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Contributions to the growth and decay of large-scale eddy available potential energy

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ABSTRACT

To study the growth and decay of eddy available potential energy associated with the large-scale atmospheric waves, we have computed composite averages of the terms in the available potential energy equation in the wave-number domain. We found that the eddy available potential energies of the ultra-long waves of wave numbers 1 and 3 grow essentially by receiving energy through the resultant meridional and vertical eddy heat transport process; whereas during the period of decay, the energy loss is essentially through the non-linear heat transport process. We also found that the eddy available potential energies of the synoptic-scale waves of wave numbers 4 to 8 increase by receiving energy and decay by losing energy through both the non-linear and the resultant meridional and vertical eddy heat transport processes. However, the main source of eddy available potential energy for wave number 2 is the contribution of diabatic processes, although diabatic processes contribute to the maintenance of the seasonal mean but not to the growth and decay of the eddy available potential energy of wave number 2. Its time change is also determined by the eddy heat transport processes.

It is also found that the amplitude oscillations of eddy available potential energy are generally in-phase with those of eddy kinetic energy in the same wave number. However, the growth of the former usually occurs slightly behind that of the latter. The growth of eddy available potential energy may be interpreted as an indirect consequence of the growth of the eddy kinetic energy of the wave, which enhances the eddy heat transport, thereby increasing the available potential energy of the wave; whereas the growth of the eddy kinetic energy is a result of the receipt of kinetic energy from other waves through barotropic non-linear exchange processes as described by Tsay and Kao (1978).

1 Introduction

The study of transfer and conversion of available potential energy is essential to the understanding of the growth and decay of atmospheric waves in middle latitudes. According to the linear baroclinic instability theory, an amplifying wave of small amplitude receives available potential energy from the zonal mean motion, and at the same time, converts it to kinetic energy. However, the energy transfers become complex when the waves become finite in amplitude. The examination of the energy transfer processes in the real atmosphere with the use of the Fourier transformed governing equations has been considered as one of the fruitful approaches for the study of the maintenance of large-scale wave motions. We will use the spectral available potential energy equation in wave-number domain to study transfers and conversion of available potential energy in the real atmosphere.

The seasonal average values of various terms in the spectral available potential energy equation in wave-number domain have been computed by Saltzman and Fleisher (1961), Brown (1964), Yang (1967) and others. Saltzman (1970) reviewed these studies and constructed an energy flow diagram. These observational studies revealed that available potential energy is generated by diabatic processes in the zonally averaged component and that it flows

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to waves of all wave numbers by eddy heat transfer processes, particularly to the longer waves. Part of the available potential energy in the longer waves (wave numbers 2 to 5) is transferred to the shorter waves (wave numbers 6 to 15) through interactions among them. The available potential energy in waves of all wave numbers is reduced by diabatic processes occurring on the same scale as the waves and by conversions to eddy kinetic energy.

These studies have provided valuable information for the seasonal averages of the atmospheric processes of the large-scale wave motions. However, to understand the mechanism for the growth and decay of the wave motion, it is necessary to analyze time changes of the energy transfers in the atmospheric waves. Tsay and Kao (1978) have studied the mechanisms for the kinetic energy change in waves of each wave number by analyzing the composite average of each term in the spectral kinetic energy equation at various stages in the life cycle of these waves. They found that except for wave number 2 both the synoptic-scale waves and the ultra-long waves grow by receiving and decay by losing kinetic energy through non-linear energy exchanges among waves. The conversion of eddy available potential energy to kinetic energy was found to be positive in all stages and a maximum in the stage of maximum wave energy. In addition, its contribution was found to be generally cancelled by the energy dissipation. Therefore, the conversion term contributes little to oscillations of wave amplitudes. On the contrary, in all stages of the life cycle of waves of wave number 2, the contribution of non-linear interactions is negative and the conversion term always has a large positive value. However, its amplitude oscillations may also be attributed to the fluctuations of the non-linear interaction contribution and the non-linear boundary term.

The composite average study by Tsay and Kao (1978) proves to be effective in analyzing the changes of kinetic energy in the life cycles of atmospheric waves. The main objective of this study is to further investigate the mechanism for oscillations of available potential energy in wave-number domain.

2. Notation and equations

All the notation used in this study will be the same as that introduced in an earlier paper (Tsay and Kao, 1978). In order to avoid repetition, only the notation which is new in the present paper will be defined. For an atmospheric quantity q, we define

$$\bar{q} = \frac{1}{2\pi} \int_0^{2\pi} q \, d\lambda$$

zonal average of q;

$$\tilde{q} = \frac{1}{b} \int_{\phi_s}^{\phi_N} q \cos \phi \, d\phi$$

meridional average of q;

$$q'' = q - \tilde{q}$$

deviation from the meridional average. The equation of available potential energy per unit mass in wave-number domain in a computational form may be expressed as follows:

$$\frac{\partial}{\partial t} P(n) = S(n) - R(n) - C(n) + G(n) + BS1(n) + BVP(n) + BWP(n) \qquad (1)$$

where

$$P(n) = \frac{C_p \gamma}{b} \int_{\phi_s}^{\phi_N} |\Theta(n)|^2 \cos \phi \, d\phi$$

is the available potential energy of waves of wavenumber n;

$$S(n) = S_1(n) + S_2(n),$$

and

$$S_1(n) = \frac{C_p \gamma}{b} \int_{\phi_s}^{\phi_N} \left[\frac{n}{a \cos \phi} \Psi^2_{\Theta\Theta U}(n) + \frac{1}{a} \Psi^{(1)}_{\Theta\Theta_\phi V}(n) \right] \cos \phi \, d\phi$$

and

$$S_2(n) = \frac{C_p \gamma}{b} \int_{\phi_s}^{\phi_N} \left[\Psi^{(1)}_{\Theta\Theta_p \Omega}(n) + \frac{R}{pC_p} \Psi^{(1)}_{\Theta\Theta\Omega}(n) - \frac{\partial}{\partial p} \Psi^{(1)}_{\Theta\Theta\Omega}(n) \right] \cos \phi \, d\phi,$$

are contributions from non-linear heat transport process to eddy available potential energy of wave number n;

$$R(n) = R_1(n) + R_2(n)$$

and

$$R_1(n) = \frac{C_p \gamma}{b} \int_{\phi_s}^{\phi_N} [\bar{T}_\phi \bar{\Phi}_{V\Theta}(n)] \cos\phi \, d\phi$$

and

$$R_2(n) = \frac{C_p \gamma}{b} \int_{\phi_s}^{\phi_N} \left[\Phi''_{\Omega\Theta}(n) \left(\bar{T}_p - \frac{R}{pC_p} \bar{T} \right)'' \right] \cos\phi \, d\phi,$$

are the rates of transfer of zonal available potential energy to eddy available potential energy in wave number n;

$$C(n) = -\frac{R}{bp} \int_{\phi_s}^{\phi_N} \Phi_{\Omega\Theta}(n) \cos\phi \, d\phi$$

is the rate of conversion between available potential and kinetic energies in wave number n;

$$G(n) = \frac{\gamma}{b} \int_{\phi_s}^{\phi_N} \Phi_{H\Theta}(n) \cos\phi \, d\phi$$

is the rate of generation of available potential energy of wave number n due to diabatic processes;

$$BS1(n) = -\frac{C_p \gamma}{b} [\Psi^{(1)}_{\Theta\Theta V}(n) \cos\phi] |_{\phi_s}^{\phi_N}$$

is the net non-linear boundary fluxes;

$$BVP(n) = -\frac{1}{ba} [\bar{v} C_p \gamma |\Theta(n)|^2 \cos\phi] |_{\phi_s}^{\phi_N}$$

and

$$BWP(n) = -\frac{1}{b} \int_{\phi_s}^{\phi_N} \left[\frac{\partial}{\partial p} \bar{\omega} C_p \gamma |\Theta(n)|^2 \right] \cos\phi \, d\phi$$

are the net boundary fluxes of available potential energy of wavenumber n;

and

$$\gamma = \left[\tilde{T} - \frac{pC_p}{R} \frac{\partial \tilde{T}}{\partial p} \right]^{-1}$$

$b = \sin\phi_N - \sin\phi_s$

C_p = specific heat of air at constant pressure,

h = diabatic heating rate,

$H(n)$ = the Fourier transform of h.

3. Data and computations

Northern hemisphere octagonal grid data at 850, 500 and 200 mb from the National Meteorological Center are used in this study. The period selected is from 00 GMT 1 December 1975 through 12 GMT 29 February 1976. This is the same data set used by Tsay and Kao (1978) for the study of kinetic energy changes. Further descriptions of the data can be found in that study.

Equation (1) is computed for wave energetics at the 500 mb level. The domain of integration is 30° to 60°N. The vertical derivatives have been computed by using data for 850 and 200 mb. The terms $R_2(n)$, $BVP(n)$, $BWP(n)$, and $G(n)$ are not computed. Therefore, the residue term will include contributions from generation of eddy available potential energy due to diabatic processes, data and computational errors, and all other uncomputed terms. The contributions from the errors and all the other uncomputed terms should be small for the residue term generally behaves similarly to the effect of diabatic processes.

Composite studies as in Tsay and Kao (1978) were conducted in order to investigate the mechanisms involved in increases and decreases of eddy available potential energy. Time oscillations of available potential energy are generally in-phase with those of kinetic energy as can be seen in Fig. 2. Therefore, the divisions of five stages in each cycle of time oscillations of available potential energy are the same as those of kinetic energy in Tsay and Kao (1978). Detailed descriptions of the composite method can be found in that study.

4. Maintenance of eddy available potential energy

Seasonal averaged spectra for eddy available potential and kinetic energies in wave-number domain are shown in Fig. 1. We found that the characteristics of spectrum of available potential energy are similar to those of kinetic energy. Both spectra generally decrease with increasing wave number and are approximately proportional to the minus third power of the wave number in the range between 7 and 20. This result agrees with the temperature spectrum in the wave-number domain found earlier (Kao, 1970).

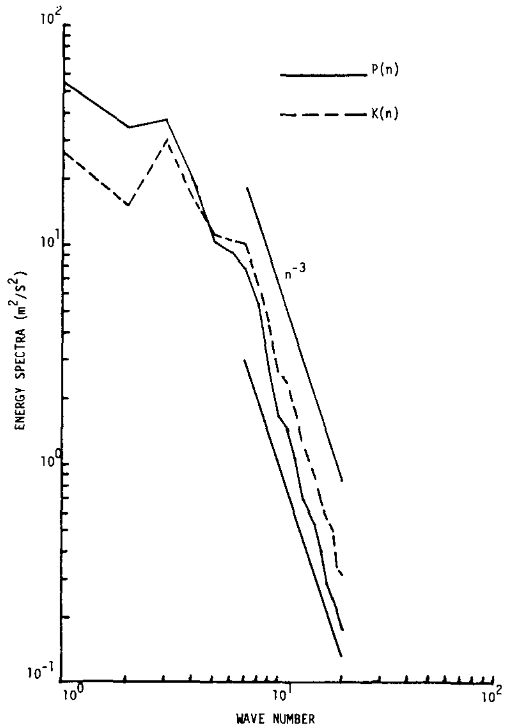

Fig. 1. Seasonal averaged available potential energy (solid) and kinetic energy (dashed) spectra for 30°–60° N, 500 mb, winter 1975–76. The two solid straight lines have n^{-3} dependence.

In order to discuss the maintenance of eddy available potential energy, we have listed the seasonal average values of the terms in Eq. (1) in Table 1. It is noted that the characteristics of the mean transfers and conversion of available potential energy are in general agreement with those edited by Saltzman (1970). Waves of wave numbers 1 to 12, particularly the longer waves, receive available potential energy from the zonally averaged component ($-R_1$ is positive). This positive contribution through the meridional eddy heat transport process is largely cancelled by the conversion to eddy kinetic energy through the vertical eddy heat transport process ($-C$ is negative). In addition, the shorter waves of wave numbers 6 to 20 gain available potential energy from the longer waves of wave numbers 1 to 5 (S is positive for energy gaining waves and negative for energy losing waves).

Contributions of the residue term are generally positive. This result is different from that reported by Brown (1964) and cited in Saltzman's review paper (1970). However, we should point out that Brown's results are vertically integrated values for the 1000–200 mb layer, while ours are at the 500 mb level. According to Hayashi and Golder's (1977) recent computation of generation of eddy available potential energy by diabatic processes in a general circulation model of Geophysical Fluid Dynamics Laboratory at Princeton, the values for wave number 1 and the integrated values for wave numbers 4 to 10 are generally positive at latitudes between 30° and 60° N at the 500 mb level. It may also be noted that the large negative values in the lower troposphere in that model probably would contribute to negative vertically integrated values. Therefore, the results of the present study are not necessarily in contradiction with those of Brown. Since the residue term behaves like a generation term due to diabatic processes, we may infer that data and computational errors and contributions from all the other uncomputed terms are small. The residue term, therefore, may also be considered as the generation term due to diabatic processes. It is noted in Table 1 that the generation term at 500 mb

Table 1. *Seasonal average contributions to the rate of change of eddy available potential energy in wave number domain for 30° –60° N, 500 mb, winter 1975-76. Unit in* $(m/s)^2$ *day*$^{-1}$

Wave number	Terms dP/dt	S	$-R_1$	$-C$	$BS1$	RES
1	–0.1	–3.3	6.7	–4.1	–1.7	2.3
2	–0.1	–3.3	8.6	–13.8	1.1	7.3
3	–0.2	–0.1	13.8	–8.3	–6.1	0.5
4	0.1	–0.2	8.6	–5.3	–2.3	–0.7
5	–0.0	–0.5	4.9	–5.9	0.7	0.8
6	–0.0	0.5	4.6	–6.3	–0.0	1.2
7	–0.2	0.7	6.3	–8.1	–0.3	1.2
8	–0.0	0.8	5.1	–7.7	0.1	1.7
9	–0.0	1.2	2.0	–4.1	–0.3	1.2
10	0.0	0.6	0.7	–2.2	–0.1	1.0
11	–0.0	0.4	0.2	–1.6	0.1	0.9
12	0.0	0.7	0.3	–1.5	0.1	0.4
13	0.0	0.6	0.0	–0.9	–0.1	0.4
14	0.0	0.3	–0.3	–0.4	–0.0	0.4
15	0.0	0.2	–0.1	–0.5	–0.0	0.4
16	0.0	0.0	–0.1	–0.4	0.0	0.5
17	0.0	0.1	–0.0	–0.2	0.0	0.1
18	0.0	0.1	–0.1	–0.2	0.0	0.2
19	0.0	0.2	–0.1	–0.2	0.0	0.1
20	0.0	0.1	–0.0	–0.1	0.0	0.0

has two peak values in wavenumber ranges of 1 to 2 and 6 to 11. The generation of eddy available potential energy of wave numbers 1 and 2 may be attributed to diabatic processes related to land–sea distribution in the northern hemisphere, while the generation of eddy available potential energy of wave numbers 6 to 11 is probably related to the synoptic-scale latent heat release in the mid-troposphere.

5. Contributions to the growth and decay of eddy available potential energy in wave-number domain

The time evolutions of available potential and kinetic energies of waves of wave numbers 1 to 8 are shown in Fig. 2. In order to eliminate diurnal oscillations, daily average energy values are plotted in this figure. We notice that the oscillations of available potential energy are generally in-phase with those of kinetic energy. Therefore, it would be constructive to study the contributions to the growth and decay of available potential energy as well as to those of kinetic energy. In the earlier study, Tsay and Kao (1978) investigated the contributions to the growth and decay of kinetic energy associated with the large-scale atmospheric waves. In the present study, the composite averaged contributions to the rate of change of available potential energy at the five stages are computed and shown in Table 2. Discussion of these contributions is made in the following subsections.

5.1 *Ultra-long waves of wave numbers 1 and 3*

The composite averages of various terms in the available potential energy equation for wave numbers 1 and 3 are shown in Tables 2a and 2c, respectively. The rate of change of eddy available potential energy, dP/dt, is positive in stages I and II, and negative in stages IV and V. This indicates that oscillations of eddy available potential energy for wave numbers 1 and 3 are approximately in-phase with those of kinetic energy. We should point out, however, that the positive value of dP/dt is small in stage I of wave number 3, which indicates that the growth of eddy available potential energy in wave number 3 occurs slightly later than that of the kinetic energy in the same wave number. Contributions to the rate of change of eddy available potential energy are also shown in these tables. It is seen that the contribution from non-linear heat transport processes, S, is generally negative and has smaller magnitude in the energy increasing stages (I and II) than in the energy decreasing stages (IV and V). The contribution from the meridional eddy heat flux, $-R_1$, is positive in all five stages. Its contribution is partly cancelled by the conversion of the available potential energy to kinetic energy in the same wavenumber through the vertical eddy heat transport process, $-C$. It is interesting to note that the resultant contribution of these two eddy heat transport processes, $(-R_1-C)$, is positive in stages I and II, and negative in stages IV and V.

Oscillations of eddy available potential energy of the ultra-long waves of wave numbers 1 and 3 may be explained as follows: In the first 4 to 5 days of their life cycle (stages I and II), the eddy available potential energy grows essentially by receiving energy through the resultant meridional and vertical eddy heat transport process, $(-R_1-C)$. The loss of available potential energy through non-linear heat transport processes is generally small in these stages. As a consequence of the balance between the energy supplied by the resultant meridional and vertical eddy heat transport process and the energy lost through the non-linear processes, there is no increase or decrease in the amplitude of eddy available potential energy for 3 to 4 days (stage III). During the next 3- to 4-day period (stages IV and V), the loss of the available potential energy through non-linear eddy heat transfers is generally greater than the gain through the resultant meridional and vertical eddy heat transfers.

5.2 *Synoptic-scale waves of wave numbers 4 to 8*

Contributions to the rate of change of available potential energy of wave numbers 4 to 8 are shown in Tables 2d to 2h. Like the ultra-long waves, the rate of change of eddy available potential energy is positive in stages I and II, and negative in stages IV and V. However, the positive values in stage I are generally small as compared with those in stage II. Therefore, we may infer that the growth of eddy available potential energy of the synoptic-scale waves usually occurs slightly behind that of the eddy kinetic energy. The non-linear eddy heat transport process, S, generally provides positive

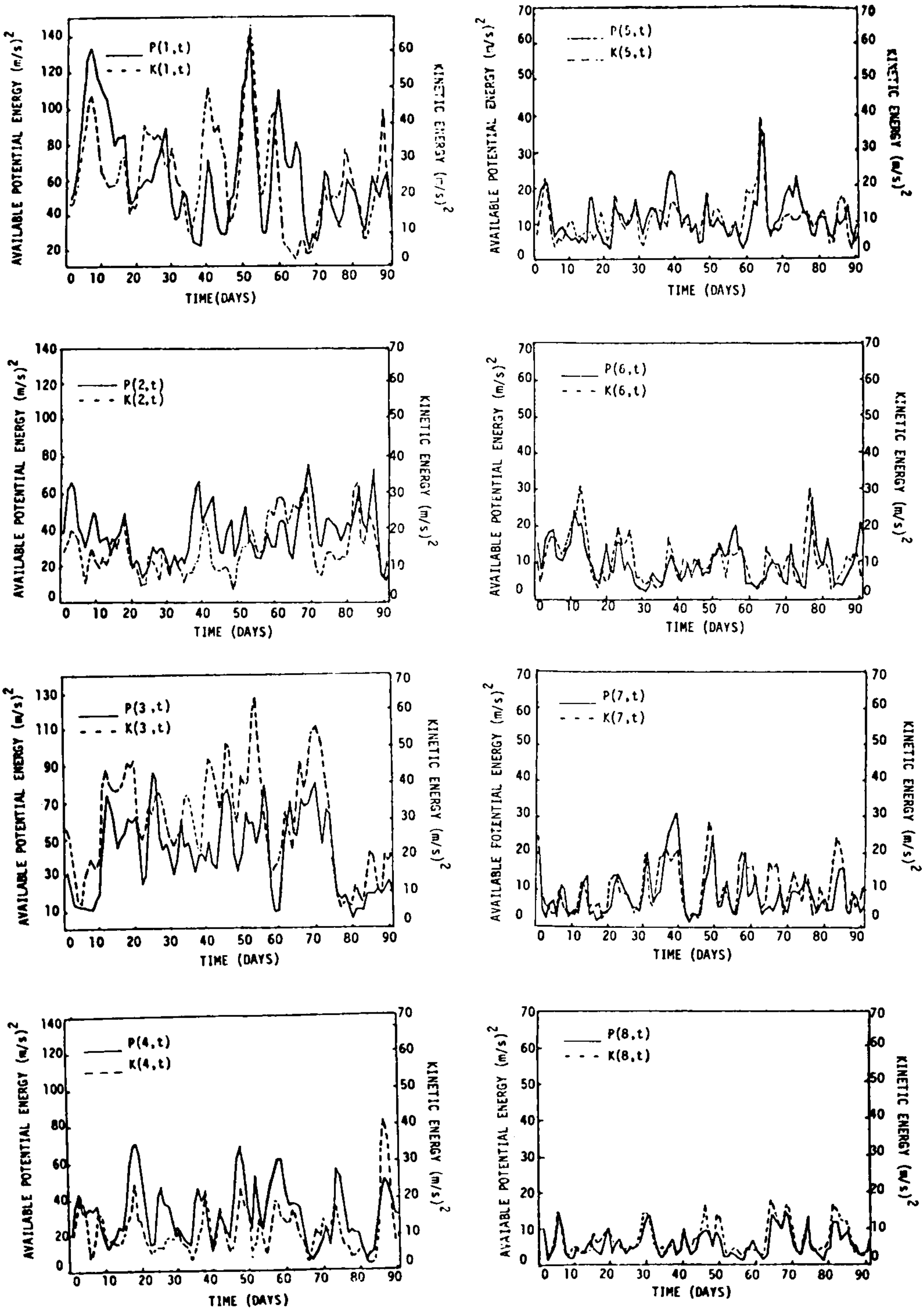

Fig. 2. Time evolutions of available potential energy (solid) and kinetic energy (dashed) of large-scale waves in the period of 1 December 1975 to 29 February 1976.

Table 2. *Contributions to the rate of change at 500 mb eddy available potential energy at individual stages. Unit in* $(m/s)^2$ *day*$^{-1}$.

(a) $n = 1$ (7 cases)

	Stages (number of days)				
Term	I (2.4)	II (1.9)	III (3.0)	IV (2.3)	V (1.9)
dP/dt	6.5	9.0	0.2	−3.2	−6.9
S	1.1	−3.2	−4.0	−1.9	−6.4
$-R_1$	7.7	9.9	8.6	5.2	4.4
$-C$	−0.5	−3.0	−5.3	−4.1	−4.5
$BS1$	−4.4	0.6	0.0	−1.5	−1.8
RES	2.6	4.7	0.9	−0.9	1.4

(b) $n = 2$ (7 cases)

	Stages (number of days)				
Term	I (1.2)	II (1.7)	III (2.8)	IV (1.2)	V (2.3)
dP/dt	4.3	4.6	3.2	−10.0	−7.0
S	−2.2	−2.5	−0.8	−9.4	−5.4
$-R_1$	10.6	12.6	13.9	5.8	4.1
$-C$	−12.7	−17.4	−15.8	−16.7	−15.1
$BS1$	4.0	−1.3	−3.5	0.9	3.9
RES	4.6	13.2	9.4	9.4	5.5

(c) $n = 3$ (7 cases)

	Stages (number of days)				
Term	I (1.4)	II (1.9)	III (3.9)	IV (1.8)	V (1.1)
dP/dt	0.1	10.5	−2.4	−2.3	−12.7
S	−0.5	−0.8	−3.5	−8.7	2.9
$-R_1$	9.3	20.5	19.5	18.4	0.0
$-C$	0.0	−8.4	−10.8	−11.4	−7.1
$BS1$	−6.7	−5.0	−4.9	−6.0	−8.4
RES	−2.0	4.2	−2.7	5.4	−0.1

(d) $n = 4$ (8 cases)

	Stages (number of days)				
Term	I (1.5)	II (1.1)	III (2.2)	IV (2.3)	V (1.3)
dP/dt	1.2	13.3	−0.4	−3.7	−6.6
S	−1.1	1.8	1.7	−2.2	−5.3
$-R_1$	4.4	20.3	11.8	8.1	7.1
$-C$	−5.1	−7.6	−6.1	−7.2	−2.4
$BS1$	−1.7	0.9	−4.8	−1.2	−0.9
RES	4.7	−2.1	−3.0	−1.2	−5.1

(e) $n = 5$ (8 cases)

	Stages (number of days)				
Term	I (1.6)	II (1.0)	III (1.5)	IV (1.4)	V (1.4)
dP/dt	1.4	6.0	−0.1	−1.9	−3.5
S	0.1	0.9	−0.6	−4.2	−3.2
$-R_1$	4.9	9.2	11.0	4.4	0.2
$-C$	−5.0	−3.1	−6.1	−4.5	−5.3
$BS1$	0.0	1.9	0.5	0.0	3.0
RES	1.4	−2.9	−4.9	2.4	1.8

(f) $n = 6$ (8 cases)

	Stages (number of days)				
Term	I (1.2)	II (1.3)	III (1.5)	IV (1.4)	V (2.1)
dP/dt	2.6	3.2	−0.9	−2.9	−1.3
S	3.7	0.9	−2.1	1.5	−2.5
$-R_1$	4.8	10.5	12.2	3.6	−0.4
$-C$	−4.6	−8.5	−10.4	−5.9	−4.5
$BS1$	−2.5	−1.7	0.4	−1.7	3.7
RES	1.2	2.0	−1.0	−0.4	2.4

(g) $n = 7$ (9 cases)

	Stages (number of days)				
Term	I (1.1)	II (1.0)	III (1.9)	IV (1.1)	V (1.2)
dP/dt	1.3	5.4	0.9	−3.3	−4.2
S	2.2	4.6	−0.5	−3.5	−2.8
$-R_1$	3.8	13.4	16.8	7.2	2.0
$-C$	−3.5	−9.6	−17.1	−10.4	−7.7
$BS1$	−0.7	−1.5	−0.1	−0.7	1.0
RES	−0.5	−1.5	1.8	4.1	3.3

(h) $n = 8$ (7 cases)

	Stages (number of days)				
Term	I (1.6)	II (0.7)	III (1.1)	IV (1.1)	V (1.4)
dP/dt	1.1	6.0	0.8	−3.3	−2.6
S	0.4	3.3	3.1	0.2	0.8
$-R_1$	4.2	12.8	15.1	9.5	2.4
$-C$	−4.8	−13.3	−17.3	−14.0	−6.1
$BS1$	0.3	−0.7	0.3	−0.2	−1.4
RES	1.0	3.9	−0.4	1.2	1.7

contribution in the energy increasing stages (I and II), but negative contribution in the energy decreasing stages (IV and V). The contribution by the resultant meridional and vertical eddy heat flux ($-R_1-C$) is also generally positive in stages I and II, and negative in stages IV and V. Therefore, the growth and decay of the eddy available potential energy in wave numbers 4 to 8 are determined by both the non-linear and the resultant meridional and vertical eddy heat transport processes.

5.3 *Waves of wave number 2*

As in the kinetic energy changes, the mechanism for the life cycle of eddy available potential energy of wave number 2 is rather different from that of the other waves. Table 2b shows the composite contributions to the rate of change of eddy available potential energy of wave number 2. The rate of change of eddy available potential energy, dP/dt, is also positive in stages I and II, and negative in stages IV and V. An in-phase relationship between oscillations of eddy available potential and kinetic energies may also be established. It is noted that the contributions from both the non-linear (S) and the resultant meridional and vertical ($-R_1-C$) eddy heat transport processes are negative in all five stages. However, the negative values of both terms are smaller in the energy increasing stages than those in the decreasing stages. Since the contribution by the residue term is positive in all five stages, we may infer that the contribution by the resultant diabatic process is the main source of available potential energy of wave number 2. However, the diabatic process contributes to the maintenance of the wave but not to the growth and decay of the eddy available potential energy of wave number 2. The available potential energy of wave number 2 grows and decays essentially through the non-linear and the resultant meridional and vertical heat transport processes.

6. Conclusion and discussion

We have analyzed the mechanisms for the growth and decay of eddy available potential energy in wave number domain. It is found that the time evolution of the available potential energy is practically in-phase with that of the kinetic energy. In general, the growth and decay of the eddy available potential energy in all wave numbers is contributed by the non-linear, S, and the resultant meridional and vertical, $-R_1-C$, eddy heat transport processes. Both heat transport processes involve velocity fluctuations as well as temperature fluctuations. In the earlier study of Tsay and Kao (1978), it was found that the growth and decay of eddy kinetic energy was essentially governed by the non-linear interactions among velocity waves, L_1, which is a barotropic process not involving temperature fluctuations. Therefore, eddy available potential energy (temperature fluctuations) has little effect on the growth and decay of the eddy kinetic energy.

The mechanism for simultaneous evolution of eddy kinetic and available potential energies may be explained. In stage I, the kinetic energy of a wave number starts to grow by receiving kinetic energy from other waves through the barotropic non-linear exchange processes. As soon as the velocity fluctuations become finite, these eddy velocities provide a vehicle for the non-linear and the resultant meridional and vertical eddy heat transports to contribute to the growth of eddy available potential energy in the same wavenumber. In Fig. 2, the growth of eddy available potential energy appears approximately in-phase but slightly lags behind that of eddy kinetic energy. It may be noted that dP/dt is small in stage I for wave numbers 3 to 8. The wave kinetic energy continues to grow in stage II by receiving kinetic energy from other waves through the non-linear exchange processes. In the meantime, the eddy available potential energy of the same wave number continues to grow by the contributions from the non-linear and the resultant meridional and vertical heat transport processes. Then, both the eddy kinetic and available potential energies maintain their amplitudes for a few days. In the next few days the wave kinetic energy decays by losing its kinetic energy to other waves through non-linear exchange processes. At the same time, heat transports decrease greatly and contribute to the decay of eddy available potential energy of the same wave number. In the life cycle of energy changes, the conversion from eddy available potential to kinetic energy primarily serves to balance the kinetic energy loss by dissipation; whereas the loss of eddy available potential energy through the energy conversion is supplied by the zonally averaged component.

The mechanism for simultaneous evolution of wave available potential and kinetic energies seems reasonably well explained. However, the reason for the approximate in-phase contributions between the non-linear exchange processes to the rate of change of kinetic energy and eddy heat transport processes to the rate of change of available potential energy remains to be studied. We also note that the energy in a particular wave number is not related, in a direct or simple way, to the energy of specific atmospheric disturbances on weather charts. The application of our results to individual disturbances also remains to be further studied.

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ВКЛАДЫ В РОСТ И УМЕНЬШЕНИЕ КРУПНОМАСШТАБНОЙ ВИХРЕВОЙ ДОСТУПНОЙ ПОТЕНЦИАЛЬНОЙ ЭНЕРГИИ

Для изучения роста и уменьшения вихревой доступной потенциальной энергии, связанной с крупномасштабными атмосферными волнами, мы подсчитали средние значения членов в уравнении для доступной потенциальной энергии в пространстве волновых чисел. Мы нашли, что вихревая доступная потенциальная энергия ультрадлинных волн с волновыми номерами 1 и 3 растет в основном за счет процессов меридиального и вертикального вихревого переноса тепла, в то время как в период уменьшения энергии ее потеря происходит за счет процессов нелинейного переноса тепла. Мы также нашли, что вихревая доступная потенциальная энергия волн синоптического масштаба в волновыми номерами 4–8 растет за счет получения энергии и уменьшается за счет потери энергии вследствие как нелинейного, так и меридионального и вертикального переносов вихревого переноса тепла. Однако главным источником доступной вихревой потенциальной энергии для волнового числа 2 является вклад неадиабатических процессов, хотя последние в основном влияют на поддержание сезонного среднего, а не на рост или падение доступной потенциальной энергии волнового переноса тепла. Найдено также, что амплитудные колебания вихревой доступной потенциальной энергии как правило находя тся в Фазе с колебаниями вихревой кинетической энергии для того же волнового числа. Однако рост первых обычно имеет место несколько после роста вторых. Рост вихревой доступной потенциальной энергии можно интерпретировать как косвенное следствие роста вихревой кинетической энергии волны, который усиливает вихревой перенос тепла, увеличивая тем самым доступную потенциальную энергию волны. В то же время рост вихревой кинетической энергии является результатом получения кинетической энергии от других волновых чисел посредством баротропного нелинейного обмена согласно Цэй и Као/1978/.