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A modified two-stream approximation for computations of the solar radiation budget in a cloudy atmosphere

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ABSTRACT

A two-stream method with variable parameters for backward scattering of the diffuse and direct radiant flux densities has been developed to calculate accurately the solar heating rates and radiation fluxes in vertically inhomogeneous model atmospheres with cloud layers included. The spectrum covers the region between 0.2 and 3.580 μm and the model atmospheres reach up to 70-km altitude. Comparisons between this approximate method and an accurate iterative method yield to maximum relative deviations of less than 3% for the upward- and downward-going fluxes.

1. Introduction

The diabetic heating of the atmosphere and of the ground by absorption of solar radiation is primarily modulated by clouds. All other constituents (aerosols and gases) and the ground reflectivity, though they cannot be neglected, play a comparatively minor role in the solar radiation budget. Therefore in any model for the atmospheric circulation the interaction between clouds and the ambient radiation field needs to be taken into account.

There are known various accurate methods to solve the equation of radiative transfer. They are based on methods with spherical harmonics (e.g. Dave and Canosa, 1974), matrix operators (e.g. Howell and Jacobwitz, 1970), discrete ordinates (e.g. Liou, 1976) and successive orders of scattering (e.g. Raschke, 1972). A thorough summary of such procedures has been prepared recently by Lenoble (1977).

The calculation time of such methods becomes unbearable if the atmosphere contains clouds with larger optical thicknesses (cf. Coakley and Chylek, 1975). Monte Carlo methods (e.g. Plass et al., 1968) suffer even more than the other methods from the disadvantage of quick extending computation time for the case of detailed considerations of vertically inhomogeneous atmospheres.

As an alternative for the routine use there remains only a parameterization of the radiative transfer with an approximative method. Such approximations should be also applicable to rather relastic models of the atmospheric structure—at least in its vertical direction—and provide results on the diabetic heating rates with a minimum of computational time. If used as routines in the framework of numerical circulation, forecast or climate models, they should require only an extremely small fraction of the computational time of those models.

There have been developed many approximative methods. The principles of them have been summarized by Özisik (1973). The basis of our two-stream approximation is the method described in detail by Zdunkowski and Korb (1974). Their original approximation, like many other methods (Liou, 1974; Sagan and Pollack, 1967; Coakley and Chylek, 1975), yields to results which differ considerably from exact calculations especially for highly non-isotropical radiation fields. Joseph et al. (1976) avoided such systematic errors in a modification of the well-known "Eddington approximation" (Chandrasekhar, 1950) where they introduced a delta approximation for the peak of the phase function. We solve this problem, which is caused by the strong non-isotropical scattering, with special parameters for the backward scattered portion of radiation and consider also their dependence on the solar zenith angle and the optical thickness of the medium. In this paper the method has been applied for a model atmosphere above a rough sea surface. It can also be used for complete atmosphere-ocean models and is, therefore, also of interest for remote sensing of ocean properties.

The symbols and nomenclature of this paper are in agreement with recent recommendations for the use of S.I. units (e.g. Page and Vigoureux, 1974).

2. Theory

2.1. *Basic equations*

The basic equations of radiative transfer for a plane-parallel scattering and absorbing atmosphere are:

$$\pm\mu\frac{dL^{\pm}(\tau,\mu,\varphi)}{d\tau} = L^{\pm}(\tau,\mu,\varphi) - \frac{\tilde{\omega}}{4\pi}\int_0^{2\pi}\int_0^1 L^{\pm}(\tau,\mu',\varphi') \times p(\mu,\varphi;\mu',\varphi')d\mu'\,d\varphi' - \frac{\tilde{\omega}}{4\pi}\int_0^{2\pi}\int_0^1 L^{\mp}(\tau,\mu',\varphi') \times p(\mu,\varphi;\mu',\varphi')d\mu'\,d\varphi' - \frac{\tilde{\omega}}{4\pi}\cdot S_0\cdot p(\mu,\varphi;\mu',\varphi')\cdot e^{-\tau/\mu_0} \tag{1}$$

where $L^{\pm}(\tau, \mu, \varphi)$ is the upward- (+) or downward- (—) going radiance, τ the vertical optical depth, μ and μ' the cosine of the zenith angle, μ_0 the cosine of the solar zenith angle, φ and φ' the azimuth of the radiance with respect to the principle plane. S_0 is the extra-terrestrial solar irradiance, $p(\mu, \varphi)$ the phase-function and $\tilde{\omega}$ the single-scattering albedo, i.e. the fractional extinction due to scattering. The integrals in eq. (1) describe the up- (+) and downward- (—) going portion of scattered diffuse radiation only, the last term the scattered portion of direct solar radiation. This equation applies for each wavelength, where thermal emission can be omitted.

In a first step towards a two-stream approximation, i.e. only the consideration of up and downward fluxes, the azimuthal dependence is removed with the assumption that the diffuse radiances are axial-symetric. In a second step, the integration over zenith angle leads with the relation for the radiative flux across a horizontal unit area

$$\Phi(\tau) = 2\pi \int_0^1 L(\tau, \mu)\mu\, d\mu \tag{2}$$

to differential equations for the up- and downward-going radiation fluxes Φ^+ and Φ^- respectively:

$$\frac{d\Phi^+(\tau)}{d\tau} = (1-\tilde{\omega}(1-\beta))\cdot\frac{\phi^+(\tau)}{\bar{\mu}_+} - \tilde{\omega}\beta\frac{\Phi^-(\tau)}{\mu^-} - \tilde{\omega}\beta_0\cdot S_0\cdot e^{-\tau/\mu_0} \tag{3}$$

$$\frac{d\Phi^-(\tau)}{d\tau} = \tilde{\omega}\beta\frac{\Phi^+(\tau)}{\bar{\mu}_+} = (1-\tilde{\omega}(1-\beta))\frac{\Phi^-(\tau)}{\bar{\mu}_-} + \tilde{\omega}(1-\beta_0)\cdot S_0\cdot e^{-\tau/\mu_0} \tag{4}$$

The parameters β, β_0 describe the fraction of backward scattering for the diffuse and direct solar radiation, respectively. Their definitions are discussed in Section 2.4 of this paper.

The mean effective zenith angles $\bar{\mu}_+(\tau)$ and $\bar{\mu}_-(\tau)$ for the up and downward radiation are obtained from (1) by

$$\bar{\mu}_\pm(\tau) = \frac{\int_0^1 \mu L^{\pm}(\tau,\mu)d\mu}{\int_0^1 L^{\pm}(\tau,\mu)d\mu} \tag{5}$$

Those mean values $\bar{\mu}_\pm(\tau)$ are essentially a function of the optical depth. We use for optically thick clouds a value of $\bar{\mu}_\pm = 0.5$ (i.e. for complete isotropical diffuse radiation), whereas the identity $\mu_0 = |\mu_\pm|$ is valid for clear atmospheres. This approximation has been found sufficiently accurate in comparison with more complex treatments (cf. Fig 1, 2 and 3).

2.2. *Formal solution of the equations for the upward/downward-going radiation fluxes*

For an absorbing and scattering layer of the atmosphere the formal solutions of the eqs. (3) and (4) can be written:

$$\Phi^+(\tau) = C_1\cdot e^{r_1\tau} + C_2\cdot e^{r_2\tau} + X_1 e^{-\tau/\mu_0} \tag{6}$$

and

$$\Phi^-(\tau) = C_1\cdot A\cdot e^{r_1\tau} + C_2\cdot B\cdot e^{r_2\tau} + X_2\cdot e^{\tau/\mu_0} \tag{7}$$

The constants C_1 and C_2 are obtained from the boundary conditions, which are explained in Section 2.3. The other variables are functions of $\tilde{\omega}$,

β, β_0, $\mu_\pm$ and μ_0:

$$r_{1,2} = \frac{a_1 + a_2}{2} \pm \sqrt{(a_1 - a_4)^2 + 4a_2 a_3}$$

where

$$\begin{aligned}
a_1 &= \frac{1}{\bar{\mu}_+}(1 - \tilde{\omega}(1 - \beta)) \\
a_2 &= \frac{1}{\bar{\mu}_-}\tilde{\omega}\beta \\
a_3 &= \frac{1}{\bar{\mu}_+}\tilde{\omega}\beta \\
a_4 &= \frac{1}{\bar{\mu}_-}(1 - \tilde{\omega}(1 - \beta))
\end{aligned} \tag{8}$$

and

$$X_1 = \frac{S_0 \cdot (a_5 \cdot B + a_6)}{(B - A)\left(r_1 + \dfrac{1}{\mu_0}\right)}$$

$$X_2 = \frac{S_0(a_5 \cdot B + a_6)}{(A - B) \cdot \left(r_2 + \dfrac{1}{\mu_0}\right)}$$

with

$$a_5 = \tilde{\omega}\beta_0, \quad a_6 = \tilde{\omega}(1 - \beta_0)$$

$$A = \frac{r_1 - a_1 - a_2}{a_2 + a_4 - r_1}, \quad B = \frac{r_2 - a_1 - a_3}{a_2 + a_4 - r_2}$$

2.3. *Boundary conditions*

The boundary conditions at the top and the bottom of the atmosphere can be expressed by the equations

$$\Phi^-(\tau = 0) = 0 \tag{9}$$

$$\Phi^+(\tau = \tau_A) = \rho_{\mathrm{dif}} \cdot \Phi^-(\tau = \tau_A) + \rho_{\mathrm{dir}} \cdot \mu_0 \cdot S_0 \cdot e^{-\tau_A/\mu_0} \tag{10}$$

where ρ_{dif} and ρ_{dir} are the reflectivity or albedo for the diffuse and direct downward solar radiation at The ground, respectively, and τ_A is the optical depth of the whole atmosphere. The albedo for diffuse radiation is assumed to be isotropical while the albedo for the direct solar radiation depends on the sun's zenith angle, as it can be computed for the sea surface with Fresnel's formulae.

In case of a multi-layered inhomogeneous atmosphere two equations of the form (6), (7) exist for each layer. At each boundary between two layers (i and $i + 1$) the up and downward fluxes, respectively, must be constant.

This coupling of the atmospheric layers yields to a system of linear equations for the constants C_1^i and C_2^i in the flux equations (6) and (7).

2.4. *Backward scattering*

The correct parameterization of the anisotropic scattering of solar radiation into a forward and a backward fraction is of essential interest for the accuracy of a two-stream approximation. Here the expressions "forward" and "backward scattering" describe in a somewhat misleading manner the scattering of an incident beam of radiation into the upward or downward hemisphere, respectively. In most of the former approximative methods, as discussed extensively by Wiscombe et al. (1976), the scattering in the atmosphere has been treated without consideration of its dependence on the sun's zenith angle. This simplification caused considerable errors if highly anisotropical scattering functions have been used (Liou, 1973).

Therefore, in this paper, the backward scattering has been treated for the direct solar radiation in a cloud-free atmosphere with consideration of its dependence on the solar zenith angle. The ratio of backward to total scattering for the direct solar radiation incident with a zenith angle θ_0 is approximately given by

$$\beta_0(\theta_0) = \frac{\int_{\pi/2-\theta_0}^{\pi} p(\theta) \sin\theta \, d\theta + \int_{\pi/2-\theta_0}^{\pi} p(\theta) \sin\theta \, d\theta}{2\int_0^{\pi} p(\theta) \sin\theta \, d\theta} \tag{11}$$

More exact calculations with consideration of the azimuthal anisotropy of the scattering function, if fitted with respect to a horizontal plane, show (last column in Table 1) that the error due to this simplification is less than 3% for $\theta_0 \angle 60°$. Liou (1973) used a similar approximation.

$\beta_0(\theta_0)$ increases steadily with increasing zenith angle. Some values computed for the phase functions of Haze L and the cloud models C1 and C2, as defined by Deirmendjian (1969), are summarized in Table 1.

For grazing incidence ($\theta_0 = 90°$) $\beta_0(\theta_0)$ must reach the value 0.5, since equal amounts of radiation are scattered into the up and downward hemispheres. Also the values of $\beta(\theta)$ computed for the *diffuse* up and downward fluxes vary with

Table 1. *Backward scattering coefficients for phase functions of Deirmendjian of a wavelength of 0.700* μm

	Approximated by (II)			Exact
θ_0	Haze L	C1	C2	C1
0°	0,044	0,043	0,046	0,043
10°	0,046	0,044	0,047	0,045
20°	0,051	0,047	0,051	0,048
30°	0,061	0,055	0,059	0,056
40°	0,079	0,069	0,072	0,070
50°	0,112	0,091	0,094	0,094
60°	0,170	0,128	0,129	0,132
70°	0,269	0,183	0,182	0,189
80°	0,410	0,247	0,247	0,256
90°	0.5	0.5	0.5	0.5

increasing θ_0 as the fraction and angular distribution changes. But various comparative calculations with an accurate iterative method (Raschke, 1972; Kerschgens et al., 1976) showed that inside and below clouds the approximations $\beta(\theta) = \beta_0(\theta_0 = 0°)$ and for a cloud-free atmosphere $\beta(\theta) = \beta_0(\theta_0)$ yielded to the best results. The first approximation is understandable due to the fact that multiple scattering inside of clouds causes a symmetry of downward radiation field with respect to the zenith. Such asymptotic behavior is often also discussed for the fields of overlight in deeper ocean layers (e.g. in Jerlov, 1968).

3. Results

3.1. *The model atmosphere*

The accuracy of the approximative method has been tested with comparative calculations using the iterative method mentioned above. A variety of comparisons among results obtained by other authors with exact or with approximative methods for homogeneous layers have been coordinated by Lenoble (1977).

In our case the model atmosphere is an average mid-latitude summer atmosphere. The radiation fields have been calculated for two values (37.6° and 72.2°) of the sun's zenith distance.

A complete description of the model atmosphere and the input parameters has been published in an earlier paper (Kerschgens et al., 1976). In summary the absorption by O_2, O_3, H_2O, CO_2, aerosols and the scattering by air molecules (Rayleigh scattering) and aerosols (maritime profile of Shettle and Fenn, 1975) is considered here in high spectral detail.

Due to fits of the transmission functions for CO_2 H_2O by Moskalenko and Golubitskiy (1968) by finite series of exponentials (Wiscombe and Evans, 1976) the number of spectral subintervals of the solar spectrum increases from originally 37 to 87.

Table 2. *Fractional amounts (in per cent) of absorption of solar radiation in a clear model atmosphere and of its albedo at its top calculated with the approximative and exact iterative method for a clear atmosphere*

	Solar absorption (%)		Planetary albedo (%)	
θ_0	Approx.	Exact	Approx.	Exact
37.6°	19.7	20.5	10.2	9.8
72.2°	26.5	26.4	27.1	25.9

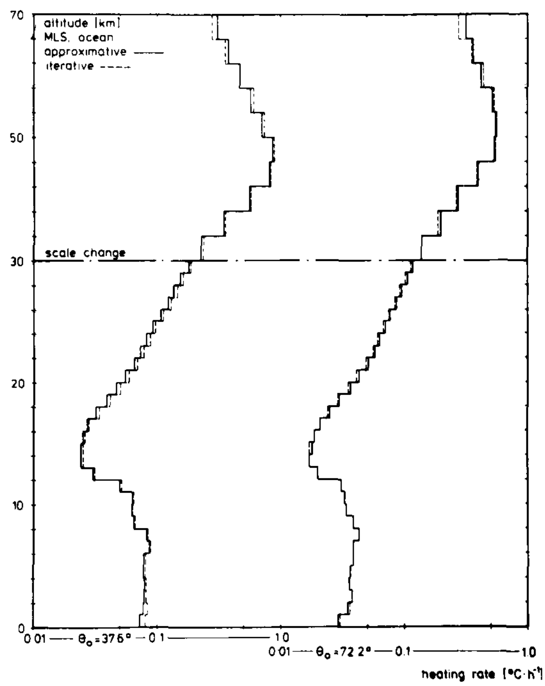

Fig. 1. Radiative heating in a clear model atmosphere with underlying ocean for two solar zenith distances, θ_0, calculated with an exact and the approximate method. MLS = Midlatitude summer.

In the vertical direction the atmosphere is divided into 70 homogeneous layers with a constant geometrical thickness of 1 km.

The cloud model has been simulated with absorption and scattering coefficients computed for an altostratus by Yamamoto (1972) and with the phase function C2 by Deirmendjian (1969) assuming saturated water vapor within the cloud. The lower boundary is a rough ocean surface.

3.2. *Cloud-free atmosphere*

Some results of comparative runs with the iterative and approximative methods obtained for the clear model atmosphere described above are compared in Table 2 and Fig. 1.

The absorption obtained here is largely due to the gaseous components in the atmosphere, as it has been shown to some extent by Raschke (1972).

The upward and downward fluxes in each layer (Table 3) show a relative deviation of less than 3% while the total absorption within the atmosphere and the planetary albedo agree even within 1%. In the troposphere the agreement of calculated flux divergences is better than 5%. Only in the strato- and mesosphere deviations up to 10% may be reached. But in both cases these small amounts cannot be verified experimentally with the technology available at present.

The approximative method needs only a very small amount of computer time, as shown in Table 3, in comparison with the time required for the iterative method by a CDC 7600. In each program about 0.5 sec is required to build up the model

Table 3. *Computation time in seconds for both methods*

θ_0	37.6°	72.2°
Exact	111.7 s	116.0 s
Approx.	2.7 s	2.7 s

Table 4. *Absorption (in per cent) of the radiation incident at the top of the atmosphere in different layers in the presence of a stratiform cloud at 1–2 km. τ_c = optical depth of the cloud at $\lambda = 0.55$ μm, θ_0 = solar zenith distance*

		$\theta_0 = 37.6°$		$\theta_0 = 72.2°$	
Layer	Method	$\tau_c = 3$	$\tau_c = 12$	$\tau_c = 3$	$\tau_c = 12$
0–70 km	Iterative	22.9	24.8	26.9	27.4
	Approx.	22.5	23.8	27.5	28.4
2–70 km	Iterative	16.6	17.6	22.6	23.2
	Approx.	16.0	16.7	22.9	23.7
1–2 km	Iterative	4.8	6.5	3.6	3.9
	Approx.	5.2	6.5	3.2	3.8
0–1 km	Iterative	1.5	0.7	0.7	0.3
	Approx.	1.3	0.6	1.4	0.8

Table 5. *Planetary and cloud albedo as ratio of upward- to downward-going fluxes computed with both methods in an atmosphere with a cloud layer between 1–2 km for two solar zenith distances. τ_c = optical depth of the cloud at $\lambda = 0.55$ μm*

		$\theta_0 = 37.6°$		$\theta_0 = 72.2°$	
Altitude (km)	Method	$\tau_c = 3$	$\tau_c = 12$	$\tau_c = 3$	$\tau_c = 12$
70	Iterative	0.23	0.44	0.40	0.54
	Approx.	0.23	0.43	0.37	0.49
2	Iterative	0.26	0.54	0.47	0.67
	Approx.	0.25	0.51	0.43	0.62

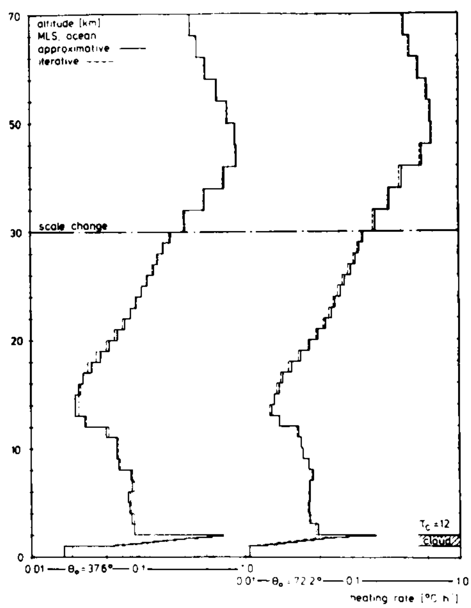

Fig. 2. Radiative heating of the atmosphere model with a cloud layer of $\tau_c = 12$ (at $2 = 0.55$ μm) between 1 and 2 km altitude. Comparison between iterative and approximative solution.

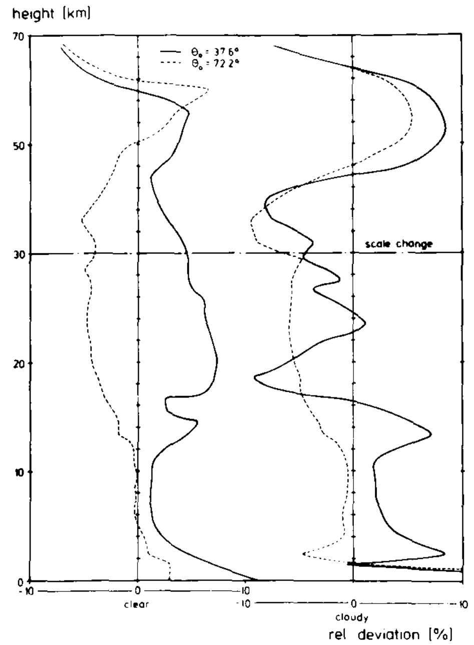

Fig. 3. Relative deviations (in percent) of the iterative value for the model calculations of Figs. 1 and 2.

atmosphere before the integration of the transfer equation. In both cases a model with 70 layers and 87 spectral subintervals has been used. These time requirements drop linearly as the number of layers and sub-intervals is decreased.

3.3. *Cloudy atmospheres*

The errors of the fluxes and divergences computed for an atmosphere with a cloud layer between 1 and 2 km are of the same magnitude as for the cloud-free case (cf. Fig. 3). Some results are compared in Tables 4 and 5 and in Fig. 2.

The agreement between both methods is slightly better for high sun ($\theta_0 = 37.6°$) than for low sun ($\theta_0 = 72.2°$). It decreases with increasing optical thickness of the cloud where the iterative methods starts to become unstable.

4. Conclusions

With the presented two-stream approximation it is possible to study to a large extent the more complicated coupled effects between clouds and the radiation fields in the atmosphere with sufficient accuracy for energetic consideration. A loss in accuracy of less than 10% in the heating rates is counterbalanced by a decrease of computational time by the factor 0.03.

Since the accuracy of the monochromatic transmission functions is not better than 12% (cf. Moskalenko et al., 1968) this loss in accuracy may be tolerated.

This short computation time could be reduced further to a few milliseconds by reducing the vertical and spectral resolution of the model.

The approximative method described here can also be applied to studies of the solar radiation fields in coupled atmosphere-ocean models with clouds in the atmosphere and detailed plankton layers in the upper ocean layers.

In a forthcoming paper more detailed consideration of the influence of various cloud parameters such as the liquid water content, cloud thickness, height, and droplet size distribution on the solar radiation fields in the atmosphere will be discussed. Such studies are necessary for the preparation of field experiments, when the radiation fields near stratiform cloud fields will be measured from aircraft and ships for their parameterization in terms of physical cloud properties.

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МОДИФИЦИРОВАННОЕ ДВУХПОТОКОВОЕ ПРИБЛИЖЕНИЕ ДЛЯ РАСЧЕТОВ БАЛАНСА СОЛНЕЧНОЙ РАДИАЦИИ В ОБЛАЧНОЙ АТМОСФЕРЕ

Для аккуратных расчетов скоростей солнечного разогрева и потоков радиации в вертикально неоднородных моделях атмосферы с облачными слоями развит двухпотоковый метод с переменными параметрами для обратного рассеяния прямых и диффузионных радиационных потоков. Спектр охватывает область между 0,2 и 3,580 мкм, а модельная атмосфера простирается до 70 км по высоте. Сравнение между этим приближенным методом и точным итерационным методом показывает, что для восходящего и нисходящего потоков максимальные относительные отклонения меньше 3%.