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On the splitting of atmospheric and oceanic turbulent billows

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ABSTRACT

The evolution of a density stratified turbulent shearing layer in the presence of a gravity field is traced from an initial state as a discontinuity in density and velocity to a final state where the Richardson number approaches $\frac{1}{4}$ for a turbulent Prandtl number of unity. For the more realistic case of turbulent Prandtl number greater than unity, a model for the splitting of the initially single turbulent shearing layer into two separate turbulent layers is discussed.

1. Introduction

In what follows we propose a mechanism capable of describing the turbulent flow field associated with billows in the atmosphere and oceans and the observed splitting of these turbulent shearing layers. In the oceans and atmosphere the formation of billows as a result of Kelvin–Helmholtz instability constitutes an important mixing mechanism. According to Woods and Wiley (1972) "... At present there exists no detailed theory of the changes in density and velocity profiles due to billow turbulence ...". Furthermore, Woods and Wiley conjectured that billow turbulence is *the* dominant mechanism for mixing and for transferring heat and momentum which involves a diffusive competition between heat and salt can only be responsible for ocean microstructure formation found in rather special localities like the Mediterranean outflow, undersea ice, and over high-salinity pools found at the bottom of the Red Sea. They carried out an extensive field program off the continental shelf of Malta during the summer of 1970 and used dye tracers as well as microstructure recording devices to obtain depth profiles of the salinity and temperature.

Down to a depth of some 100 m the microstructure is characterized by: (1) a summer thermocline some 10 m thick which is due to the downward transport by billow turbulence of heat absorbed at the surface layers; (2) layers some 1–2 m thick which appear to slide on one another; (3) sheet ensembles with thicknesses of some 10–20 cm in which enhanced vertical temperature and shear gradients are found. These sheets lie at the interface between layers described in (2).

According to Woods and Wiley large-scale geostrophic motions as well as internal gravity waves traveling as a wave packet can produce an intensification in the shear found across one of these sheets and therefore can reduce the local Richardson number below its critical value of $\frac{1}{4}$. Kelvin–Helmholtz instabilities start as a long-wave undulation of one of these sheets; typically their wavelength is about 7.5 times the sheet thickness. Under the influence of that instability, the sheets roll up and vorticity becomes concentrated in billows which develop secondary instabilities due to shear and/or unstable density stratification. The lifetime for the formation of such a turbulent patch may be 2–3 min. In a comparable time scale of some 5–10 min the shear smears these turbulent patches into a turbulent layer having thickness, H, which is found to be nearly double the thickness of the sheet on which the waves initially grew. Turbulent diffusion proceeds in that layer of thick-

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ness H, until its local Richardson number reaches values of about unity. Woods and Wiley reported that the density is not uniform across the thickness H, but that sharp gradients tend to be found near the edges of that layer with weaker gradients inside it. Molecular diffusion operates between two "billow events" which might repeat on time scales of 1–6 hr. Therefore, at the start of the next billow event, scales of 5–15 cm are found to be those not yet smoothed by diffusion. Woods and Wiley argued that these newly formed density sheets are the seat of Kelvin–Helmholtz waves which will set in during the next billow event. Yet, Thorpe (1973) pointed out what we feel is the correct mechanism: Due to the wave-induced Reynolds stresses, momentum diffused faster than density. As a result, slightly above and below the edges of the newly formed sheets of thickness H, the local Richardson number was positive and much lower than at the center of the sheet which implied that values of $Ri < 0.25$ were prone to be found on the edges of the sheets rather than at their mid-level. Woods and Wiley found that "... There is no detectable variation of billow size with depth and billows in the surface layer lying between the wave-mixed layer and the thermocline have the same size.... This narrow range of billow sizes is found using the dye streak method for sheets with density differences ranging from one part in a thousand in the thermocline to one part in a million in the surface layer over the thermocline and below the thermocline".

In the atmosphere, short-wavelength radar can be used to probe atmospheric motions taking place in clear air, i.e., in which the usual radar radiation scatter particles such as rain droplets are absent. Browning and Watkins (1970), Atlas et al. (1970) and Browning (1971) confirmed some of the findings in the ocean; the principal end-product of billow turbulence appears to be the splitting of the layer on which the Kelvin–Helmholtz instability grew. The reader is referred to Fig. 1 of the paper of Atlas et al. which displays a time-height record of the radar echo. Vertical resolution is 2 m. An unstable wave is seen to grow in time, its envelope spreading in a wedge-like fashion until a limiting amplitude is reached after some 15 min. The resulting billows last for a comparable length of time after which the single layer splits into two distinct layers separated by some 100 m. A similar behavior is found in Fig. 4 of Browning and Watkins (1970) which depicted the evolution of these flows: A single layer becomes wavy, the waves grow and break, cat's eyes form, the shear stretches the filaments and the initial layer splits into two layers.

Browning and Watkins (1970), Atlas et al. (1970) and Browning (1971) all associated large-amplitude turbulent billows with clear air turbulence. Browning (1971) dealt with radar echoes from heights of some 6–11 km, i.e., in the middle of the troposphere while the Atlas et al. investigation dealt with billows occurring near the top of the planetary boundary layer. Also, the billows recorded by Browning had amplitudes of several hundred meters (radar resolution of about 100 m). He noticed that the large amplitude billow generation was generally preceded by an increase in the wind shear, the billows formed when the Richardson number reached values close to 0.2 and found at levels where the wind profile possessed a point of inflexion. Most of his observations were biased towards occasions of strong shear associated with the passage of the Jet Stream or a front. Billows have a lifetime of some 30 min although Browning reported an unusual case which lasted for 4 h; they occur in rows of some 2–8 billows.

2. The turbulent shearing layer of Prandtl number unity and marginal instability

The growth of a turbulent shearing layer was considered by Lessen, Barcilon and Butler (1977); using the principle of marginal stability (cf. Lessen and Singh, 1974; Lessen and Paillet, 1976) they investigated the time evolution of a vortex sheet coincident with a stable density discontinuity for a Prandtl number equal to unity. If

$$(\delta_M^2, \delta_T^2) = \int_0^t dt' \, (\nu(t'), \kappa(t')) \tag{1}$$

are respectively the square of the momentum and temperature thicknesses, the solution to the above diffusion problem reads

$$\left.\begin{aligned} u/U_0 &= \operatorname{erf}(z/2\delta_M) \\ \rho_0 - \rho &= \hat{\rho}_0 \operatorname{erf}(z/2\delta_T) \end{aligned}\right\} \tag{2}$$

$\hat{\rho}_0$ and U_0 are measures of the density and velocity discontinuities; the time dependence of the turbulent eddy coefficients of viscosity, ν, and diffusivity,

κ, is yet unknown and is to be found consistent with the ideas of marginal stability. For Prandtl number of unity $\delta_M = \delta_T$ and we can introduce a single similarity variable $\eta = z/2\delta_M$. For any given time the above basic state resembles the one described by

$$\left.\begin{aligned} \bar{u}/U_0 &= \tanh z \\ \bar{\rho} &= \exp(-\beta \tanh z) \end{aligned}\right\} \tag{3}$$

The viscous stability of laminar velocity and density distributions corresponding to (3) was investigated by Maslowe and Thompson (1971). Such a flow is characterized by a vertical scale, δ_*, which relates to δ_M via

$$\delta_* = \frac{2}{\sqrt{\pi}}\delta_M \tag{4}$$

The quantity β which appears in (3) enters the expression of the Richardson number, J_0, (evaluated at $z = 0$) which can be expressed as

$$J_0 = g\beta\delta_*/U_0^2 \tag{5}$$

It is now considered that the turbulent shearing layer is marginally unstable at the minimum critical turbulent Reynolds number. Since the *e*-folding time scale of the turbulent shear-layer growth is large compared to that of disturbances growing on the basic state described approximately by (3), we feel justified in using results of the Maslowe and Thompson study in our present problem. From these results, which are displayed in Fig. 2 of the Maslowe and Thompson paper, we infer that the minimum critical turbulent Reynolds number,

$$R_c = U_0\delta_*/\nu \tag{6}$$

and the corresponding dimensionless disturbance wavenumber, α_c, can be closely approximated in terms of the Richardson number by

$$R_c = 6J_0/(\tfrac{1}{4} - J_0) \quad \text{and} \quad \alpha_c = 2J_0 \tag{7}$$

Now, from (1), (4), (5) and (6) we can write

$$\frac{d\delta_*}{dt} = \frac{2}{\sqrt{\pi}}\frac{d\delta_M}{dt} = \frac{1}{\sqrt{\pi}}\left(\frac{\nu}{\delta_M}\right) = \frac{2U_0}{\pi R_c}$$

but since

$$\frac{dJ_0}{dt} = \left(\frac{g\beta}{U_0^2}\right)\frac{d\delta_*}{dt}$$

we can write using (7) and the above equations

$$\frac{dJ_0}{dt} = \frac{2g\beta}{\pi U_0 R_c} = \frac{g\beta}{3\pi U_0}\left(\frac{\frac{1}{4} - J_0}{J_0}\right)$$

upon separation of variables and a time integration, the above equation yields

$$\tau = -3\ln|1 - 4J_0| - 12J_0 \tag{8}$$

where τ is a non dimensional time defined as

$$\tau = 4g\beta t/\pi U_0 \tag{9}$$

Fig. 2 of Lessen, Barcilon and Butler (1977) shows that after an initial infinite growth rate J_0 *approaches* $\frac{1}{4}$ *as* τ *tends to infinity*. The merits and shortcomings of that solution are discussed in the above paper but suffice it to say that after a short time theoretical predictions compare well with available experimental evidence. It is of importance to note that throughout the growth process, the location of the minimum Richardson number is in the middle of the layer (i.e. at $z = 0$) for this case which dealt with a Prandtl number of unity.

3. The turbulent shearing layer with Prandtl number greater than unity

Though a theoretical model of a stratified turbulent shearing layer with a Prandtl number of unity reproduces some of the growth and dominant eddy-scale characteristics of the corresponding observed flows in nature, the model is unrealistic in that mass and momentum transport actually do not take place over the same scale distances. Mass transport takes place through the mechanism of sheet roll up or wave breaking and hence is limited to the wave-breaking zone of a turbulent shearing layer; momentum transport takes place through the mechanism of Reynolds stress which is not limited to the wave-breaking region and indeed is present well outside it. The "momentum thickness" of the shearing layer will therefore always be greater than the "density thickness" and the Richardson number for this actual case may develop a maximum value in the center of the layer.

Hazel (1972) studied the *inviscid* stability of the *laminar* shearing layer and found that "... When the scale of the density variation is much less than that of velocity variation in a shear layer, the flow is unstable for some α, however large J_0 may be. The unstable modes are moving ones for large J_0, but in the region $0 \leqslant J_0 \leqslant 0.25$, they are stationary for some α. . . ." For Prandtl number greater than unity, the flow is unstable. When the Richardson number in the middle of the layer is less than 0.25, the phase velocity of the wave corresponds to the unperturbed flow velocity at the middle of the layer (point of inflexion of the velocity distribution); for this case, there is therefore only a single critical layer. When the Richardson number in the middle of the layer exceeds 0.25, there exist two traveling-wave disturbances moving in opposite directions relative to the point of inflexion, hence for this case, there exist two critical layers above and below the center of the shearing layer, respectively. The significance of the critical layer is that the disturbance amplitudes are greatest at or near the critical layer and that *wave breaking can therefore be associated with the critical layer.* If there are two critical layers, there are two wave-breaking regions.

4. Conclusions

The results of Hazel (1972) along with the evolution modeled in Section 3 can be utilized to understand "splitting" of stratified, turbulent shearing layers in terms of the following sequence of events. A shearing layer forms with an initially low Richardson number (everywhere much less than 0.25) as in Fig. 1a. The resulting wave breaking at the center causes the momentum- and density-change layers to grow in thickness with the density layer growing less rapidly than the momentum layer. Throughout the process, the average flow is marginally unstable. When the Richardson number in the middle of the layer exceeds 0.25, the single critical layer is replaced by two critical layers above and below the original one resulting in two wave-breaking zones. The Richardson number in the center of the layer continues to grow and the pair of wave-breaking zones move further apart, each generating new critical layers farther away from the central layer (see Fig. 1b). It is probable that the new wave-breaking zones continue to further generate new layers resulting finally in a multistepped density stratification.

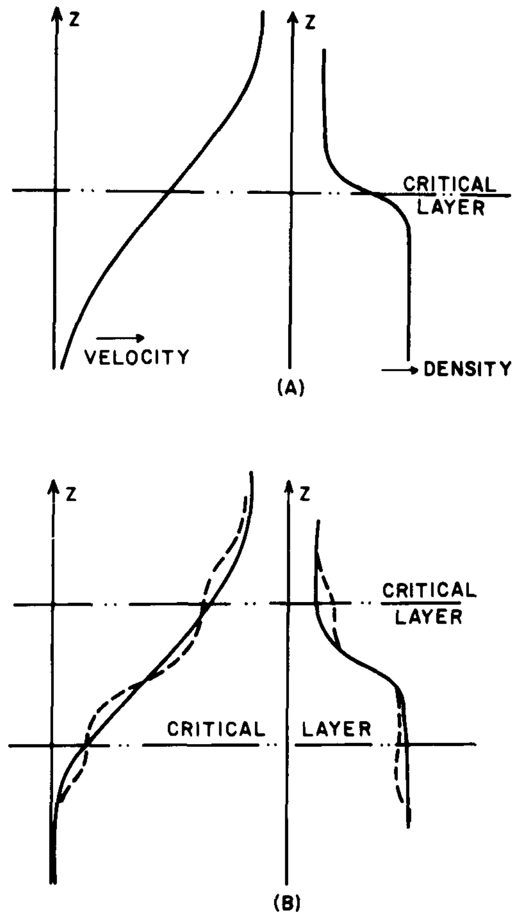

Fig. 1. Developing "average" velocity and density distributions. Dotted lines show modification in "average" velocity and density distribution due to wavebreaking. (a) $J_0 < 0.25$, single critical layer; (b) $J_0 > 0.25$, two critical layers.

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О РАЗДЕЛЕНИИ АТМОСФЕРНЫХ И ОКЕАНИЧЕСКИХ ТУРБУЛЕНТНЫХ СЛОЕВ

Прослежена эволюция стратифицированного по плотности турбулентного слоя со сдвигом скорости от начального состояния в виде разрыва в плотности и скорости до конечного состояния, где число Ричардсона приближается к $\frac{1}{4}$ при турбулентном числе Прандтля, равном единице. Для более реалистического случая турбулентного числа Прандтля, большего единицы, обсуждается модель, описывающее разделение первоначально одного турбулентного слоя со сдвигом на два отдельных турбулентных слоя.