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A parameterization of large-scale heat transport in mid-latitudes. Part I. Transient eddies

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ABSTRACT

The heat transport by large-scale transient eddies in mid-latitudes is formulated in terms of the basic external parameters—primarily the horizontal temperature gradient, the vertical static stability, and Rossby's radius of deformation. A mid-latitude baroclinic instability combined with a geostrophic turbulence model is the basis for the parameterization, whereby the wind variance both in the horizontal and vertical planes is computed. The result of heat flux computation agrees both with the earlier results of Stone (1974), and observations with reasonable accuracy.

1. Introduction

The global mean state of the atmosphere is maintained by a statistical balance between sources and sinks of the circulation which remove energy and momentum from the atmosphere (Oort and Rasmusson, 1971; Newell et al., 1972). Observational studies together with recent numerical simulations of the global circulation have elucidated the mechanisms producing this balance in the atmosphere. One of the notable findings from these studies is the similarity between the statistical behavior of atmospheric motions and turbulence which has long been a major subject in general fluid dynamics. Turbulence models have recently been used in atmospheric dynamics to assess the predictability of weather forecasts (Lorenz, 1969; Leith, 1971) and parameterize sub-grid scale processes in numerical weather prediction (Leith, 1968).

Turbulence models have a number of other meteorological applications which include interpretation of inherent internal variability and mixing in the atmosphere due to nonlinear interactions among motions involved in various scales. For example, if the energy power spectrum is known for the atmospheric motions, one can estimate the total velocity variance in the system which yields the atmospheric diffusivity for heat and momentum. In a recent study Charney (1971) has raised this possibility by introducing into the model the constraints due to gravity, the earth's rotation, and stratification. His geostrophic turbulence model is capable of prescribing the power spectrum for available potential energy as well as for horizontal kinetic energy. Although his model in its present form may be limited to middle and high latitudes, it immediately suggests several possibilities. First of all, the forcing by baroclinic unstable waves can be incorporated into the turbulence model which makes it possible to determine the turbulence spectrum in terms of external parameters, primarily meridional baroclinicity. Secondly, it is possible to formulate the power spectrum for vertical velocities based on their high correlation

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with geopotential height. The result is particularly useful for parameterization of hydrological processes discussed in Sasamori (1975). Finally by combining these two possibilities we may be able to model transport of heat from equator to pole through mid-latitudes. This paper describes the method of these parameterizations and compares their predictions with observations.

2. Baroclinic unstable wave and the cascade of potential enstrophy

Spectral energy distributions for large-scale atmospheric motions have been analyzed by a number of authors using observed data (Benton and Kahn, 1958; Ogura, 1958; Wiin-Nielsen, 1967; Kao, 1970; Kao and Wendell, 1970; Julian et al., 1970; Steinberg et al., 1971; among others), and for simulated data by the general circulation experiments (Wellck et al., 1971; Manabe et al., 1970). These analyses agree that the energy spectrum in mid-latitudes varies with approximately minus three power of the longitudinal wavenumber in the high-wavenumber region and that the energy density depends more weakly on the wavenumber in the low-wavenumber region. The theory, based on a two-dimensional turbulence model, was discussed by Kraichnan (1967) and postulates that the spectrum in the high-wavenumber region is the result of inertial cascade of enstrophy (one half of squared vorticity) from excited wavenumbers, whereas in the low-wavenumber region it is maintained by horizontal kinetic energy cascaded from the excited wavenumbers. Steinberg et al. (1971) evaluated nonlinear interactions for basic second-order moments of the large-scale atmospheric variables in the domain of wavenumber, based on the actual data. It was found that kinetic energy is cascaded from intermediate wavenumbers to both small and large wavenumbers, while available potential energy and potential enstrophy are cascaded from small to large wavenumbers with relatively little accumulation in the intermediate range with the cascade rate of potential enstrophy being approximately 1.7×10^{-16} s^{-3}. Lilly (1972) carried out a series of numerical simulations of the two-dimensional turbulence and produced results supporting Kraichnan's theory. More recently Thompson (1973) derived the spectral shape function from the basic equations of motion through an analytic deductive mathematical method.

Charney's study (1971), which includes the effects of gravity and the earth's rotation, shows that atmospheric motions smaller than the Rossby radius of deformation form a three-dimensional homogeneous isotropic turbulence when observed from a proper coordinate system stretched in the vertical direction. He further shows that, because of the tendency to be isotropic, the energy is apportioned equally into each degree of freedom so that the available potential energy and all components of the horizontal kinetic energy are equal at the individual wavenumber. This conclusion is validated by comparison with the observation.

Ogura (1958) also pointed out earlier that statistical isotropy is indeed a good approximation for horizontal atmospheric motions whose scales are smaller than about longitudinal wavenumber six which is close to the radius of deformation. In general, statistical isotropy is a good approximation only when the spatial extent of the mean state is sufficiently large compared to the scales of individual eddies. In a rotating atmosphere restrained by gravity, however, the horizontal scale for the mean state is prescribed by the Rossby radius of deformation and not by the geometrical size of the earth. In fact, a major feature of the mean state is confined to this scale, i.e. the meridional width of the zonal westerly jet in the mid-latitude troposphere. Therefore it is pertinent to apply a turbulence model only for wavelengths smaller than the radius of deformation.

The two-dimensional turbulence model is a heuristic prototype for large-scale atmospheric turbulence. The energy spectrum in the inertial subrange is given by (Lilly, 1972):

$$E_2(k_2) = \alpha_2 \eta^{2/3} k_2^{-3}, \tag{1}$$

where k_2 is the two-dimensional wavenumber, η is the cascade rate of enstrophy per eddy, and α_2 is a constant whose magnitude is approximately one. In a stationary state η is constant, independent of k_2, but it depends on the spectral width which a single eddy may exhibit in wavenumber space. Tennekes (1976) has proposed a wave packet model for turbulent eddies, such that they possess an organized structure only within a length approximately equal to $2\pi/k_2$. In the course of the eddy's motion the structure remains unchanged only for a limited time interval which is equal to

$\eta^{-1/3}$. The effect of the eddy in the wavenumber space is thereby dispersed over $\Delta k_2 \approx k_2$. Tennekes notes that this idea has been used, explicitly or otherwise, in most current theories of turbulence. We will use his model, but with a slight modification by conjecturing that an eddy has an organized structure in physical space within a length $2\pi\alpha_2/k_2$, so that the eddy has the spectral width,

$$\Delta k_2 \approx k_2/\alpha_2 \tag{2}$$

in the wavenumber space.

Suppose that we can collect at any time all such eddies which have a specific central wavenumber k_2. We refer to this ensemble of eddies as the subensemble with k_2. First we assume that k_2 is separated from both excitation wavenumber k_e and dissipation wavenumber k_d where molecular dissipation of enstrophy into heat becomes important (Fig. 1). The subensemble of these eddies is

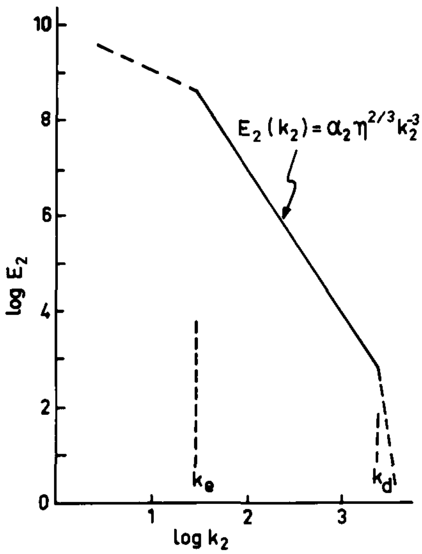

Fig. 1. A schematic energy spectrum for the two-dimensional turbulence.

schematically shown by a box in Fig. 2a. Nonlinear inertial interactions among all eddies in the system continually feed the enstrophy into this box. The rate of feeding is denoted by $\eta_{\rm in}$. At the same time the enstrophy of the subensemble is leaking from the box by a rate of $\eta_{\rm out}$ per eddy in the box. In the stationary state

$$\eta_{\rm in} = \eta_{\rm out}. \tag{3}$$

The influx $\eta_{\rm in}$ is not necessarily made up from the

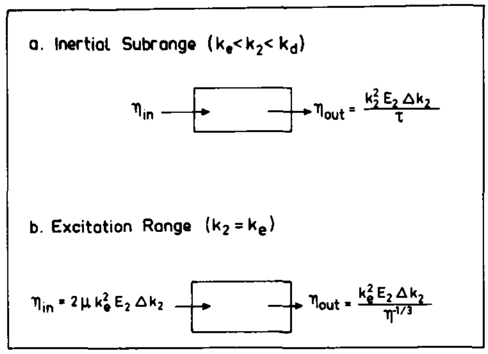

Fig. 2. The enstrophy budget for the subensemble of eddies with wavenumber k_2 in the inertial subrange (a) and with the wavenumber of the baroclinic instability (b).

eddies with the wavenumbers adjacent to k_2, but almost all the eddies in the turbulence system may contribute to producing $\eta_{\rm in}$. However, if the leakage of the enstrophy is essentially due to diffusion by eddies whose horizontal scales are smaller than $2\pi\alpha_2/k_2$, we may assume that the rate of leakage is proportional to the enstrophy density in the box:

$$\eta_{\rm out} = \frac{k_2^2 E_2(k_2)\Delta k_2}{\tau}, \tag{4}$$

where τ is the proportionality constant with dimension of time. By substituting (1), (2), and (3) into (4) we identify τ as

$$\tau = \eta^{-1/3}. \tag{5}$$

We consider next the enstrophy budget at the excitation wavenumber k_e, where the enstrophy feeding rate is known. The most important mechanism feeding the enstrophy into the large-scale atmospheric turbulence is the linear baroclinic instability. The rate of feeding is proportional to the enstrophy itself and the proportionality constant is the growth rate of the unstable eddy:

$$\eta_{\rm in} = 2\mu k_e^2 E_2(k_e)\Delta k_e, \tag{6}$$

where μ is the amplitude growth rate and the factor 2 is necessary since $\eta_{\rm in}$ refers to the rate of feeding of squared vorticity. The leakage from the excitation wavenumber may be approximately given by (4) with k_e replaced for k_2. By equating $\eta_{\rm in}$ and $\eta_{\rm out}$ we obtain

$$2\mu = \eta^{1/3}. \tag{7}$$

One of Lilly's experiments was designed to simulate the turbulence system maintained by a baroclinic instability. A simple forcing with $\mu = 0.5$ was introduced at wavenumber eight. The diagnostic analysis of the spectrum determined α_2 to be 4.0 ± 0.5. By using this value, together with his spectrum (shown in his Fig. 12c), we found $\eta^{1/3}$ to be 0.86 (nondimensional), which favorably compares with $2\mu = 1.0$.

The relation (7) can also be compared to the energy spectrum observed in the atmosphere and the theoretical growth rate for realistic baroclinic unstable waves. To do so it is desirable to consider the effect of water vapor condensation which is possibly involved in the observed energy spectrum as well as in the growth rate of the unstable wave. In general the condensed water falls out when the air parcels are moving upward, whereas air parcels when moving downward contain only a small amount of liquid water to evaporate. Since this directional dependency of heating makes it difficult to treat the problem exactly, we will use the following approximation.

Consider a large area in the atmosphere where the mean vertical stability is conditionally unstable. In the region of upward motion the saturated air tends to be gravitationally unstable while the area moving downward remains stable if the liquid water in the air is negligible. In an unstable wave system, however, the vertical motion appears always as a pair of fields of upward and downward motions which are closely adjacent and, therefore, coupled dynamically through mass continuity, so that the instability in the region of upward motion may be suppressed by the stability in the region of downward motion.

In a turbulence system, if the individual eddy is approximately a doublet of vortices which is modeled by a non-propagating wave packet with a spatial extent only slightly larger than the central wavelength, upward and downward motion regions may also exist in pairs. It is assumed that a mean static stability exists for the eddy as a whole which is given by an average of two static stabilities defined separately for the upward and downward motions. We further assume that upward motion is saturated with water vapor and the downward motion is dry. Thus the static stability for each portion of the motions is

$$S^{\uparrow} = \frac{g}{\bar{\theta}_e} \frac{\partial \bar{\theta}_e}{\partial z}, \tag{8}$$

and

$$S^{\downarrow} = \frac{g}{\bar{\theta}} \frac{\partial \bar{\theta}}{\partial z} \tag{9}$$

where $z = -H \ln (p/p_s)$ is the log-pressure coordinate, and H is the scale height of a standard atmosphere with the surface pressure p_s. $\bar{\theta}$ is the basic potential temperature and $\bar{\theta}_e$ is the basic equivalent potential temperature. The mean static stability for an eddy as a whole is thus given by

$$S = (S^{\uparrow} + S^{\downarrow})/2$$

and its Brunt–Väisälä frequency by

$$N = \sqrt{S}.$$

If we follow Charney (1971) it is found that the potential vorticity

$$\nabla^2 \psi + \frac{f}{\bar{\rho}} \frac{\partial}{\partial z} \left(\frac{\bar{\rho}}{N^2} \frac{\partial \psi}{\partial z} \right) + f \tag{10}$$

is conserved following the horizontal projection of the eddy's motion, where ψ is the geostrophic stream function, $\bar{\rho}$ the density of the basic mean state, f the Coriolis parameter, and ∇ the horizontal Lapace operator. In the inertial subrange where the scale of eddy is smaller than Rossby's radius of deformation, Charney concluded that the spectrum for total energy including the horizontal kinetic and available potential energy is a function of the three-dimensional wavenumber through

$$E(k) = \alpha_3 \eta_p^{2/3} k^{-3}, \tag{11}$$

where k is the wavenumber in the three-dimensional space which is stretched in the vertical direction by

$$\zeta = \frac{N}{f} z.$$

η_p is the cascade rate of potential enstrophy per eddy (one half of squared potential vorticity) and α_3 is a constant. Since (1) and (11) are formally the same, we anticipate that μ and η_p are related by

$$2\mu = \eta_p^{1/3}. \tag{7a}$$

The growth rate μ may be estimated by a quasi-geostrophic two-layer model. For an adiabatic frictionless atmosphere Wiin-Nielsen (1963) derived the frequency equation in the form:

$$(1 + q^2)\xi^2 + \frac{q^2}{l^2} \beta\xi + (q^2 - 1)(\Delta u)^2 = 0, \tag{12}$$

where

$$\xi = \bar{u}(500 \text{ mb}) - c - \beta/l^2$$

with $\bar{u}$(500 mb) the mean zonal wind at 500 mb, c the complex phase velocity, β the meridional derivative of the Coriolis parameter, $\Delta u = \bar{u}$(250 mb) $- \bar{u}$(750 mb), l the longitudinal wavenumber and q the normalized wavelength defined by

$$q = l_e/l. \tag{13}$$

Here

$$\pi/l_e = \pi\sqrt{R\sigma/2^{R/c_p+1}}/f$$

is the Rossby's radius of deformation for the two-layer version and σ is a measure of static stability. We assume that

$$\sigma = (\sigma^\uparrow + \sigma^\downarrow)/2$$

with

$$\sigma^\downarrow = -0.25 p_s\, \partial\bar{\theta}/\partial p$$

and

$$\sigma^\uparrow = \sigma^\downarrow\, (\partial\bar{T}/\partial z + \gamma_s)/(\partial\bar{T}/\partial z + \gamma_d),$$

where γ_s and γ_d are the temperature lapse rate for saturated and dry air, respectively. The imaginary part of ξ gives the growth rate as

$$\mu = l\,\mathrm{Im}\,(\xi). \tag{14}$$

The numerical values of μ were calculated from (12) using the same parameters suggested by Wiin-Nielsen (1963) for winter mid-latitudes but with the vertical static stability $\sigma = 10°$ (Fig. 3). Since the instability is significant only in a narrow interval $q = 1$–3, we took an average over this interval and found

$$2\mu = 3.1 \times 10^{-5}\ \mathrm{s}^{-1}. \tag{15}$$

The mean growth rate is approximately equal to the growth rate at $q = 1.1$. By neglecting the stabilizing effect of the β-term we find that (14) reduces to

$$\mu = 0.3 l_e\, \Delta u$$

or reduces to

$$\mu = -0.3\, \frac{l_e R}{f}\, \frac{\partial\bar{T}}{\partial y} \tag{16}$$

with use of the thermal wind relation for Δu. Eq. (15) will be compared to the cascade rate of the potential enstrophy based on the spectral data of Kao and Wendell (1970) as summarized by Charney (1971). Since we are not able to determine η_p independently from α_3 the result is given only as a function of possible values of α_3:

α_3	$\eta_p^{1/3}$ (s^{-1})
1	2.6×10^{-5}
2	1.8×10^{-5}
4	1.3×10^{-5}.

The comparison of these to (15) indicates that $\eta_p^{1/3}$ with $\alpha_3 \approx 1$ gives the best agreement to the relation (7a).

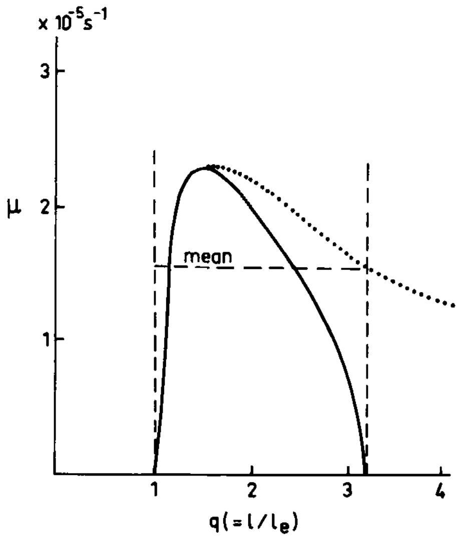

Fig. 3. The amplitude growth rate as a function of the longitudinal wavenumber for the two-layer baroclinic unstable wave in mid-latitude winter. The dotted line shows a growth rate without stability due to the β-effect.

3. Local equilibrium and power spectrum for vertical velocity

In a quasi-geostrophic system the relevant vertical velocity is calculated from the ω-equation based on the thermodynamic and vorticity equations. Although the ω-equation is exact in the quasi-geostrophic context, the solution does not seem suitable for a turbulence model because of the nonlinear forcing term in the equation. Hence, we propose here an alternative method that gives the power spectrum indirectly from the known spectrum for the geopotential.

Let us return to Fig. 2, which shows the budget of enstrophy for the subensemble of eddies whose central wavenumber is specified by k. The air particles in this subensemble change continually: new members are coming in with enstrophy from other subensembles and old ones are leaving for other subensembles carrying out the enstrophy from the box. The mean residence time of an air particle in the box may be of the order of $\eta_p^{-1/3}$ (Tennekes, 1976). The metabolism is essentially due to the horizontal nonlinear advection in both vorticity and thermodynamic equations. In the quasi-geostrophic system the vertical advection of vorticity is disregarded and the effect of vertical motion is retained only as a linear term in the vorticity equation in order to relate it with the horizontal wind divergence. The thermodynamic equation also retains the vertical motion as a linear term. Since the linear term has no effect on the scale-interactions we may postulate that a primary role of the large-scale vertical motion is not to transfer the potential vorticity from one scale to another but to convert the potential energy into kinetic energy at the specific scale of motion. If the time-scale needed for this energy conversion is sufficiently short compared to $\eta_p^{-1/3}$, we may assume that the field of vertical motion of the air particles in the subensemble is statistically in equilibrium with other atmospheric variables. We will now validate this assumption by using a numerical value.

The tendency to generate vertical motion during the course of energy conversion may be considered using the linearized system of atmospheric equations. With hydrostatic equilibrium they are:

$$\frac{\partial u}{\partial t} - fv + \frac{\partial \phi}{\partial x} = 0, \tag{17}$$

$$\frac{\partial v}{\partial t} + fu + \frac{\partial \phi}{\partial y} = 0, \tag{18}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{N}{f}\frac{\partial w}{\partial \zeta} - \frac{w}{H} = 0, \tag{19}$$

$$\frac{\partial}{\partial t}\left(\frac{\partial \phi}{\partial \zeta}\right) + fNw = 0, \tag{20}$$

where from the top are the two equations for horizontal motions, the equation of continuity, and the thermodynamic equation. Most of the symbols are standard, except $w = (d/dt)[-H \ln (p/p_s)]$ is the vertical velocity in the log-pressure coordinate, and ϕ is the geopotential. The solution for the above equations is assumed to be in the form

$$X = X_k \exp\{i(\mu_f t + lx + my + n\zeta)\}, \tag{21}$$

where X represents any atmospheric variable, X_k is the proportionality constant, l, m, and n are the wavenumbers in x, y, and ζ directions, respectively, and μ_f is the frequency. Substitution of (21) into eq. (17)–(20) yields a set of homogeneous linear equations for X_k whose determinant should vanish in order to have a non-zero solution. Thus we obtain the frequency equation:

$$\mu_f^2 = f^2 k^2/n^2; \tag{22}$$

a well-known inertial-gravitational frequency, where

$$k^2 = l^2 + m^2 + n^2. \tag{23}$$

If the turbulence system is isotropic, l, m, and n should take values of the same order of magnitude, yielding $\mu_f \approx \pm\sqrt{3}f$. This is significantly larger than the $\eta_p^{-1/3}$ estimated before, since $f \sim 10^{-4}$ s^{-1} in middle latitudes. Thus in the subensemble of eddies the vertical motion may be in equilibrium with the geopotential through the thermodynamic eq. (20). By assuming statistical isotropy, we obtain from (20)

$$E_w(k) = \frac{k^2}{N^2} E_\phi(k), \tag{24}$$

where E_w and E_ϕ are one half of the spectral variance of vertical motion and geopotential height, respectively.

In the quasi-geotrophic system the geopotential height is a single scalar quantity which generates the total energy $E(k)$, the sum of horizontal kinetic and available potential energy. Therefore

$$\begin{aligned} E_\phi(k) &= f^2 E(k)/k^2 \\ &= \alpha_3 f^2 \eta_p^{2/3} k^{-5}. \end{aligned} \tag{25}$$

Combining (24) and (25) we obtain

$$E_w(k) = \alpha_3 \left(\frac{f}{N}\right)^2 \eta_p^{2/3} k^{-3}. \tag{26}$$

The equipartition of the total energy into the horizontal kinetic energy and available potential energy (Charney, 1971) determines the energy spectra for the u- and v-components to be

$$E_u(k) = E_v(k) = \frac{\alpha_3}{3} \eta_p^{2/3} k^{-3}. \tag{27}$$

Eqs. (26) and (27) suggest that $[u, v, (N/\sqrt{3}f)w]$ is an isotropic vector in three-dimensional space (x,y,ζ).

The above results can be examined by comparison with observations. A one-dimensional longitudinal energy spectrum for each component of the vector may be obtained by the transform given by Charney (1971):

$$U(l) = \frac{\alpha_3}{15}\,\eta_p^{2/3} l^{-3}, \tag{28}$$

$$V(l) = 3U(l), \tag{29}$$

$$W(l) = 3\left(\frac{f}{N}\right)^2 U(l). \tag{30}$$

The geopotential height spectrum E_ϕ can also be transformed to the longitudinal spectrum by (Kovasznay et al., 1949):

$$\Phi(l) = 4\pi \int_l^\infty \frac{E_\phi}{k}\,dk$$

$$= \frac{4\pi}{5}\,\alpha_3 f^2 \eta_p^{2/3} l^{-5}. \tag{31}$$

Some years ago Syono et al. (1955) analyzed the geopotential height of the 500-mb level in the northern hemisphere winter at several latitudes. Ogura and Miyakoda (1954) made a theoretical analysis for the spectrum and suggested that the geopotential height variance be proportional to minus seven-thirds power of the three-dimensional wavenumber. The result was, however, based on the assumption that the energy spectrum is proportional to minus five-thirds power of the wavenumber which is incorrect in the light of updated observation and theory. Indeed the spectra shown in Syono et al. can be fit very well by a function with minus five power of the longitudinal wavenumber (Fig. 4). The spectrum shown in the figure (at lat. 40° N) is applied to eq. (31) to estimate η_p. For $\alpha_3 = 1$, we found $\eta_p^{1/3} = 1.7 \times 10^{-5}$ s^{-1}, which again compares favorably with the previous estimate.

The energy spectrum of the vertical velocity was analyzed by Saltzman and Fleisher (1960, 1961). The power spectrum of the individual pressure change at the 600-mb level for February 1959 at 50° N is reproduced in Fig. 5. The spectrum is fit very well by the theoretical distribution (26) proportional to the minus three power for the eddies having the longitudinal wavenumber larger than eight. Also the simulated data in the general circulation experiments (Kasahara and Washington, 1971) are suitable for the present analysis.

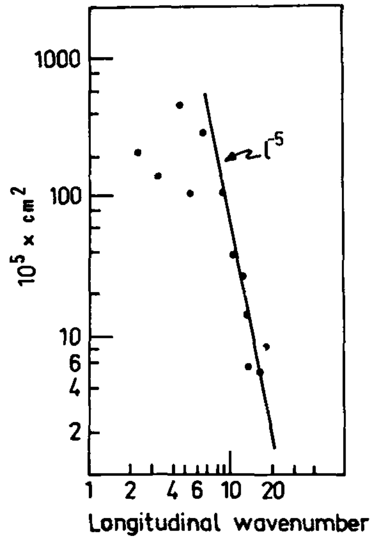

Fig. 4. Variance of the 500-mb geopotential height at 40° N during winter, analyzed by Syono et al. (1955).

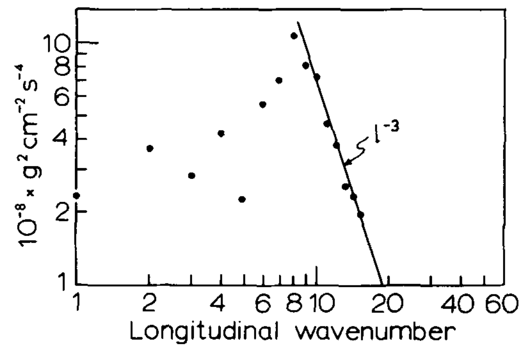

Fig. 5. Variance of the individual pressure change at 600-mb level, analyzed by Saltzman and Fleisher (1960) for February 1959 at 50° N. Unit: 10^{-8} gram^2 cm^{-2} s^{-4}.

From a history tape obtained for a January simulation with 5° horizontal resolution, full mountains, and hydrological cycle, the variance of the vertical velocities was calculated through Fourier analysis (Washington, 1976). The examples are shown in Fig. 6 for latitude 40° N at 6 and 9 km above sea level. The spectral shape is largely unchanged for the other mid-latitudes. The results are compared favorably with the theoretical function (26) for wavenumbers larger than eight. The ratio between $W(l)$ and $U(l)$ was also computed by the simulated data at 40° N. The mean value in the inertial subrange is 1.2×10^{-4}, which is compared to the theoretical value $3(f/N)^2 = 1.9 \times 10^{-4}$.

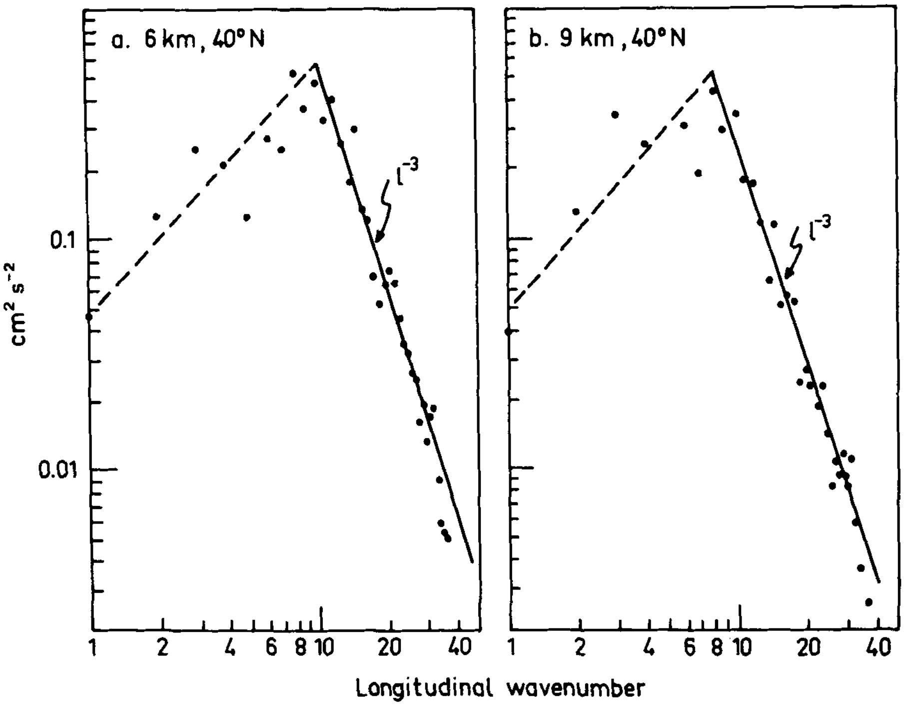

Fig. 6. Variance of the vertical velocities calculated from the data simulated by the NCAR global general circulation model: (a) 40° N at 6 km level and (b) 40° N at 9 km level.

4. Transient eddies and stationary waves

There are three basic time-scales in mid-latitude atmospheric motions:

the Rossby-wave frequency

$$\mu_r = \beta/\sqrt{l^2 + m^2}, \tag{32}$$

the nonlinear interaction rate

$$\mu_n = \eta_p^{1/3}/2, \tag{33}$$

and the inertial-gravitational frequency

$$\mu_f = \pm fk/n. \tag{34}$$

μ_r represents the rate of the stabilizing effect for long waves against the baroclinic instability. Similarly μ_f is the rate of the stabilizing effect for short waves against the baroclinic instability. Since μ_n is the growth rate of baroclinic instability against these stabilizing effects, $\mu_r > \mu_n$ for the long waves, and $\mu_f > \mu_n$ for the short wave disturbances (transient eddies). More specifically, we divide the whole spectrum of atmospheric motions into two regions at the spectral boundary l_e.

The relationship between the basic time-scales are summarized in Table 1. For small-scale disturbances (region 1), smaller than the radius of deformation, μ_n is larger than μ_r, the nonlinear effect being faster than the linear effect. As time goes on the disturbances may represent eddies in which the organized structure is confined only within the central wavelength. However, the nonlinear interaction is still slow compared to μ_f so that the statistical equilibrium is maintained among individual degrees of freedom of motion.

For large-scale disturbances (spectral region 2), horizontal scale being larger than the radius of deformation, the vertical scale of motion is still confined within the scale height owing to the solid lower boundary and the stable stratosphere on the top. In this spectral region we assume that the flux of potential enstrophy is at most equal to η_p which is identified in the spectral region 1. Thus the linear Rossby's β-effect is faster than the nonlinear effect, so that each disturbance may tend to be a planetary wave which ultimately reaches equilibrium with stationary external boundary conditions.

Table 1

	Spectral region	Time-scale relationship	Remark
1.	$l, m, n > l_e$	$\mu_r < \mu_n, \mu_n < \mu_f$	Transient eddies
	$l, m, n \sim l_e$	$\mu_r \sim \mu_n, \mu_n < \mu_f$	Unstable wave
2.	$l, m < l_e, n \sim l_e$	$\mu_r > \mu_n$	Stationary waves

Since the baroclinic instability takes place within a narrow spectral region, the whole spectrum may be divided into two parts: the transient eddy regime and the stationary wave regime. In the next section we will formulate the heat transport by the transient eddies and the heat transport by the stationary waves will be discussed in Part II of this paper.

5. Transient eddy transport and conclusion

Consider a longitudinal belt at the center of the mid-latitudes denoted by y_0 and consisting of many eddies originating from various latitudes. Since the individual eddies have the life time $\eta_p^{-1/3}$ on the average, the path length of an eddy may be correlated with the meridional velocity of the eddy by

$$y' = v' \eta_p^{-1/3}. \tag{35}$$

By assuming that the meridional change of the mean temperature over the path length is not large, the temperature of the eddy at the original point may be approximated by

$$T' = \bar{T}_0 - \frac{\partial \bar{T}}{\partial y} y', \tag{36}$$

where $\bar{T}_0$ is the temperature at $y = y_0$. Let $P(v')dv'$ be the probability that a specific eddy in the slab at y_0 has the passing velocity between v' and $v' + dv'$. The mean heat flux through the slab may then be given by

$$\overline{v'T'} = \int_{-\infty}^{\infty} T'v'P(v')\, dv'. \tag{37}$$

By substituting (35) and (36) into (37) we obtain

$$\overline{v'T'} = -\eta_p^{-1/3}\sigma_v^2 \frac{\partial \bar{T}}{\partial y}, \tag{38}$$

where

$$\sigma_v^2 = \int_{-\infty}^{\infty} v'^2 P(v')\, dv'$$

is the variance of the meridional wind velocity for the ensemble of eddies in the slab. We assume that the statistics for the eddy ensemble in the slab is equal to the statistics taken from the spectral analysis, so that the variance may be calculated from

$$\sigma_v^2 = 2 \int_{\sqrt{3}l_e}^{\infty} E_v(k)\, dk = \frac{\alpha_3}{9} \eta_p^{2/3} l_e^{-2}. \tag{39}$$

By substituting (39) into (38), the heat flux is given by

$$\overline{v'T'} = -K_y \frac{\partial \bar{T}}{\partial y} \tag{40}$$

with the diffusion coefficient

$$K_y = -\frac{0.067\alpha_3 R}{f l_e} \frac{\partial \bar{T}}{\partial y}, \tag{41}$$

where (7a) and (16) have been used.

It is to be noted that the result given by (40) with (41) is almost identical with the earlier result by Stone (1974), if we let $\alpha_3 = 2$. The present parameterization is based on the baroclinic instability model as Stone did, but the approach to close the amplitude of the baroclinic unstable wave is somewhat different. Stone assumed that the baroclinic unstable wave grows until its kinetic energy becomes equal to the kinetic energy of the basic flow. These two parameterizations seem to support each other in regard to the basic assumptions made in each study. Since the diffusion coefficient defined by (41) is a bulk diffusivity for the mid-latitude troposphere as a whole, it is not very appropriate to apply the parameterization at every latitude. Nevertheless, the result of computation is shown in Fig. 7 where the rate of heating due to flux divergence is computed at the 500-mb level. The largest discrepancy is seen in the subtropics. Stone (1974) has pointed out that the meridional heat flux should be parameterized with variable zonal wind with latitude.

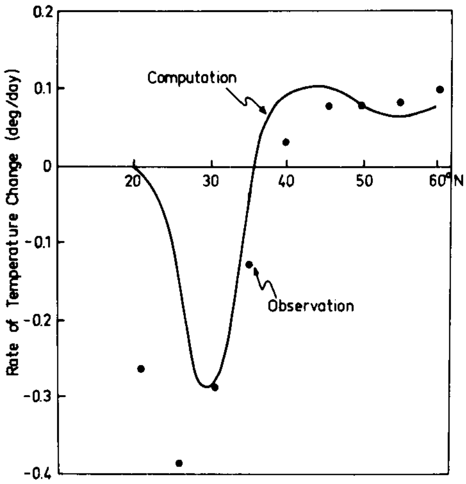

Fig. 7. The rate of temperature change due to divergence of the horizontal sensible heat flux transported by the transient eddies. The black dots were computed from the data in Oort and Rasmusson (1971) by their density weighted vertical averages.

Similar consideration for the vertical direction may lead us to the formulation of the vertical diffusivity, but a difference should be considered due to the solid boundary at the lower surface and the stable stratosphere at the top. The presence of these boundaries should impede the vertical free movement for the large eddies whose vertical size is comparable to the scale height. It may be then assumed that the vertical mixing due to eddies whose size is larger than $2\pi\kappa/l_e$ is negligible, where κ may be identified with the Karman constant. Thus the effective variance contributing to the vertical diffusion is given by

$$\sigma_w^2 = 2\int_{\sqrt{3}l_e/\kappa}^{\infty} E_w(k)\,dk. \qquad (42)$$

By the same approach as discussed for the horizontal diffusion, we find the vertical diffusion coefficient to be

$$K_z = \eta_p^{-1/3}\sigma_w^2 = -\frac{0.2\alpha_3\kappa^2 R}{f l_e}\left(\frac{f}{N}\right)^2 \frac{\partial \bar{T}}{\partial y}. \qquad (43)$$

The ratio of K_z versus K_y is

$K_z/K_y = 3\kappa^2 f^2/N^2$.

For the mid-latitude means we set $f = 10^{-4}\ \mathrm{s}^{-1}$ and $N = 2\times 10^{-2}\ \mathrm{s}^{-1}$, which yields $K_z/K_y = 1.0\times 10^{-5}$ if $\kappa = 0.4$. Louis (1975) summarized the eddy diffusivity distribution based on various sources of data. For the northern hemisphere winter at the 5 km level, his table (Table 2) shows (in SI units):

Table 2

Lat (°N)	K_y	K_z	K_z/K_y
30	1.16×10^5	1.87	1.6×10^{-5}
40	2.85×10^5	3.35	1.2×10^{-5}
50	4.68×10^5	4.09	0.9×10^{-5}
60	5.27×10^5	3.51	0.7×10^{-5}

Although there is a tendency for K_z/K_y to decrease with latitude, the ratio is favorably compared with the result obtained from our model.

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ПАРАМЕТРИЗАЦИЯ КРУПНОМАСШТАБНОГО ПЕРЕНОСА ТЕПЛА В СРЕДНИХ ШИРОТАХ. ЧАСТЬ I. НЕСТАЦИОНАРНЫЕ ВИХРИ

Перенос тепла крупномасштабными вихрями в средних широтах определяется через основные внешние параметры — главным образом, через горизонтальный температурный градиент, вертикальную статическую устойчивость и радиус деформации Россби. Бароклинная неустойчивость средних широт в комбинации с моделью геострофической турбулентности является основой для параметризации, с помощью которой вычисляется распределение ветра по горизонтали и вертикали. Результаты вычисления потока тепла согласуются как с вычислениями Стоуна (1974), так и с наблюдениями с разумной точностью.