" or dimer absorption of $H_2O$ in the HR model. This tropospheric difference is not evident in Fig. 4 since the very dry subarctic winter atmosphere is not influenced by the dimer absorption.

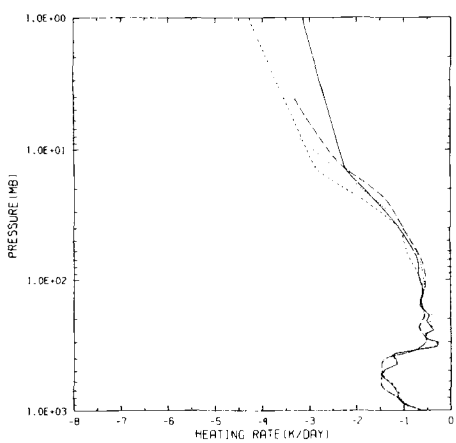


*Fig. 4.* Same as Fig. 3a except using subarctic winter rrofiles.

The heating rate differences in Fig. 3b also show a small change in the troposphere. This is due to absorption at the centre of the 15 $\mu$m $CO_2$ band. Although it is only a small decrease in the cooling it affects a comparatively large atmospheric mass. The significance of this change will be examined in the following section.

As might be expected the HR results in Figs. 3 and 4 exhibit a more detailed structure than those of the MS model since it has a considerably finer vertical resolution. This structure arises from the fact that the profiles are constructed by interpolating between data points which are given at 1 km intervals. Consequently the model actually has a finer resolution than the atmospheric data. This can lead to small turning points in the heating rate profile in regions where the temperature profile changes slope abruptly. Since these turning points are accurate computations of the heating rate for the given temperature profile, assuming that the data points represent actual temperatures at the specified height, the heating rate profiles were not smoothed.

An example of the actual upwards and downwards fluxes computed by the models is shown in Fig. 5. The HR model fluxes are slightly larger due to the dimer parameterization used in the window region. This larger flux has a small effect on flux differences within the atmosphere, as the previous

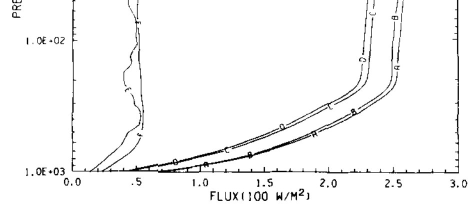


*Fig. 5.* Net fluxes and flux differences computed using mid-latitude summer profiles. Curve A = HR code, $CO_2$ concentration of 300 p.p.m.; B = HR code, 600 p.p.m.; C = MS code, 300 p.p.m.; D = MS code, 600 p.p.m.; E, F are the flux differences between A and B and C and D respectively. Note that both E and F have been multiplied by a factor of 10 for display purposes.

results indicated, and may have slight significance in terms of an overall radiative balance. The curves of difference in net flux (note that the abscissa values of these curves have been multiplied by 10) are very similar for the two models with again more structure in the HR model curve.

## 4. A radiative-convective calculation

The preceding results suggest that there are two regions of the atmosphere most affected by the change in $CO_2$ amount and which in turn most affect the surface equilibrium temperature. These regions are, of course, the stratosphere with its marked cooling, and the lower tropopause with its slight warming. In order to assess the relative importance of these two effects we ran a radiative-convective model using the MS code for three cases: a standard $CO_2$ (300 p.p.m.) case, a doubled $CO_2$ (600 p.p.m.) case, and a hybrid case in which we used the standard $CO_2$ heating rates below 100 mb and the doubled $CO_2$ heating rates above. This latter case removes the tropospheric heating due to $CO_2$ and allows the surface to respond to the stratospheric cooling only. It should be mentioned that the radiative-convective model used in this study is slightly different from that used by Manabe and Wetherald (1967) in that it assumes a moist adiabatic lapse rate adjustment rather than a constant 6.5 K/km, and also has a modified surface atmosphere coupling. The calculations were made assuming mean annual climatology at 35° latitude and a land surface.

The results of these calculations, plotted as differences in the equilibrium temperatures, are shown in Fig. 6. The effect of the tropospheric warming is immediately apparent. Without it the change in surface temperature is only 0.71 K as opposed to 1.82 K in the original case. Upper tropospheric temperatures are 3 to 4 K warmer than in the standard case when the lower tropospheric warming is included, but only 1 K warmer in the hybrid case. The rather large effect on the surface temperature of this small heating is due not only to radiative exchange processes but also to a reduction of the sensible heat flux because of the increased atmospheric temperature. Also this heating reflects a sizable energy input since it occurs in the densest part of the atmosphere.

There are two additional points of interest to note in Fig. 6. First of all, the $\Delta T_s$ for doubled $CO_2$

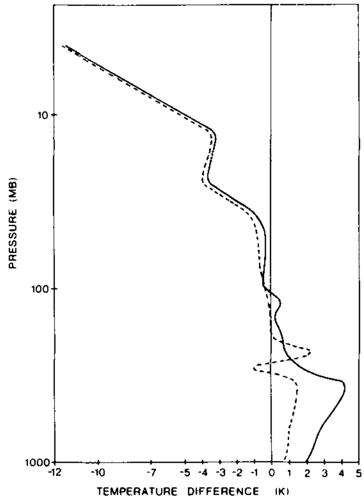


*Fig. 6.* Difference in equilibrium temperature profiles computed using radiative-convective model. Solid curve is doubled $CO_2$ (600 p.p.m.) minus normal $CO_2$ (300 p.p.m.) and dashed curve is doubled $CO_2$ above 100 mb only minus normal $CO_2$.

minus single $CO_2$ is 1.82 K here as against the $\Delta T_s$ = 2.36 quoted by Manabe and Wetherald (1967). The difference in these values is due to the different lapse rates used in the convective adjustment schemes and to the different surface–atmospheric couplings. This difference is also a further indication of the sensitivity of the problem to model assumptions. Secondly, the oscillation in the difference curve for the hybrid case is a response to the truncation of the $CO_2$ cooling. Of course since this is a difference curve, the oscillation is not evident in the temperature curves themselves, which are smoothly varying functions of height, but appears when the temperatures are subtracted.

It was also instructive to perform a calculation with the radiative-convective model similar to that described in Section 2 and shown in Fig. 1a. In this case we multiplied the $CO_2$ emissivities in the model by 0.9 and 1.1 and considered the change in the surface temperature for single and doubled $CO_2$ amounts. The results are not unexpected in that, although multiplying the emissivities by 1.1 produces a surface temperature ($T_s$ = 295.02) higher than that of the control ($T_s$ = 293.55), the $\Delta T_s$ for the two cases is nearly the same ($\Delta T_s$ = 1.93 for the larger emissivities). This implies that if the errors in $\tau_r$ are purely systematic, their effect on the calculation of $\Delta T_s$ is minimal. However, if the errors have a dominant random component, it is not clear that the effect on $T_s$ will be the same for all $CO_2$ concentrations.

## 5. Conclusions

The results described in the sections above can be summarized in four main points. Firstly there is good agreement between the HR and MS codes in the computation of heating rates. The HR code produces more detail as expected but the curves generally exhibit the same behaviour below 50 mb. Above 50 mb there is some divergence in the values of the cooling predicted by the two models. While it is difficult to determine the absolute accuracy of either of the models it is reasonable to assume that the HR model is the more accurate due to better resolution. This would apply particularly in the troposphere since both models have substantial errors in the stratosphere. The fluxes computed by the HR code are slightly larger due to dimer absorption.

Secondly, doubling the $CO_2$ concentration produces similar results in both models. A small decrease in the cooling rate of less than 0.1 K $day^{-1}$ is observed in the lower troposphere and a large increase of as much as 2.0 K $day^{-1}$ is observed in the stratosphere. The increases in surface temperature computed in doubled $CO_2$ experiments are due to both these effects with the tropospheric warming dominating to a slight degree.

Thirdly, in view of the results produced by modifying the transmission functions with constant or random multipliers, it is apparent that the absolute accuracy of the HR model in computing heating rates is about 0.1 K $day^{-1}$. It is extremely unlikely that the errors in the MS model are smaller and in all probability they are larger.

And fourthly, in comparing the estimated accuracy of the calculations with the difference in heating rates for the two $CO_2$ concentrations, it is apparent that the difference, particularly in the troposphere, is on the order of or less than the absolute accuracy of the models. As was mentioned before, it can be argued that the accuracy of the difference is not dependent on the absolute accuracy of the model but rather on the relative accuracy. This point was demonstrated before with the radiative-convective calculations which used uniformly changed emissivities, and is true to the extent that model errors for a given set of atmospheric conditions are systematic and produce errors in the heating rate of a similar sign for both $CO_2$ concentrations. Unfortunately, it cannot be shown that this latter statement is indeed true. Because the measurement and fitting errors are to a large degree random, it is quite possible that the model errors are not systematic and that, consequently, the heating rates have a random error component which significantly affects the values of the heating rate differences.

Taken together, these points suggest that the quantitative results of models used to study the effects of changing $CO_2$ concentrations should be treated with considerable care. Although the figure of about a 2 K change in surface temperature for a doubled $CO_2$ concentration has been produced by several models (most recently by Augustsson and Ramanathan, 1977, using a radiative-convective model), these models all have similar accuracy problems to those described here. Because it is very difficult to estimate the magnitude of these errors it is equally difficult to put error limits on the 2 K figure. Consequently, while we can have considerable faith in the qualitative results of the model we should remain cautious about the quantitative results. It is indeed unfortunate that so much uncertainty is evident in what is a reasonably straightforward theoretical problem, particularly since it implies that similar errors must be present in many of the studies of climate response. This is not to suggest that these model studies are of little value, which is obviously not true, but rather to point out that a great deal of work remains to be done, particularly in the area of radiative transfer, before we can rely on the quantitative results of climatic models.

## 6. Acknowledgements

I am indebted to Mr B. G. Hunt for supplying the version of the Manabe–Strickler radiation code and the radiative-convective model which was used in this study, and to Dr G. Stevens for several enlightening discussions concerning the material presented here. I am also grateful for the helpful comments of both of them in reviewing an earlier version of this paper.

## REFERENCES

Augustsson, T. and Ramanathan, V. 1977. A radiative-convective model study of the $CO_2$ climate problem. *J. Atmos. Sci. 34*, 448–451.

Bignell, K. J. 1970. The water-vapour infra-red continuum. *Quart. J. R. Met. Soc. 96*, 390–404.

Goody, R. M. 1964. *Atmospheric radiation, I. Theoretical basis.* London: Oxford University Press.

Manabe, S. and Hunt, B. G. 1968. Experiments with a stratospheric general circulation model: I Radiative and dynamic aspects. *Mon. Wea. Rev. 96*, 477–539.

Manabe, S. and Strickler, R. F. 1964. Thermal equilibrium of the atmosphere with a convective adjustment. *J. Atmos. Sci. 21*, 361–385.

Manabe, S. and Wetherald, R. T. 1967. Thermal equilibrium of the atmosphere with a given distribution of relative humidity. *J. Atmos. Sci. 24*, 241–259.

Manabe, S. and Wetherald, R. T. 1975. The effects of doubling the $CO_2$ concentration on the climate of a general circulation model. *J. Atmos. Sci. 32*, 3–15.

McClatchey, R. A., Bolle, H. J. and Kondratyev, K. Ya. 1976. Report of the IAMAP Radiation Commission Working Group on a Standard Radiation Atmosphere. Presented at: Symposium on Radiation in the Atmosphere, Garmisch-Partenkirchen, Federal Republic of Germany.

Ramanathan, V. 1976. Radiative transfer within the earth's troposphere and stratosphere: A simplified radiative-convective model. *J. Atmos. Sci. 33*, 1330–1346.

Rodgers, C. D. and Walshaw, C. D. 1966. The computation of infra-red cooling rate in planetary atmospheres. *Quart. J. R. Met. Soc. 92*, 67–92.

Schneider, S. H. 1975. On the carbon dioxide—climate confusion. *J. Atmos. Sci. 32*, 2060–2066.

Smith, W. 1969. A polynomial representation of carbon dioxide and water vapour transmission. ESSA Technical Report NESC 47, National Environmental Satellite Center, Washington, D.C.

Stone, H. M. and Manabe, S. 1968. Comparison among various numerical models designed for computing infrared cooling. *Mon. Wea. Rev. 96*, 735–741.

## О ВЛИЯНИИ $CO_2$ НА СКОРОСТИ НАГРЕВА АТМОСФЕРЫ

Развита модель переноса длинноволновой радиации с высоким разрешением для исследования влияния увеличения концентраций $CO_2$ на длинноволновые потоки и скорости нагрева. Для оценки функций пропускания в полосе $CO_2$ 15 мкм с допущением частичного перекрывания полосами $H_2O$ используется высокое спектральное разрешение. Сравнение результатов этой модели и результатов модели Манабе и Везеролда показывает, что модель Манабе-Везеролда достаточно точная и что обе модели дают аналогичные измерения и скорости нагрева для удвоенных концентраций $CO_2$. Эти изменения достаточно малы (<0.1 °K/демь) в тропосфере и одного порядка величины или меньше, чем оцененная точность моделей. Большие различия замечены в стратосфере, но модельные ошибки такие больше в этой области. С использованием радиационно-конвективной модели было найдено, что малые изменения в тропосферной скорости нагрева ответственны за более чем половину рассчитанного изменения температуры поверхности.