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Linear parameterization of large-scale eddy transports

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ABSTRACT

A method for the parameterization of the large-scale meridional transports of heat and vorticity is proposed which rests entirely upon data and does not invoke theories of large-scale turbulence. This method is tested for the case of a two-dimensional (zonally averaged) quasi-geostrophic model (M2). The necessary data are gained from the three-dimensional Lorenz model (M3). The parameterization is a linear one and the coefficients of the parameterization are determined so that the results of M2 come as close as possible to the results of M3. It turns out that M2 simulates the behaviour of M3 with great accuracy as long as the heat forcing does not depend on time. Once we introduce a variation of the heating in a rough similarity to the annual variation of the solar heating, our parameterization scheme behaves less satisfactorily. It then exhibits about the same quality as parameterization methods based on the Austausch concept.

1. Introduction

It has been recognized that progress in the field of climate modelling on time scales of decades and more is closely linked to our ability to develop simplified models as compared to general circulation models (GCM) which can hardly be run over such time spans (e.g. GARP Publication Series, 1975). There have been several attempts to build such models. In most cases one has tried to achieve the required simplicity and economic structure by some kind of averaging of the basic equations of atmospheric motion. In turn the closure problem comes up as the price to be paid for this procedure. Several proposals have been made how to solve this problem. These range from more or less elaborate applications of the Austausch concept (e.g. Wiin-Nielsen and Fuenzalida, 1975) to attempts which invoke the baroclinic stability theory (e.g. Stone, 1972; Saltzman and Vernekar, 1971) in order to parameterize the sought eddy transports in terms of average quantities. Experiments with parameterizations gave encouraging results but it is generally recognized that we are far from a satisfactory solution of the problem. In particular, we have to be aware that the theoretical foundations of these parameterizations are somewhat shaky. For example, the Austausch concept rests on a rather hypothetical analogy of large-scale turbulence and molecular processes. The baroclinic wave theory is essentially linear and has not much to say when fully developed nonlinear turbulence has to be dealt with. Under such circumstances one might be well advised to take a more pragmatic point of view. We know a lot about the large-scale turbulence of the atmosphere and are able to learn more by data analysis. On the other hand we do not have a reliable theory of these turbulent flows. Should we then not try to develop a parameterization of the transport properties of this kind of turbulence which rests entirely on observations and not at all on theoretical reasoning? In other words, the parameterization should be derived from the data. This idea will be worked out and tested in this paper.

Since this approach appears to be novel we should solve a somewhat simpler problem as a first step. We shall use the well-known Lorenz model as a three-dimensional analogue of the atmosphere. Model data obtained from this model are used to arrive at parameterizations for a two-dimensional (zonally averaged) simplified model.

The basic features and the "climate" of the three-dimensional model will be outlined in Section 2. The zonally averaged model and its parameterizations are described in Section 3. Results

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obtained with this model are displayed in Section 4 as well as a discussion and presentation of difficulties inherent to our approach.

2. The three-dimensional model atmosphere

As mentioned above we shall not try to build a simplified model of the atmosphere. Instead we shall replace the atmosphere by a three-dimensional model which will be simulated by a zonally averaged model. The three-dimensional model yields data. Based on these data a parameterization of the northward fluxes of heat and vorticity will be derived which will be built into the zonally averaged model. This way we have all the data we need and are able to assess the skill of our simplified model with considerable accuracy. An extension of our technique to atmospheric data and to models of the atmosphere is not considered here.

The three-dimensional model (M3) is based on the vorticity equation and the first law of thermodynamics in the two-level formulation as used by Lorenz (1960):

$$\frac{\partial}{\partial t}\nabla^2\psi = -J(\psi, \nabla^2\psi) - \frac{1}{B^2}J(\theta, \nabla^2\theta) - k\left(\nabla^2\psi - \frac{1}{B}\nabla^2\theta\right) \tag{2.1}$$

$$\frac{\partial}{\partial t}\left(\frac{1}{B}\nabla^2 - \frac{f_0}{\sigma_0}\right)\theta = -\frac{1}{B}J(\psi, \nabla^2\theta) - \frac{1}{B}J(\theta, \nabla^2\psi) - \frac{f_0}{\sigma_0}J(\psi, \theta) + \frac{f_0 h}{\sigma_0}(\theta - \theta^*) + k\nabla^2\psi - (k - 2k')\frac{1}{B}\nabla^2\theta \tag{2.2}$$

where ψ is the mean streamfunction of the fluid, θ the potential temperature, σ_0 the static stability, f_0 the constant coriolisparameter and B is a factor which relates the streamfunction of the vertical wind shear (τ) to θ:

$$\nabla^2\theta = B\nabla^2\tau \tag{2.3}$$

k' and $2k$ are the coefficients of internal friction and surface skin friction, respectively. θ^* is a prescribed enforced temperature and h regulates the time scale of the thermal forcing. (2.1), (2.2) is identical to the model equations used by Lorenz (1960) with the only exception that Lorenz employed an additional prognostic equation for σ_0. σ_0 is constant here. The flow described by (2.1), (2.2) is supposed to be restricted to a channel with walls at $y = 0$ and $y = \pi L$. There is periodicity in x-direction with $2\pi L$ as length of the channel.

Following Lorenz (1963) we expand ψ, θ and θ^* into truncated double Fourier series, e.g.

$$\psi = L^2 f_0 \sum_{i=1}^{M+2MN} \psi_i \phi_i \tag{2.4}$$

where the functions ϕ_i form an orthogonal set and satisfy the boundary conditions. M is the number of waves enclosed in the model for the meridional direction, N the number of waves for the zonal direction. For $i \leqslant M$, ϕ_i does not depend on x. These functions ($\sim\cos\, ny/L$) must be used to describe the zonally averaged state of the flow whereas all ϕ_i with $i > M$ are harmonic in x as well as in y. The ψ_i are the non-dimensional Fourier coefficients of ψ. Inserting (2.4) and the corresponding expansions for θ and θ^* into (2.1), (2.2) we arrive at a system of ordinary differential equations of first order in ψ_i, θ_i.

The various types of circulation exhibited by M3 have been studied extensively (Metz, 1976; Hartjenstein, 1977). Of course, the model's behaviour depends on the choice of parameters as well as on the truncation of the Fourier series. M3 possesses axisymmetric flow types, Rossby circulations, vacillating and irregular circulations. This last type of flow is the most interesting for us since it is only for such "turbulent" flow that a parameterization is called for. As an example we show the behaviour of the mean flow in an experiment with $M = 6$ and $N = 6$. All the Fourier coefficients displayed exhibit an irregular behaviour (Fig. 1). Fig. 2 shows a typical wave pattern.

With that our problem can be stated fairly precisely: Can we develop a zonally averaged model (M2) which is able to stimulate the behaviour of the zonal mean state of M3? In particular, can we reproduce the essential features of Fig. 1 by aid of M2?

3. The zonally averaged model

The equations for the mean flow of M3 can be

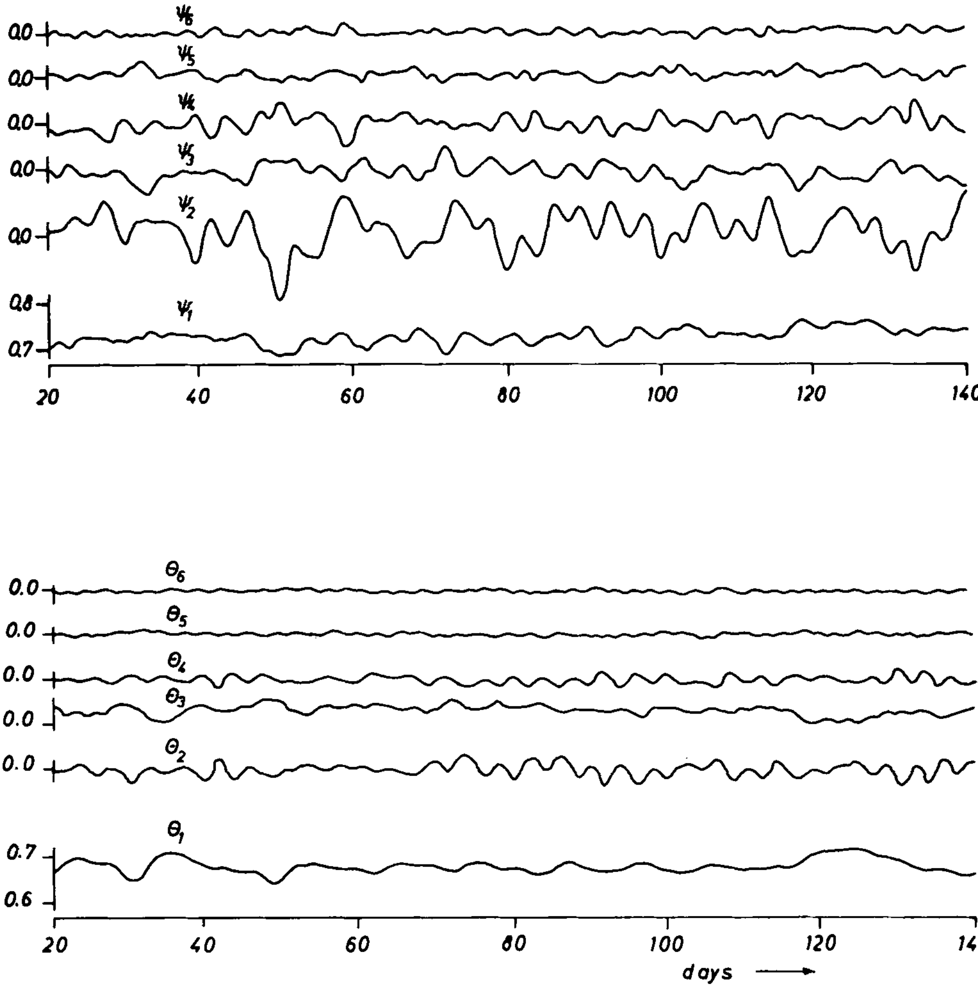


Fig. 1. The Fourier coefficients of the mean flow during a period of 120 days as obtained from M3 with constant θ^*. $M = 6, N = 6$. Relative units.

written in a symbolic form

$$\dot{\psi}_i = A_i^\psi(\psi_j, \theta_k; j, k > M) - k(\psi_i - \theta_i)$$
$$\dot{\theta}_i = A_i^\theta(\psi_j, \theta_k; j, k > M) + X_i\,\psi_i - Y_i\,\theta_i + Z_i\,\theta_i^* \qquad (3.1)$$
$$i \leqslant M$$

Here all variables and constants are non-dimensional, X_i, Y_i, Z_i are constants which depend only on the choice of the orthogonal functions.

The mean streamfunction and the mean temperature of the zonal flow are influenced by interaction of waves which create a zonally averaged meridional transport of heat and vorticity (described by the nonlinear terms A_i^ψ, A_i^θ). Also frictional terms and the heating come into play. The terms A_i^ψ and A_i^θ involve Fourier coefficients of modes with $j, k > M$ only. It is these terms which ought to be parameterized in our zonally averaged model M2.

We propose the most general linear parameterization. Hence, we have

$$A_i^\psi = \alpha_i^\psi + \sum_{j=1}^{M} (\beta_{ij}^\psi\,\psi_i + \gamma_{ij}^\psi\,\theta_j)$$
$$A_i^\theta = \alpha_i^\theta + \sum_{j=1}^{M} (\beta_{ij}^\theta\,\psi_j + \gamma_{ij}^\theta\,\theta_j) \qquad (3.2)$$

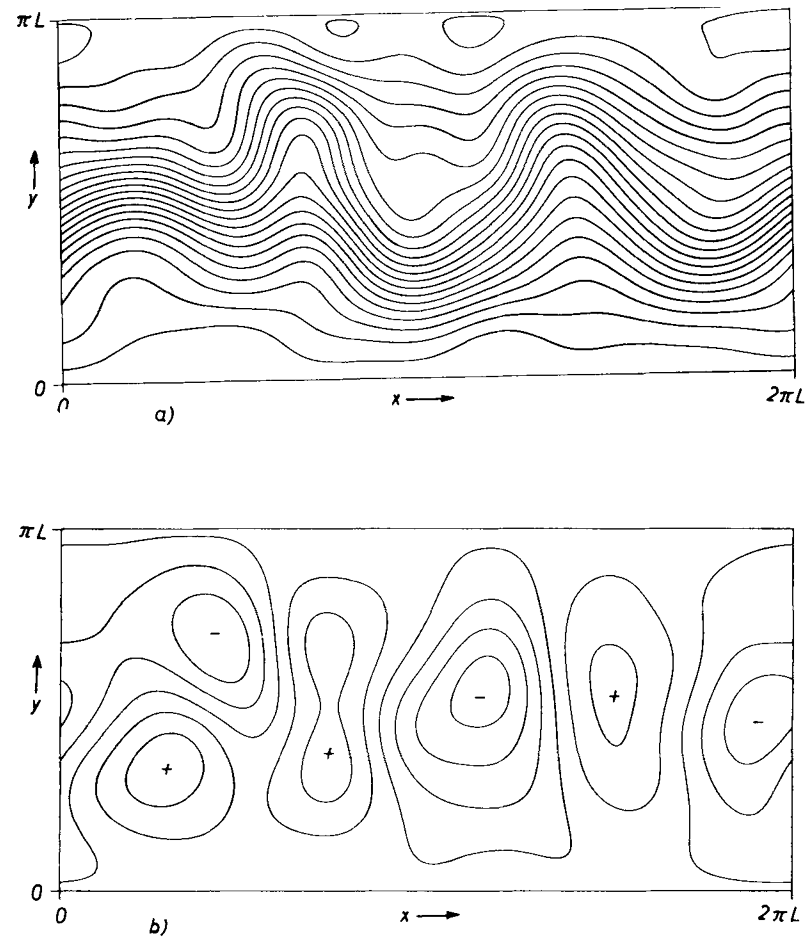


Fig. 2. The streamfunction in the upper (a) and lower (b) layer at $t = 60$ days.

where the coefficients α_i^ψ, α_i^θ, β_{ij}^ψ, β_{ij}^θ, γ_{ij}^ψ, γ_{ij}^θ may depend on time and on the parameters of the model, but not on ψ_j, θ_j. Let us, however, assume first that these coefficients are constant in time. Inserting (3.2) into (3.1) we obtain a linear system of prognostic equations for ψ_i, θ_i, $i \leqslant M$:

$$\begin{aligned} \dot{\psi}_i &= \alpha_i^\psi + \sum_{j=1}^{M} (\beta_{ij}^\psi \psi_j + \gamma_{ij}^\psi \theta_j) - k(\psi_i - \theta_i) \\ \dot{\theta}_i &= \alpha_i^\theta + \sum_{j=1}^{M} (\beta_{ij}^\theta \psi_j + \gamma_{ij}^\theta \theta_j) + X_i \psi_i - Y_i \theta_i \\ &\quad + Z_i \theta_i^* \end{aligned} \tag{3.3}$$

This is the system of equations of M2.

How are we to choose the coefficients α_i^ψ, α_i^θ, β_{ij}^ψ, β_{ij}^θ, γ_{ij}^ψ, γ_{ij}^θ in order to achieve an optimum simulation of M3?

(3.3) is a set of linear inhomogeneous differential equations with constant coefficients. The general solution of such a system of equations is well known. Given the various constants of (3.3) we can determine the eigenvalues. Given an initial state it is no problem to compute the solution of (3.3) at any time.

We may invert this statement: Given the solution of an unknown linear system of equations of the same type as (3.3) we can determine the constants α_i^ψ, α_i^θ, β_{ij}^ψ, β_{ij}^θ, γ_{ij}^ψ, γ_{ij}^θ so that (3.3) reproduces the given solution. We propose to solve the parameterization problem in accord with these considerations. Let us select from given time series of ψ_i, θ_i $(i \leqslant M)$ of M3 (as shown in Fig. 1, for example) main frequencies and the amplitudes which belong to these frequencies. If the number of these frequencies is chosen properly we can determine the parameterization coefficients of (3.3) in such a way that the solution of M2 yields just the selected main frequencies and amplitudes as found in M3.

Let us repeat this idea in a more mathematical form: The solution W_i, $i \leqslant 2M$ of a homogeneous system of $2M$ ordinary differential equations can be written

$$W_i = \sum_{k=1}^{2M} A_{ik}\, e^{\lambda_k t} \tag{3.4}$$

where λ_k are the eigenvalues of the system. The solution of an inhomogeneous system with constant forcing is obtained by adding a constant D_i on the right-hand side of (3.4). Let us suppose that all eigenvalues with $k \leqslant k_1$ are real, those with $k > k_1$ are complex:

$$\lambda_k = \lambda_k^{(r)} + i\lambda_k^{(i)}.$$

Then

$$W_i = D_i + \sum_{k=1}^{k_1} A_{ik}\, e^{\lambda_k t} + \sum_{l=1}^{(2M-k_1)/2} e^{\lambda_l^{(r)} t} \times (B_{il} \cos \lambda_l^{(i)} t + C_{il} \sin \lambda_l^{(i)} t) \tag{3.5}$$

Now, negative $\lambda_l^{(r)}$ correspond to damped motions, positive $\lambda_l^{(r)}$ to exponential growth. The oscillations of M3 show an undamped behaviour. Hence, we pose $\lambda_l^{(r)} = 0$ in (3.5). In addition, there is no need for real eigenvalues (as seen in Fig. 1) so that $k_1 = 0$. Therefore

$$W_i = D_i + \sum_{l=1}^{M} (B_{il} \cos \lambda_l^{(i)} t + C_{il} \sin \lambda_l^{(i)} t) \tag{3.6}$$

is the type of solution we want to simulate. Given a certain run of M3 we have $\psi_i = W_i$ $(i \leqslant M)$; $\theta_i = W_i$ $(M < i \leqslant 2M)$. Hence, the D_i are the time mean values of ψ_i, θ_i, whereas the $\lambda_l^{(i)}$ are the main frequencies to be chosen from the time series of M3 and the B_{il}, C_{il} are the corresponding amplitudes. Hence, all the constants in (3.6) can be determined from M3. The solution of our special system may be written

$$\begin{aligned} \psi_i &= \bar{\psi}_i + \sum_{l=1}^{M} (\psi_{il}^c \cos \nu_l t + \psi_{il}^s \sin \nu_l t) \\ \theta_i &= \bar{\theta}_i + \sum_{l=1}^{M} (\theta_{il}^c \cos \nu_l t + \theta_{il}^s \sin \nu_l t) \end{aligned} \tag{3.7}$$

Inserting (3.7) into (3.3) and separating the various Fourier modes in time we arrive at a system of $2M(2M + 1)$ equations (the summation rule for double subscripts will be used here)

$$\left.\begin{aligned} &\alpha_i^\psi + \beta_{ij}^\psi \bar{\psi}_j + \gamma_{ij}^\psi \bar{\theta}_j - k(\bar{\psi}_i - \bar{\theta}_i) = 0 \\ &\alpha_i^\theta + \beta_{ij}^\theta \bar{\psi}_j + \gamma_{ij}^\theta \bar{\theta}_j + X_i \bar{\psi}_i - Y_i \bar{\theta}_i + Z_i \theta_i^* = 0 \end{aligned}\right\} \quad (3.8a)$$

$$\left.\begin{aligned} &\nu_1 \psi_{i1}^c + \beta_{ij}^\psi \psi_{j1}^s + \gamma_{ij}^\psi \theta_{j1}^s - k(\psi_{i1}^s - \theta_{i1}^s) = 0 \\ &\nu_1 \theta_{i1}^c + \beta_{ij}^\theta \psi_{j1}^s + \gamma_{ij}^\theta \theta_{j1}^s + X_i \psi_{i1}^s - Y_i \theta_{i1}^s = 0 \\ &-\nu_1 \psi_{i1}^s + \beta_{ij}^\psi \psi_{j1}^c + \gamma_{ij}^\psi \theta_{j1}^c - k(\psi_{i1}^c - \theta_{i1}^c) = 0 \\ &-\nu_1 \theta_{i1}^s + \beta_{ij}^\theta \psi_{j1}^c + \gamma_{ij}^\theta \theta_{j1}^c + X_i \psi_{i1}^c - Y_i \theta_{i1}^c = 0 \\ &\qquad \vdots \qquad\qquad \vdots \qquad\qquad \vdots \\ &\nu_M \psi_{iM}^c + \beta_{ij}^\psi \psi_{jM}^s + \gamma_{ij}^\psi \theta_{jM}^s - k(\psi_{iM}^s - \theta_{iM}^s) = 0 \\ &\nu_M \theta_{iM}^c + \beta_{ij}^\theta \psi_{jM}^s + \gamma_{ij}^\theta \theta_{jM}^s + X_i \psi_{iM}^s - Y_i \theta_{iM}^s = 0 \\ &-\nu_M \psi_{iM}^s + \beta_{ij}^\psi \psi_{jM}^c + \gamma_{ij}^\psi \theta_{jM}^c - k(\psi_{iM}^c - \theta_{iM}^c) = 0 \\ &-\nu_M \theta_{iM}^s + \beta_{ij}^\theta \psi_{jM}^c + \gamma_{ij}^\theta \theta_{jM}^c + X_i \psi_{iM}^c - Y_i \theta_{iM}^c = 0 \end{aligned}\right\} \quad (3.8b)$$

(3.8) can be viewed as a linear system of equations for the $2M(2M + 1)$ coefficients α_i^ψ, α_i^θ, β_{ij}^ψ, β_{ij}^θ, γ_{ij}^ψ, γ_{ij}^θ. (3.8) can be solved by routine methods and the coefficients can be determined and inserted into (3.3). With that, M2 is a model which ought to have a considerable degree of similarity to M3.

There is a limit for the number of frequencies ν_l which can be chosen to determine the parameterization coefficients. From (3.7) we see that the limit must be M because the number of equations (3.8) should be equal to the number of parameterization coefficients, namely $2M(2M + 1)$.

Now one can understand the reason why we chose as many parameterization coefficients as possible in (3.2): the more coefficients, the better will be the similarity between M2 and M3.

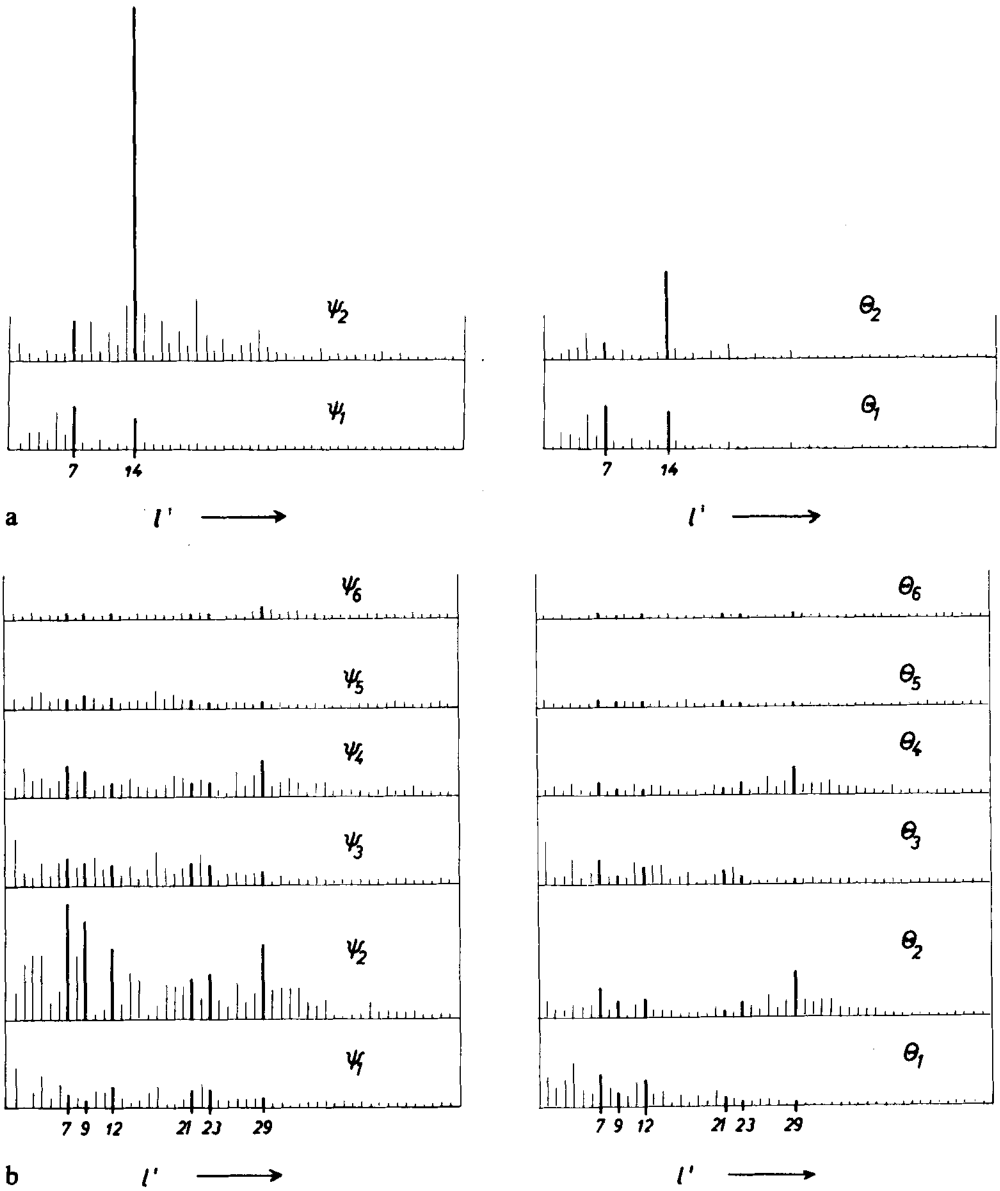


Fig. 3. The time spectra of the Fourier coefficients of the zonal mean flow. l' stands for the frequency of the oscillations, where $l' = 1$ indicates the wave with a period of 120 days. (a) Low resolution $M = 2$, $N = 4$; (b) high resolution $M = 6$, $N = 6$.

A few words should be said about the problem of how to get the various constants in (3.7) from M3. It is obvious how to extract the time averages $\bar{\psi}_i$ and $\bar{\theta}_i$ from the series of M3. The determination of the main frequencies is less clear. As an example we show the spectra of the six Fourier coefficients of the mean streamfunction and the temperature as derived from Fig. 1 through Fourier analysis (Fig. 3). It appears that there is a group of important frequencies near $l' = 7$ and another near $l' = 21$ and $l' = 29$. In this case we have chosen by inspection to elect $l' = 7, 9, 12, 21, 23, 39$ as main frequencies. It is obvious that a choice of another set of frequencies will lead to somewhat different results. Hence, an objective method to select the main frequencies is desirable but has not been developed so far, since there is quite a number of possibilities how to do that. The decision about which one to take is again subjective. On the other hand such a method should be defined when the model is applied to a large number of cases.

Once the frequencies are chosen, it is a simple matter to get the corresponding amplitudes.

4. Results and discussion

Fig. 4 shows the comparison between a low resolution run of M3 ($M = 2$, $N = 4$) and the corresponding run of the simplified model. The corresponding results of a high resolution run ($M = 6$, $N = 6$) are displayed in Fig. 5. The simulation is nearly perfect in the low resolution case. There are certain slight deficiencies. However, the overall agreement is fairly good. Note in particular that M2 is able to simulate almost exactly the mean values of the variables as well as the amplitudes of the various oscillations. The computer time reduces to 10% of the time used by M3 in the low resolution case and to about 1% in the high resolution case. It is not surprising that the simulation of the high resolution case is not as good as that of the low resolution case, for the latter has two really outstanding frequencies (Fig. 3a) whereas the former has a broad spectrum which cannot be portrayed with sufficiently high confidence when only six frequencies are free for choice. Nevertheless, one may state that the simulations are rather good. Strictly speaking, we were entitled to expect this since the parameterizations have been chosen to give an "optimum" result.

If we could construct a model of similar quality for the zonally averaged state of the atmosphere we would have a quite valuable research tool since this model would not only reproduce the time mean of the zonally averaged state of the atmosphere but also the characteristic oscillations of the mean state. This would be of considerable interest when such a model could be coupled to an ocean model which would in turn respond to the variations of the drag and heat flux at the surface.

There is, however, a main problem which has to be overcome before we may hope to develop such a model. So far we have assumed that the forcing θ^* does not depend on time. θ^* stands for some kind of differential heating. In particular, if our model M3 would be a GCM θ^* would vary according to the annual cycle. M3 has been run with

$$\theta^* = \bar{\theta}^* + \theta^{*\prime} \sin 2\pi t/T$$

with $\theta^{*\prime}/\bar{\theta}^* = 0.11$ and $T = 120$ days. We determined the parameterization coefficients as appropriate for $\theta^* = \bar{\theta}^*$. It turned out, however, that the corresponding variation of ψ_l, θ_l in M2 from "summer" to "winter" was by far too great (Fig. 6).

There is a certain remedy for this when we allow for a variation of the α_l^φ, α_l^θ follwing the variation of θ^*. It can be seen from eqs. (3.8a) that the vari-

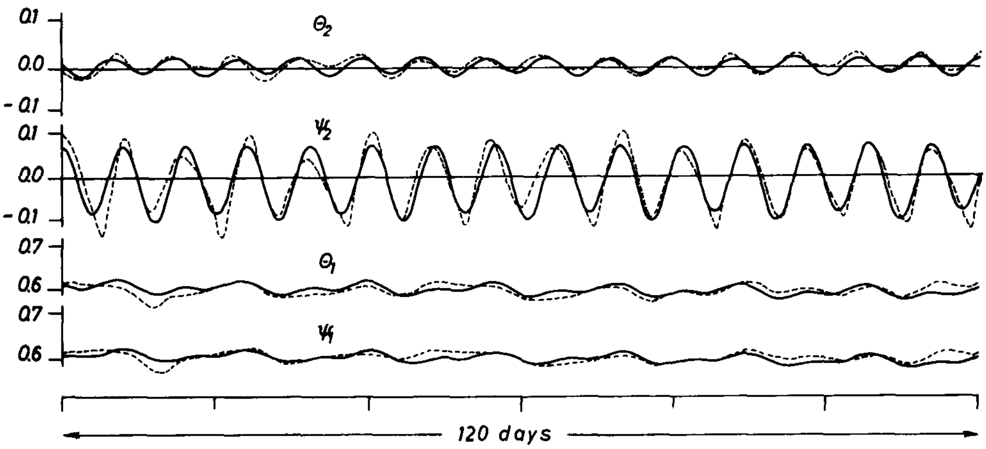


Fig. 4. Fourier coefficients of the mean flow as obtained from M2 (solid curve) and from M3 (dashed curve) with constant θ^*. $M = 2$, $N = 4$. Relative units.

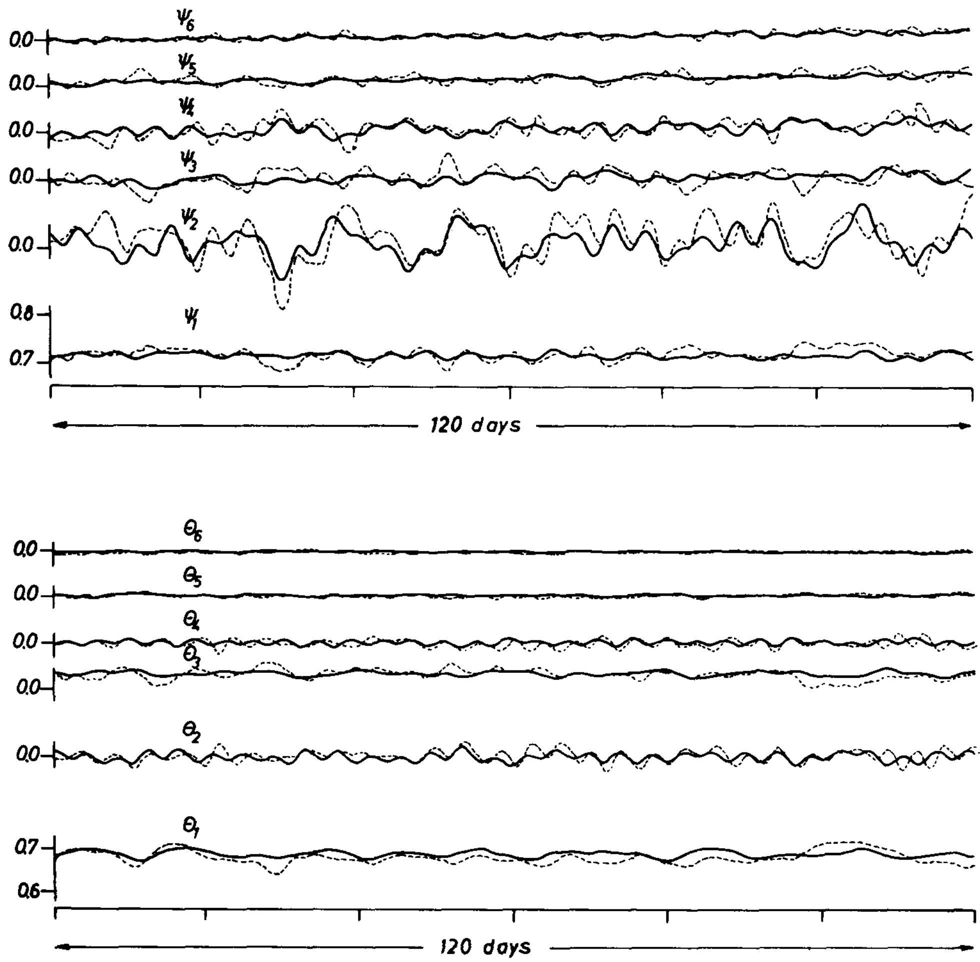


Fig. 5. Same as Fig. 4, except: $M = 6$, $N = 6$.

ables α_i^ψ, α_i^θ should be able to satisfy these equations for a sinusoidal oscillation of $\bar{\psi}_i$, $\bar{\theta}_i$, θ^* where the bar now means a time average with an averaging period $\ll T$. We have taken the amplitudes of these "annual" oscillations from M3 and repeated the above experiment.

We see from Fig. 7 that M2 is again able to simulate fairly well the "annual mean" variation of the various variables. However, M2 fails to simulate the increase of the most characteristic frequency with θ^* as well as the diminishing of the short-term amplitudes for low θ^*. This is not surprising since the β_{ij}^ψ, β_{ij}^θ, γ_{ij}^ψ, γ_{ij}^θ have been kept constant. Hence the model has to fail when such phenomena occur (variations of typical frequencies and amplitudes of the short-term oscillations). This is an expected but still disappointing outcome.

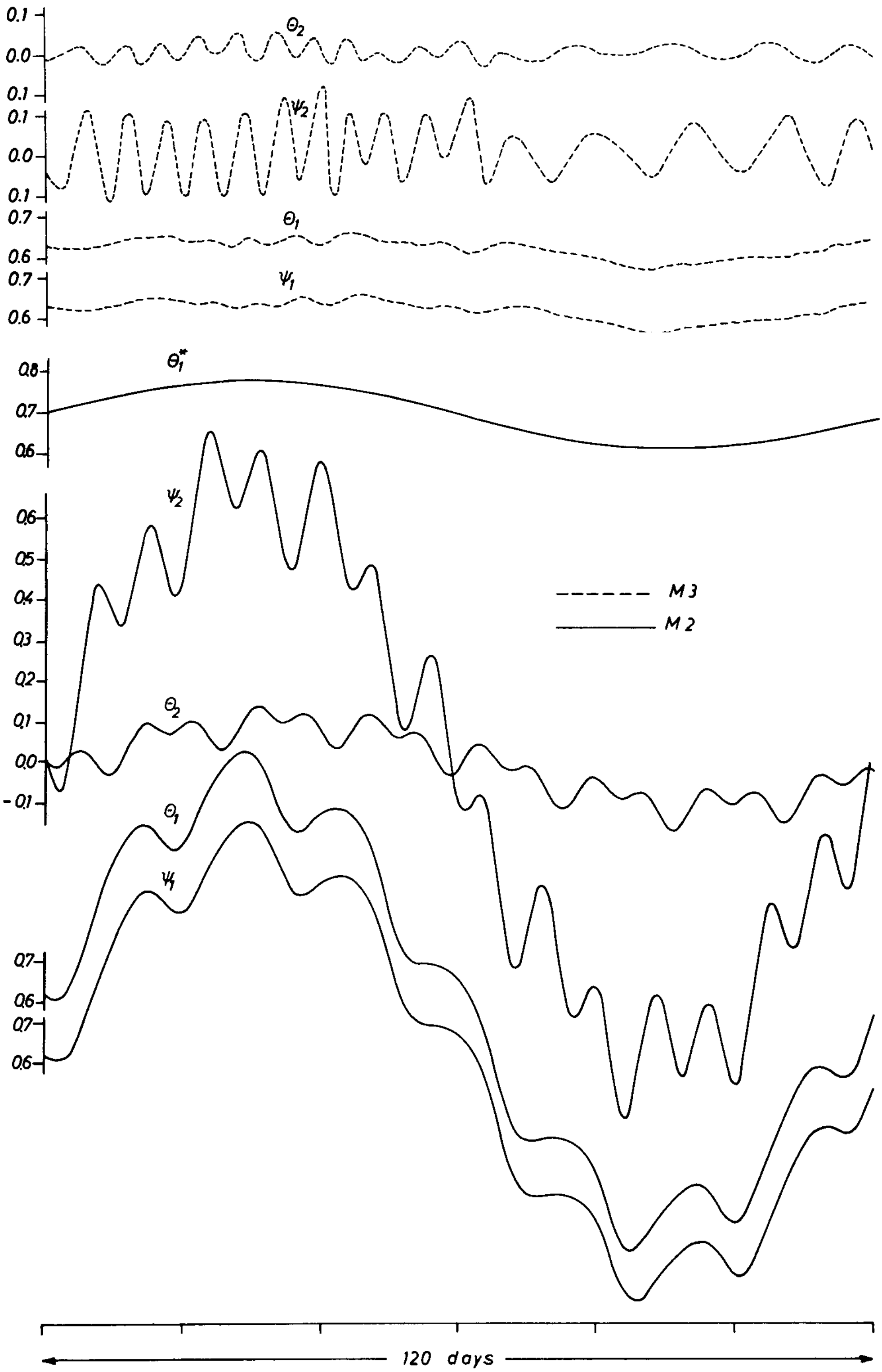
We have hoped to overcome this problem by allowing for a dependence of the coefficients β_{ij}^ψ, β_{ij}^θ, γ_{ij}^ψ, γ_{ij}^θ on time as well. M3 has been run for several constant values of the forcing

$$\theta^* = \bar{\theta}^* + \theta^{*\prime} \sin 2\pi t_n/T$$

(t_n fixed) and the corresponding $\alpha_i^\psi(t_n)$, $\alpha_i^\theta(t_n)$, $\beta_{ij}^\psi(t_n)$, $\beta_{ij}^\theta(t_n)$, $\gamma_{ij}^\psi(t_n)$, $\gamma_{ij}^\theta(t_n)$ have been evaluated according to the procedure outlined above. Then a run with M2 has been made with

$$\theta^* = \bar{\theta}^* + \theta^{*\prime} \sin 2\pi t/T$$

Fig. 6. Fourier coefficients of the mean flow. $M = 2$, $N = 4$. Time dependent θ^*, but constant parameterization coefficients. Relative units.


0.1
0.0
Θ_2
0.1
0.1
0.0
0.1
ψ_2
0.7
0.6
Θ_1
0.7
0.6
ψ_1
0.8
0.7
0.6
Θ_1^*
ψ_2
0.6
0.5
0.4
0.3
0.2
0.1
0.0
-0.1
Θ_2
Θ_1
ψ_1
0.7
0.6
0.7
0.6
M 3
M 2
120 days

and where the α_i^*, α_i^θ, β_{ij}^*, β_{ij}^θ, γ_{ij}^*, γ_{ij}^θ at time t have been interpolated between the known values of the coefficients at time t_n and t_{n+1}. Thus, the parameterization coefficients are altered at each time step. It turned out, however, that this procedure resulted in instability. Enormous amplitudes occurred after half a year or so.

We have to conclude then that the method of parameterization proposed here yields fairly good and detailed results as long as the forcing does not depend on time. Once we admit an annual cycle of heating the performance is much less convincing. Then we have to be satisfied with a proper representation of the "mean" values but we cannot hope to resolve the higher frequencies. It will be shown in the appendix that this is about the same that a model with the conventional Austausch approach can achieve (see Appendix).

5. Appendix

The general linear parameterization and the Austausch concept

It is generally accepted that the Austausch concept should be applied only to conserved quantities. In particular, quasi-geostrophic potential vorticity q is a quantity which is conserved in a geostrophic system and, consequently, the northward eddy transport of potential vorticity $\langle v' q' \rangle$ can be parameterized

$$\langle v' q' \rangle = -K \frac{\partial \langle q \rangle}{\partial y} \tag{A.1}$$

The brackets and the prime denote the zonal average and the deviation of it, respectively. This approach has been used by Sela and Wiin-Nielsen (1971) and Wiin-Nielsen and Fuenzalida (1975). Hence one may ask for the relationship between our general linear parameterization and this Austausch parameterization. If the Austausch coefficient K depends on y but not on $\langle q \rangle$ we have to expect that (A.1) is contained in (3.2) as a special case. Now

$$q = \nabla^2(\psi \underset{(-)}{+} \tau) + f_0 \underset{(+)}{-} \frac{Bf_0}{\sigma_0}\tau$$

in the upper (lower) layer. As usual, (A.1) is applied separately to each layer. Hence, if we have

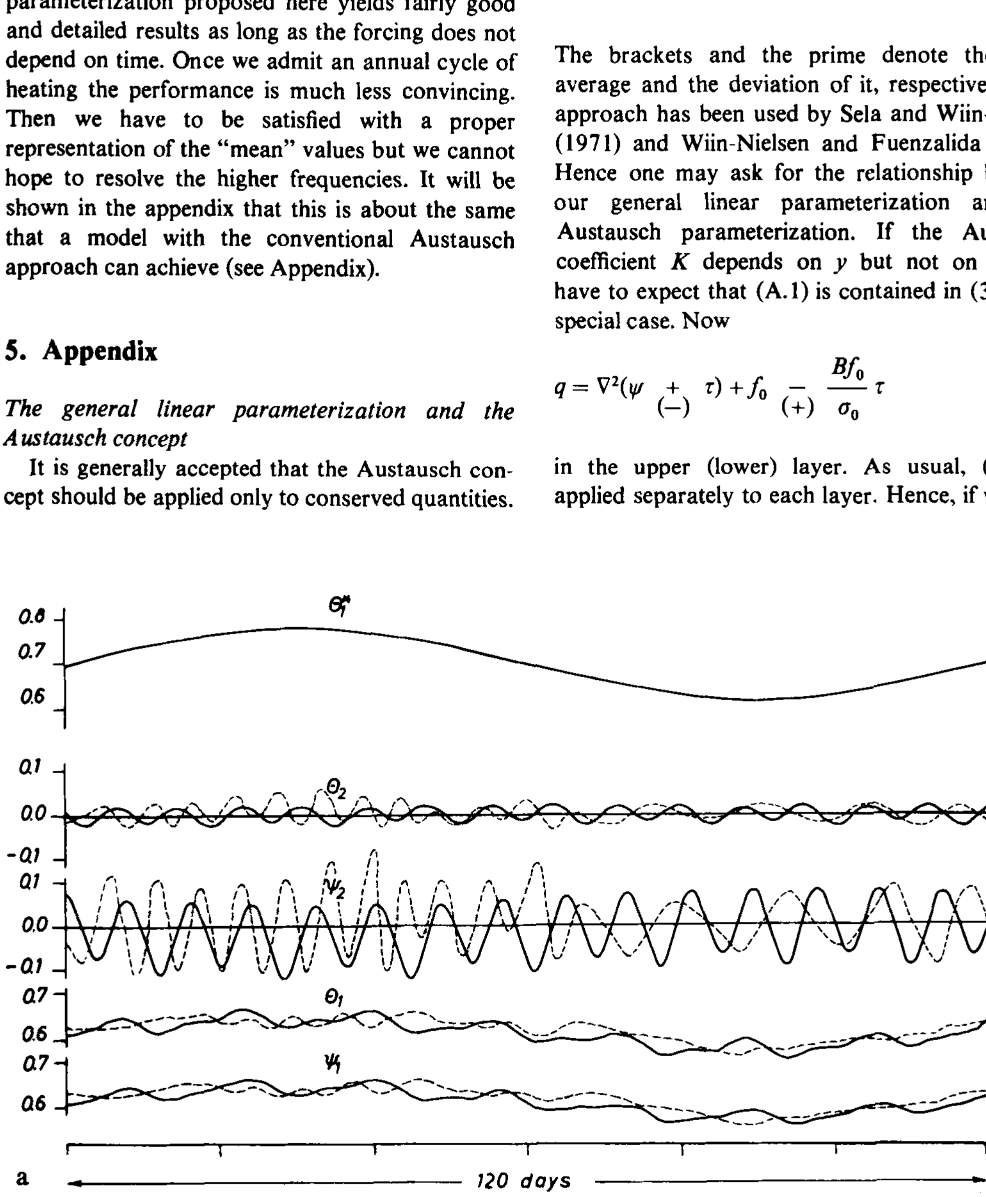


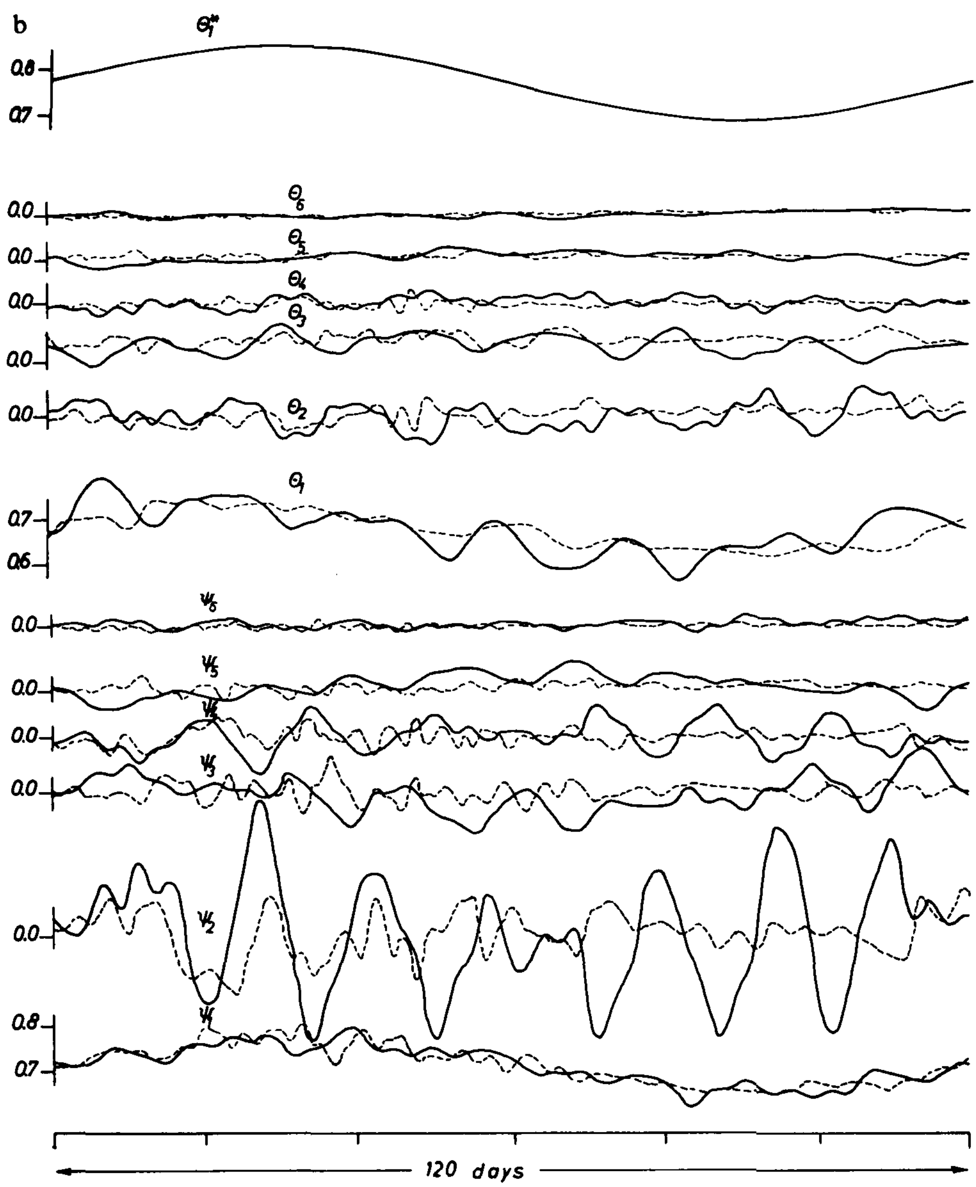
Fig. 7. Fourier coefficients of the mean flow with (a) $M = 2, N = 4$; (b) $M = 6, N = 6$. Time-dependent θ^* and time-dependent α_i^*, α_i^θ, but constant β_{ij}^*, β_{ij}^θ, γ_{ij}^*, γ_{ij}^θ. Solid curve: M2; dashed curve: M3. Relative units.

M meridional modes we have $2M$ parameterization coefficients K_{1l}, K_{2l} because

$$K_1 = L^2 f_0 \sum_{l=1}^{M} K_{1l}\,\phi_l, \quad K_2 = L^2 f_0 \sum_{l=1}^{M} K_{2l}\,\phi_l,$$

In consequence, we have only $2M$ parameters free when we try to adopt a simplified model M2 with an Austausch parameterization to M3. Thus it is left only to choose the time averages D_l, $i \leqslant 2M$ (eq. (3.5)) or $\bar{\psi}_l$, θ_l, $i \leqslant M$ (eq. (3.7)) to determine the K_{1l}, K_{2l}, $i \leqslant M$. Inserting $\psi_l = \bar{\psi}_l$, $\theta_l = \bar{\theta}_l$ into the corresponding differential equations (using (A.1)) we obtain K_{1l}, K_{2l}.

There is, however, no constraint concerning the eigenvalues of the differential equations. The system may have solutions such as (3.5 with $\lambda_K \neq 0$ and $\lambda_l^{(r)} \neq 0$, $\lambda_l^{(i)} \neq 0$. This results in an exponential growth if $\lambda_K > 0$ and $\lambda_l^{(r)} > 0$ so that in



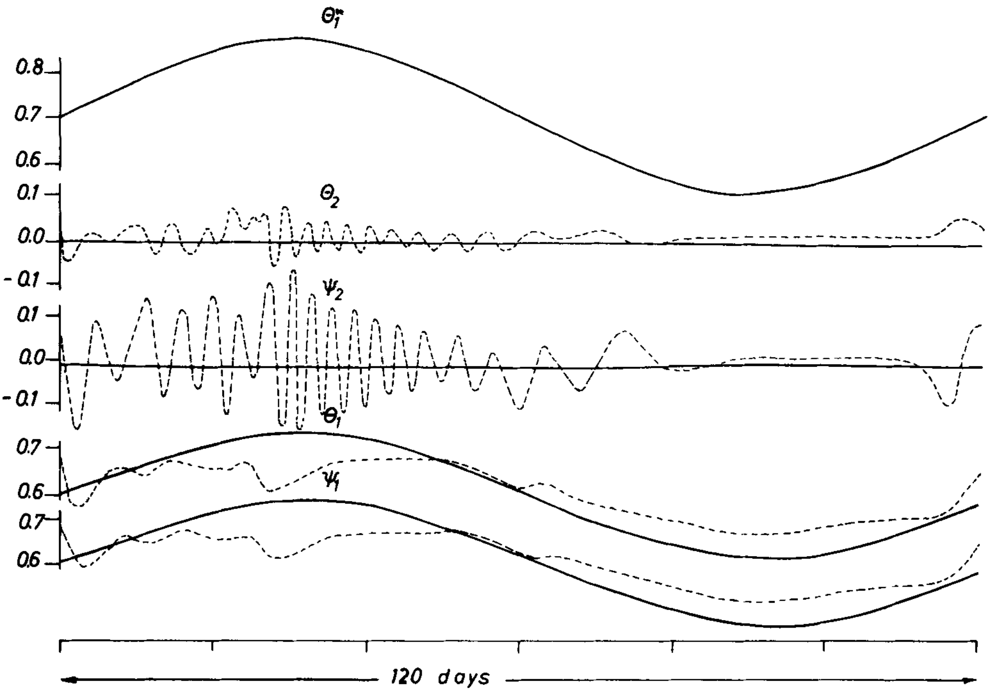


Fig. 8. Fourier coefficients of the mean flow using an Austausch approach with time-dependent θ^* but constant Austausch coefficients (solid curve). The corresponding results of M3 (dashed curve). Low resolution case. Relative units.

those cases the K_{1l}, K_{2l} are unreasonable. For example, the solutions with the high resolution case ($M = 6$, $N = 6$) exhibited such behaviour. If $\lambda_k < 0$, $\lambda_l^{(r)} < 0$ initially existing oscillations will be damped and the solutions will tend towards $\bar{\psi}_l$, $\bar{\theta}_l$ which may vary only according to a variation of θ^*. This behaviour of the solutions is observed by Sela and Wiin-Nielsen (1971) and is confirmed by our spectral model using (A.1) in the case of low resolution ($M = 2$, $N = 4$). No high frequencies appear in the results but only smoothed "annual" cycles (Fig. 8).

The special case $\lambda_l^{(r)} = 0$ (that is: the short-term oscillations are retained in the solutions) may be obtained only when as many parameterization coefficients as possible are used as in relations (3.2). This general parameterization additionally relates the eddy fluxes of the upper (lower) layer to the mean flow of the lower (upper) layer, in contrast to the Austausch approach.

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ЛИНЕЙНАЯ ПАРАМЕТРИЗАЦИЯ КРУПНОМАСШТАБНЫХ ВИХРЕВЫХ ПЕРЕНОСОВ

Предложен метод параметризации крупномасштабного меридионального переноса тепла и завихренности, основанный только на данных и не привлекающий теории крупномасштабной турбулентности. Метод испробован на примере двумерной (зонально осредненной) квазигеострофической модели (M2). Необходимые данные получены с помощью трехмерной модели Лоренца (M3). Параметризация является линейной, коэффициенты параметризации определяются так, чтобы результаты M2 были как можно ближе к результатам M3. Оказывается, что результаты M2 моделируют поведение M3 с большой точностью до тех пор, пока термическое возбуждение не зависит от времени. Как только мы вводим временну́ю зависимость нагревания в грубом соответствии с годовым изменением солнечного нагревания, наша схема параметризации ведет себя менее удовлетворительно. Тогда она выявляет те же качества, что и метод параметризации, основанный на концепции коэффициента обмена.