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Unsteady two-cell similarity solution to a convective atmospheric vortex model

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ABSTRACT

The convective atmospheric vortex model of Kuo (1966) is modifed and a time-dependent two-cell similarity solution is obtained. Predictions of flow fields and thermodynamic property distributions have been made and they compare favorably with available data measured from natural occurrences of tornadoes. The present solution predicts the dissipation of convective atmospheric vortices.

1. Introduction

Kuo's (1966) model representing convective atmospheric vortices has been modifed whereby the Boussinesq approximation is initially imposed in the derivation and the model is specified for a fixed cylindrical coordinate system. The two factors in the model which are considered most important for the formation and maintenance of the vortex motion are unstable stratification and large vorticity. The first factor acts as an energy source inducing horizontal convergence as a result of strong vertical motions due to vertical instability. The second factor provides a source of momentum which induces rapid tangential velocities in the core. A closed form two-cell similarity solution to the present time-dependent model has been obtained by utilizing a solution approach analogous to that employed by Bellamy-Knights (1970) in his extension of the steady two-cell viscous vortex solution of Sullivan (1959). The present solution exhibits certain limitations, namely, the inability to impose no-slip boundary conditions at the ground, the unrestricted linear increase of vertical velocity with height, the fact that the radial and tangential velocities do not vary with height and the fact that at large radius the vertical velocity is constant (non-zero) and the radial inflow increases indefinitely with increasing radius. However, this solution may be applied to investigations of vortex core flows in the axial region where the influence of the ground boundary layer and where lateral variations of the vortex core can be disregarded.

2. Derivation of mathematical model

The basic state equations which consist of the tropospheric temperature distribution, hydrostatic equation and equation of state become:

$$\tilde{p}_0/\tilde{p}_{00} = (\tilde{T}_0/\tilde{T}_{00})^{\tilde{g}/(\tilde{R}\tilde{\gamma})} \tag{1}$$

$$\tilde{\rho}_0/\tilde{\rho}_{00} = (\tilde{T}_0/\tilde{T}_{00})^{\tilde{g}/(\tilde{R}\tilde{\gamma})-1} \tag{2}$$

where $\tilde{T}$ is the temperature, $\tilde{p}$ is the pressure, $\tilde{\rho}$ is the density, $\tilde{R}$ (287.1 m^2 s^{-2} K^{-1}) is the gas constant for air, $\tilde{\gamma}$ is the mean lapse rate in the unstable stratification layer and $\tilde{g}$ (9.8 m s^{-2}) is the gravitational constant. Subscript 0 refers to the basic state and subscript 00 refers to the reference values. Quantities with tilde are dimensional and these quantities are represented non-dimensionally when the tilde is removed.

The thermodynamic properties are split as follows into their basic state and motion induced components:

$$\begin{aligned}
\tilde{\rho}(\tilde{r},\tilde{z},\tilde{t}) &= \tilde{\rho}_0(\tilde{z}) + \tilde{\rho}'(\tilde{r},\tilde{z},\tilde{t})\\
\tilde{p}(\tilde{r},\tilde{z},\tilde{t}) &= \tilde{p}_0(\tilde{z}) + \tilde{p}'(\tilde{r},\tilde{z},\tilde{t})\\
\tilde{T}(\tilde{r},\tilde{z},\tilde{t}) &= \tilde{T}_0(\tilde{z}) + \tilde{T}'(\tilde{r},\tilde{z},\tilde{t})\\
s(r,z,t) &= s_0(z) + s'(r,z,t)\\
\tilde{\theta}_e(\tilde{r},\tilde{z},\tilde{t}) &= \tilde{\theta}_{e0}(\tilde{z}) + \tilde{\theta}_e'(\tilde{r},\tilde{z},\tilde{t})
\end{aligned} \tag{3}$$

where $\tilde{r}$ is the radial coordinate, $\tilde{z}$ is the vertical coordinate, $\tilde{t}$ is the time, $s = \ln \theta_e$ is the dimensionless entropy, $\tilde{\theta}_e = \tilde{T}(\tilde{p}_{00}/\tilde{p})^{\kappa}$ is a modified equivalent potential temperature and $\kappa = \tilde{R}\tilde{\gamma}_m(0)/\tilde{g}$ is a constant factor where $\tilde{\gamma}_m(\tilde{t})$, the time-dependent moist-adiabatic lapse rate, has been taken at time zero.

The governing equations for vortex motion are formulated in a cylindrical coordinate system fixed at a point on the earth's surface. The coriolis and centrifugal forces due to the earth's rotation are relatively insignificant and are neglected.

A simplification is obtained by using dimensionless variables, defined in the present analysis as follows:

$$(\tilde{u}, \tilde{v}, \tilde{w}) = \tilde{v}_{00}(u, v, w), \quad \tilde{p} = \tilde{p}_{00} p$$

$$(\tilde{T}, \tilde{\theta}_e) = \tilde{T}_{00}(T, \theta_e), \quad \tilde{\rho} = \tilde{\rho}_{00} \rho$$

$$(\tilde{r}, \tilde{z}) = \tilde{r}_{00}(r, z), \quad \tilde{t} = (\tilde{r}_{00}/\tilde{v}_{00}) t$$

where u, w and v are the radial, vertical and tangential velocity components.

The continuity equation becomes

$$\frac{\partial}{\partial r}(ur) + \frac{\partial}{\partial z}(wr) = 0 \tag{4}$$

With the approximation, $\tilde{\rho}'/\tilde{\rho}_0 \ll 1$, the radial component of the momentum equation becomes

$$\pi_1 \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + w \frac{\partial u}{\partial z} - r^{-1} v^2 \right)$$
$$= -p_0^{-\tilde{R}(\tilde{\gamma}_m - \tilde{\gamma})/\tilde{g}} \frac{\partial}{\partial r} (p' p_0^{\kappa - 1})$$
$$+ \pi_1 R_e^{-1} \left(\frac{\partial^2 u}{\partial r^2} + r^{-1} \frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial z^2} - r^{-2} u \right) \tag{5}$$

where $R_e = \tilde{v}_{00}\tilde{r}_{00}/\tilde{\nu} = \tilde{\Gamma}_\infty/\tilde{\nu}$ is the Reynolds number, $\tilde{\Gamma}_\infty$ is the circulation at large radial distance, $\tilde{\nu}$ is the eddy viscosity and $\pi_1 = \tilde{v}_{00}^2/(\tilde{R}\tilde{T}_{00})$ is a measure of the ratio of inertia to viscous forces.

The tangential component of the momentum equation becomes

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial r} + w \frac{\partial v}{\partial z} + r^{-1} uv$$
$$= R_e^{-1} \left(\frac{\partial^2 v}{\partial r^2} + r^{-1} \frac{\partial v}{\partial r} + \frac{\partial^2 v}{\partial z^2} - r^{-2} v \right) \tag{6}$$

Utilizing the approximation,

$$-\tilde{\rho}^{-1} \frac{\partial \tilde{p}}{\partial \tilde{z}} - \tilde{g} \cong (\tilde{\rho}'/\tilde{\rho}_0)\tilde{g} - \tilde{\rho}_0^{-1} \frac{\partial \tilde{p}'}{\partial \tilde{z}}$$
$$= -(\tilde{R}\tilde{T}_0/\tilde{r}_{00}) \frac{\partial}{\partial z}(p'/p_0) + (T'/T_0)\tilde{g}$$
$$= -(\tilde{R}\tilde{T}_{00}\,\theta_{e0}/\tilde{r}_{00}) \frac{\partial}{\partial z}(p' p_0^{\kappa-1}) + \tilde{g}s',$$

the vertical component of the momentum equation becomes

$$\pi_1 \left(\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial r} + w \frac{\partial w}{\partial z} \right)$$
$$= -p_0^{-\tilde{R}(\tilde{\gamma}_m - \tilde{\gamma})/\tilde{g}} \frac{\partial}{\partial z}(p' p_0^{\kappa-1}) + \pi_1 \pi_2 s'$$
$$+ \pi_1 R_e^{-1} \left(\frac{\partial^2 w}{\partial r^2} + r^{-1} \frac{\partial w}{\partial r} + \frac{\partial^2 w}{\partial z^2} \right) \tag{7}$$

where $\pi_2 = \tilde{r}_{00}\tilde{g}/\tilde{v}_{00}^2$ is the reciprocal of the Froude number.

After first arranging the energy equation in the form

$$\frac{ds}{dt} = \tilde{k}/(\tilde{v}_{00}\tilde{\rho}_{00}\tilde{r}_{00}\,\tilde{c}_p'\rho T)\nabla^2 \theta_e$$

it is then found to become

$$\frac{\partial s'}{\partial t} + u \frac{\partial s'}{\partial r} + w \frac{\partial s'}{\partial z} + Nw = K \left(\frac{\partial^2 s'}{\partial r^2} + r^{-1} \frac{\partial s'}{\partial r} + \frac{\partial^2 s'}{\partial z^2} \right) \tag{8}$$

where $K = \tilde{k}\theta_{e0}/(\tilde{v}_{00}\tilde{\rho}\tilde{r}_{00}\tilde{c}_p' T)$ is the reciprocal of the Peclet number, $\tilde{c}_p' = \tilde{c}_p + 0.622\tilde{p}^{-1}\,\tilde{L}(d\tilde{e}_s/d\tilde{T})$ is a modified constant pressure specific heat, $\tilde{e}_s$ is the saturation vapor pressure, $\tilde{k}$ is the eddy conduction coefficient, $\tilde{L}$ is the latent heat of condensation, $\tilde{c}_p$ is the constant pressure specific heat, ∇^2 is the Laplacian operator and $N(t) = -(\tilde{\gamma} - \tilde{\gamma}_m)\tilde{r}_{00}/\tilde{T}_0$ is a measure of the stability.

The equation of state becomes

$$p'/p_0 = \rho'/\rho_0 + T'/T_0 \tag{9}$$

The development of (5)–(8) may be further facilitated by reference to Kuo (1966).

The expression for temperature is derived as follows:

$$\theta_e = Tp^{-\kappa}$$

$$s_0 + s' = \ln(\theta_{e0} + \theta_e')$$

$$s' = \ln(1 + \theta_e'/\theta_{e0})$$

therefore

$$T = \theta_{e0} \exp(s') p^\kappa$$

where

$$\theta_{e0} = T_0^{(\tilde{\gamma} - \tilde{\gamma}_m)/\tilde{\gamma}}$$

and

$$T_0 = 1 - \tilde{\gamma}\tilde{z}/\tilde{T}_{00}$$

3. Unsteady two-cell similarity solution

An unsteady two-cell similarity solution exhibiting vortex dissipation as a function of time is now generated. We will assume:

$$\psi = 2zF(x,t)/R_e$$
$$s' = z\sigma(x,t)/\pi_2$$
$$\Gamma = \Gamma(x,t)$$
$$p' p_0^{\kappa-1} = p''(x,t)$$

where ψ is the stream function, Γ is the circulation and $x = \frac{1}{4}r^2 R_e$.

The velocity components, when derived from the above forms, become:

$$u = -r^{-1}\frac{\partial \psi}{\partial z} = -2r^{-1}R_e^{-1}F(x,t)$$
$$w = r^{-1}\frac{\partial \psi}{\partial r} = z\frac{\partial F}{\partial x}$$
$$v = \Gamma/r$$

Substituting these expressions into (5) through (8) yields;

$$-2x\frac{\partial F}{\partial t} - F^2 + 2xF\frac{\partial F}{\partial x} - \tfrac{1}{4}R_e^2\Gamma^2 + 2x^2R_e\pi_1^{-1}\frac{\partial p''}{\partial x} + 2x^2\frac{\partial^2 F}{\partial x^2} = 0 \tag{10}$$

where $T_0^{(1-\bar{\gamma}_m/\bar{\gamma})} \sim 0(1)$

$$-\frac{\partial \Gamma}{\partial t} + F\frac{\partial \Gamma}{\partial x} + x\frac{\partial^2 \Gamma}{\partial x^2} = 0 \tag{11}$$

$$-\frac{\partial^2 F}{\partial x\,\partial t} + F\frac{\partial^2 F}{\partial x^2} - \left(\frac{\partial F}{\partial x}\right)^2 + \sigma + \frac{\partial^2 F}{\partial x^2} + x\frac{\partial^3 F}{\partial x^3} = 0 \tag{12}$$

$$-\frac{\partial \sigma}{\partial t} + F\frac{\partial \sigma}{\partial x} - \sigma\frac{\partial F}{\partial x} + \beta\frac{\partial F}{\partial x} + R_eK\left(\frac{\partial \sigma}{\partial x} + x\frac{\partial^2 \sigma}{\partial x^2}\right) = 0 \tag{13}$$

where $\beta(t) = -\pi_2 N(t)$. Let $R_eK = 1$ (i.e., a Prandtl number of unity) and consider the case where $\sigma = \delta(t)\,(\partial F/\partial x)$.

Accordingly, (12) and (13) become the following identical equation

$$-\frac{\partial^2 F}{\partial x\,\partial t} + F\frac{\partial^2 F}{\partial x^2} - \left(\frac{\partial F}{\partial x}\right)^2 + \delta\frac{\partial F}{\partial x} + \frac{\partial^2 F}{\partial x^2} + x\frac{\partial^3 F}{\partial x^3} = 0 \tag{14}$$

where

$$\frac{d\delta}{dt} + \delta^2 = \beta \tag{15}$$

In terms of the similarity variable $\eta = \eta(x,t)$, (11) becomes

$$\frac{d^2\Gamma}{d\eta^2} + \alpha(\eta)\frac{d\Gamma}{d\eta} = 0 \tag{16}$$

where

$$\alpha(\eta)x\left(\frac{\partial \eta}{\partial x}\right)^2 = -\frac{\partial \eta}{\partial t} + F\frac{\partial \eta}{\partial x} + x\frac{\partial^2 \eta}{\partial x^2} \tag{17}$$

The function $\alpha(\eta)$ will be obtained such that the boundary conditions for the circulation are satisfied. A solution of (14) is obtained by assuming the form of F as follows:

$$F = a(t)x - 2b(t)[1 - \exp(-c(t)x)] \tag{18}$$

where

$$\eta = c(t)x\,.$$

Substitution of F into (14) and equating (i) terms independent of x, (ii) coefficients of $\exp(-c(t)x)$ and (iii) coefficients of $x\exp(-c(t)x)$ yields the following three equations, respectively:

$$\frac{da}{dt} + a^2 - \delta a = 0 \tag{19}$$

$$c\frac{db}{dt} + b\frac{dc}{dt} - 2b^2c^2 + 2abc - \delta bc + bc^2 = 0 \tag{20}$$

$$-\frac{dc}{dt} + ac - c^2 = 0 \tag{21}$$

Substitution of F and η into (17) yields:

$$\alpha c\eta = -x\frac{dc}{dt} + c[ax - 2b(1 - e^{-\eta})] \tag{22}$$

and with (21)

$$\alpha(\eta) = 1 - 2b\eta^{-1}(1 - e^{-\eta}) \tag{23}$$

For α to be a function of η only, b must be a constant.

Equations (15), (19), (20) and (21) reduce to the form

$$\frac{dc}{dt} + \zeta c^2 = 0 \tag{24}$$

when

$$a = (2b - 1)c \tag{25}$$
$$\delta = (4b - 3)c \tag{26}$$

and

$$\beta = (24b^2 - 38b + 15)c^2 \quad (27)$$

where ζ is a constant.

A solution of (24) is

$$c = (c_1 + \zeta t)^{-1} \quad (28)$$

where

$$\zeta = 2(1 - b) \quad (29)$$

and c_1 is the constant of integration.

The conditions where $\beta \geq 0$ and $t \geq 0$ are considered. Therefore, $c_1 \geq 0$ and $\zeta > 0$ to preclude a singularity. Accordingly,

$$F = (2b - 1)c(t)x - 2b(1 - e^{-c(t)x}) \quad (30)$$

Expressions for the dependent variables are found to be:

$$u = -2r^{-1}R_e^{-1}[(2b - 1)cx - 2b(1 - e^{-cx})] \quad (31)$$

$$w = z[(2b - 1)c - 2bc\, e^{-cx}] \quad (32)$$

$$v = r^{-1}\Gamma(\eta) \quad (33)$$

where

$$\Gamma(\eta) = \Gamma_\infty[H_{2b}(\eta)/H_{2b}(\infty)]$$

and

$$H_m(x) = \int_0^x \exp\left\{-s + m\int_0^s [(1 - e^{-t})/t]\,dt\right\} ds$$

$$\begin{aligned} p(x,z,t) &= p_0(z) + p'(x,z,t) \\ &= (1 - \tilde{\gamma}\tilde{z}/\tilde{T}_{00})^{\tilde{g}/(\tilde{R}\tilde{\gamma})} \\ &\quad - \pi_1 R_e^{-1} p_0^{1-\kappa}\left\{\tfrac{1}{2}c^2(-4b^2 + 8b - 3)x \right. \\ &\quad + 2b^2x^{-1}[1 - \exp(-cx)]^2 \\ &\quad \left. + \tfrac{1}{8}R_e^2 \int_x^\infty \Gamma^2 x^{-2}\,dx\right\} + p_0'(t) \end{aligned} \quad (34)$$

where $p_0'(t)$ is the pressure at the stagnation point. The drop in pressure at the axis of the fully developed vortex is given by

$$p'(0,z,0) = -\tfrac{1}{8}\pi_1 \mathrm{Re}\, p_0^{1-\kappa} \int_x^\infty \Gamma^2 x^{-2}\,dx \quad \text{for} \quad p_0' = 0$$

$$T = \theta_{e0}\exp\{z\pi_2^{-1}c^2(4b - 3)[-1 + 2b(1 - e^{-cx})]\}p^\kappa \quad (35)$$

Limiting cases of the present solution will now be examined.

(a) For $t = 0$.

At $t = 0$, $c(0) = c_1^{-1} = \beta(0)^{1/2}(24b^2 - 38b + 15)^{-1/2}$ where it is assumed that $\beta(0) = -\pi_2 N(0)$ is defined for a fully developed vortex. Therefore

$$F = (2b - 1)\,c_1^{-1}x - 2b(1 - e^{-x/c_1})$$

Let

$$c_1^{-1} \to 1$$

then

$$F \to (2b - 1)x - 2b(1 - e^{-x}) \quad (36)$$

Therefore, (36) approaches the two-cell steady-state vortex solution given by Kuo (1966) as $b \to 1$. This initial condition represents the fully developed vortex.

(b) For $t \to \infty$, $\beta = 0$.

Therefore,

$$F = \frac{\partial F}{\partial x} = 0 \Rightarrow u = w = 0$$

and from (6)

$$v \to 0$$

The pressure, (34), reduces to

$$p(x,z,\infty) = p_0(z)$$

since $\varepsilon \gg 1$ and for $p_0'(\infty) = 0$.

The temperature, (35), reduces to

$$T = T_0$$

The above case clearly shows the dissipative nature of the present solution. As time approaches infinity the atmosphere approaches a neutral equilibrium state ($\beta = 0$) and the vortex dissipates.

(c) For $\tilde{r} \gg 1$, $b \to 1$, $c_1 = \tilde{v}_{00}/(2\tilde{a}\tilde{r}_{00})$.

Therefore,

$$\tilde{u} \to -\tilde{a}\tilde{r}$$

$$\tilde{w} \to 2\tilde{a}\tilde{z}$$

$$\tilde{v} \to \tilde{\Gamma}'_\infty\,\tilde{r}^{-1}(1 - e^{-\tilde{a}\tilde{r}^2/(2\tilde{\nu})})$$

$$\tilde{p} \to \tilde{p}_{00}(1 - \tilde{\gamma}\tilde{z}/\tilde{T}_{00})^{\tilde{g}/(\tilde{R}\tilde{\gamma})} - \tilde{\rho}_{00}p_0^{1-\kappa}\left[\tfrac{1}{2}\tilde{a}^2\tilde{r}^2 + \int_x^\infty \tilde{v}^2\tilde{r}^{-1}\,d\tilde{r}\right] + \tilde{p}_0'$$

where $\tilde{p}_{00} = \tilde{\rho}_{00}\tilde{R}\tilde{T}_{00}$, $x = \tilde{v}_{00}\tilde{r}^2/(4\tilde{r}_{00}\tilde{\nu})$ and $\tilde{\Gamma}'_\infty$ is the initial circulation about the origin.

This case asymptotically approaches Burgers'

(1940, 1948) and Rott's (1958, 1959) steady one-cell vortex solution. Since $\tilde{p}''$ is not a function of $\tilde{z}$, the term containing $\tilde{z}^2$ is not included in the present pressure equation.

Also, the second term in the present pressure equation is $\tilde{z}$-dependent (i.e., multiplied by $p_0^{1-\kappa}$) due to the formulation of the model.

(d) For $b = 0$, $c_1 \gg 1$.

Therefore,

$$\tilde{u}, \tilde{w} \to 0$$

and

$$\tilde{v} \to \tilde{\Gamma}_\infty \tilde{r}^{-1}(1 - e^{-\tilde{r}^2/(4\tilde{\nu}\tilde{t})})$$

This case asymptotically approaches Oseen's (1911) unsteady vortex solution and is also for a neutral equilibrium state.

4. Discussion of results

The time-dependent two-cell similarity solution is discussed in terms of predicted radial distributions of the dependent variables. Comparisons are made with available data measured from natural tornado occurrences. A difficulty arose by not having a complete collection of data for any one tornado. Typical values of the parameters in the mathematical model are specified for comparison with tornadoes as follows: $\tilde{\gamma} = 0.006$ K m^{-1}, $\tilde{\gamma}_m(0) = 0.003$ K m^{-1}, $\tilde{\Gamma}_\infty = 7500$ m^2 s^{-1} and $\tilde{\nu} = 5$ m^2 s^{-1}. Based on these values of the parameters, the following values of the non-dimensional numbers result: $R_e = K^{-1} = 1500$, $\kappa = 0.0878877$, $N(0) = -0.00883697$, $\pi_1 = 0.000888549$ and $\pi_2 = 113.16$ where the reference parameters were assigned the values $\tilde{T}_{00} = 294$ K, $\tilde{p}_{00} = 982$ mb, $\tilde{r}_{00} = 866.025$ m, $\tilde{v}_{00} = 8.66025$ m s^{-1} and $\tilde{t}_{00} = 100$ s. In the following discussion of the distributions it is also assumed that $b = 0.99$ which results in $c_1 = 0.949947$ and $\zeta = 0.02$. These values were chosen within the constraints specified in Section 3.

(a) *Radial velocity*

From (31), $u = 0$ when

$$\eta = [2b/(2b-1)]\,(1 - e^{-\eta})$$

Therefore

$$\eta \cong 1.8656.$$

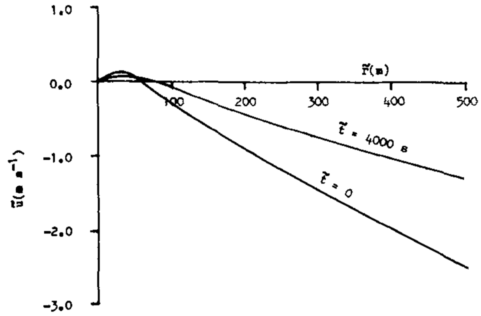

Fig. 1. Radial velocity distribution for $b = 0.99$, $c_1 = 0.949947$ and $\zeta = 0.02$ at time zero (fully developed vortex) and at a time of 4000 s.

The flow is thus divided into two cells separated by the time-dependent cylindrical surface given by $\eta \cong 1.8656$. This radial velocity is outward in the inner cell and inward in the outer cell. Fig. 1 shows the radial distribution of the radial velocity at time zero and at a time of 4000 s. It is seen that the maximum radial velocity in the inner cell decreases as time increases.

(b) *Vertical velocity*

From (32), $w = 0$ when

$$\eta = \ln[2b/(2b-1)] \cong 0.7032996.$$

This suggests another time-dependent cylindrical surface in the inner cell where the vertical velocity is zero. This cylindrical surface is defined by $\eta \cong 0.7032996$. Fig. 2 shows the radial distribution of the vertical velocity at time zero and at a time of 4000 s at an altitude of 30 m. It is seen that the maximum vertical velocity decreases as time increases.

(c) *Tangential velocity*

The radial distribution of tangential velocity resembles that of the Rankine combined vortex. Fig. 3 shows the radial distribution of tangential velocity at time zero and at a time of 4000 s. It is seen that as time increases the vortex diffuses such that the tangential velocity decreases at all finite radial positions and the radius of maximum tangential velocity increases. For a two-cell vortex, the effect of the radial inflow in the outer cell and the radial outflow in the inner cell is to cause the tangential velocity profile to become concave upwards. With reference to Fig. 3, the maximum velocity for the fully developed case is approxi-

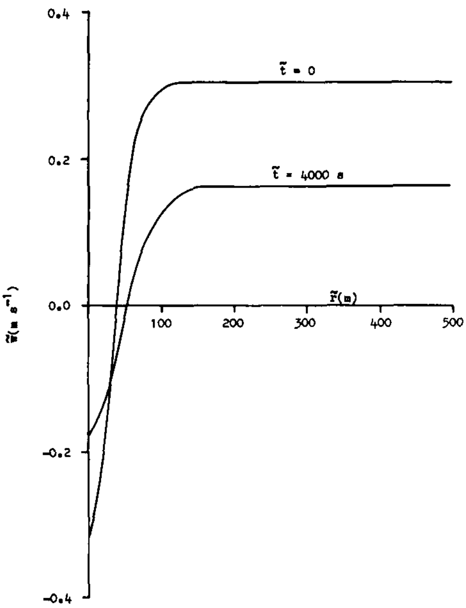

Fig. 2. Vertical velocity distribution at an altitude of 30 m for $b = 0.99$, $c_1 = 0.949947$ and $\zeta = 0.02$ at time zero (fully developed vortex) and at a time of 4000 s.

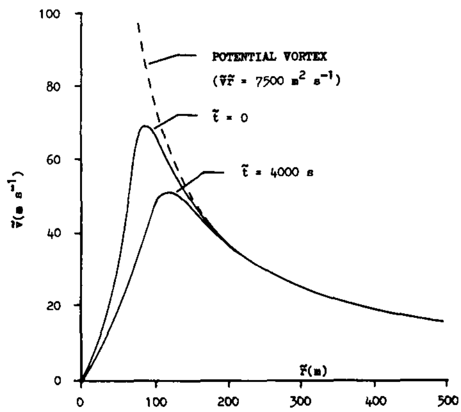

Fig. 3. Tangential velocity distribution for $b = 0.99$, $c_1 = 0.949947$ and $\zeta = 0.02$ at time zero (fully developed vortex) and at a time of 4000 s.

mately 70 m s^{-1} at a radius of about 90 m. In comparison, measurements from the April 2 1957 tornado given by Hoecker (1960) result in a maximum tangential velocity of 75 m s^{-1} at a radius of 70 m and an altitude of 300 m.

(d) *Pressure*

Fig. 4 provides the predicted radial pressure distribution at ground level ($z = 0$) and at time

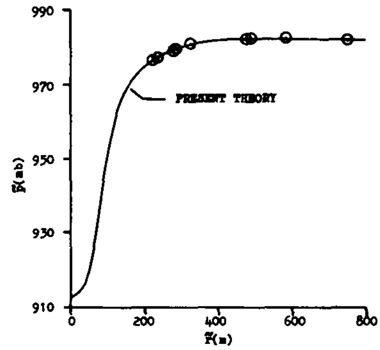

Fig. 4. Pressure distribution for $b = 0.99$, $c_1 = 0.949947$ and $\zeta = 0.02$ at ground level ($z = 0$) and at time zero (fully developed vortex) as compared to data measured in the June 8 1953 Cleveland tornado (encircled dots).

zero. This result is similar to that reported by Kuo (1966). It is seen that very good agreement is made in comparison with data obtained from direct observations in the outer part of the June 8 1953 Cleveland tornado given by Lewis and Perkins (1953). The present theory predicts a total pressure drop of about 69 mb for the fully developed case. Unfortunately, no direct measurements have been made in the central parts of tornadoes and therefore a comparison with this predicted value cannot be made.

(e) *Temperature*

Fig. 5 gives a predicted radial distribution of temperature at an altitude of 30 m and at time zero. It is seen that the temperature is lowest at the axis and increases with an increase in radius. The lower temperature in the core is attributed to expansional cooling.

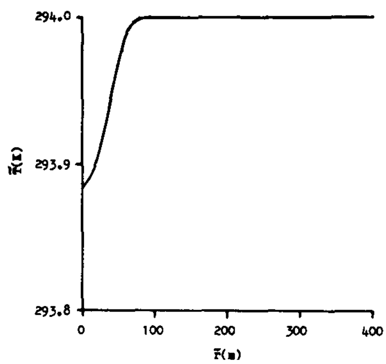

Fig. 5. Temperature distribution for $b = 0.99$, $c_1 = 0.949947$ and $\zeta = 0.02$ at an altitude of 30 m and at time zero (fully developed vortex).

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НЕСТАЦИОНАРНОЕ ДВУХЯЧЕЙКОВОЕ АВТОМОДЕЛЬНОЕ РЕШЕНИЕ МОДЕЛИ КОНВЕКТИВНОГО АТМОСФЕРНОГО ВИХРЯ

Модифицируется модель конвективного атмосферного вихря Куо (1966) и получено зависящее от времени автомодельное решение, описывающее две ячейки. Получены поля течений и распределения термодинамических величин, которые хорошо согласуются с доступными данными измерений в естественных торнадо. Это решение предсказывает диссипацию конвективных атмосферных вихрей.