SVENSKA GEOFYSISKA FÖRENINGEN

VOLUME 2, NUMBER 3 **Tellus** AUGUST 1950

A QUARTERLY JOURNAL OF GEOPHYSICS

Experimental Studies of a Polar Vortex I

By DAVE FULTZ, University of Chicago[1]

(Manuscript received 22 May, 1950)

Abstract

A description is given of experiments carried out during 1949 and early 1950 that were designed to produce some phenomenon exhibiting effects of the resultant horizontal Coriolis force on a symmetrical vortex that results from the latitude variation of the Coriolis force. A circular cap of dense liquid at the base of a small hemispherical shell filled with liquid was forced to rotate about a vertical axis independently of the rate of rotation of the remaining fluid and containing vessels. Within the ranges of 20 to 150 r.p.m. for the vessel and 0 to 120 relative r.p.m. for the cap vortex, the center of mass of the polar cap never showed appreciable displacements away from the polar axis exept for a range of sufficiently high relative rotations when the vortex was *anticyclonic* in sense. Numerical checks are given with a simplified calculation in spherical coordinates of the torque equilibrium for the vortex as a whole.

1. Introduction

In two recent papers Prof. C.-G. Rossby (1948, 1949) has advanced the idea that a certain type of instability of anticyclonic vortices may have important dynamic and synoptic applications. He obtains a resultant force (or better a resultant torque) acting on various vortex models in a direction away from the pole as a consequence of only somewhat restricted hypotheses concerning the pressure-wind field relation. The same computation, though with some differences in results, has been carried out by Davies (1948). Such an effect, if not obscured by some of the processes not fully taken care of, would have many consequences for the pole-to-equator transfer of air within the anticyclones and for related mechanisms in the general circulation.

In view both of the importance of the implications of this result and of the delicacy of conclusions which depend on finer aspects of the pressure-wind relation, Prof. Rossby suggested in the spring of 1949 that an attempt be made to get experimental observations of some phenomenon as nearly analogous as possible. This has been done by making modifications of the rotating hemispherical shell equipment used for an earlier investigation (Fultz 1949) and the following constitutes a preliminary report on results.

A little reflection soon shows that it would be impossible to set up an experimental barotropic vortex with complete dynamic and geometric similarity to the case contemplated by Rossby. It was therefore decided to attempt to use a two-fluid system in the available apparatus and modify the theoretical analysis to correspond. A lower circular cap of denser fluid (shown in cross-section in fig. 1) is caused to rotate about the vertical axis of the system by a fan which is driven independently of the rotating containing shells. The lower cap is overlain by water that fills the hemispherical space to the equator. The fan is thus able to produce a relative rotational motion of the

[1] This research has been supported in part by the Geophysical Research Directorate. Air Force Cambridge Research Laboratories, Air Materiel Command, U. S. Air Force.

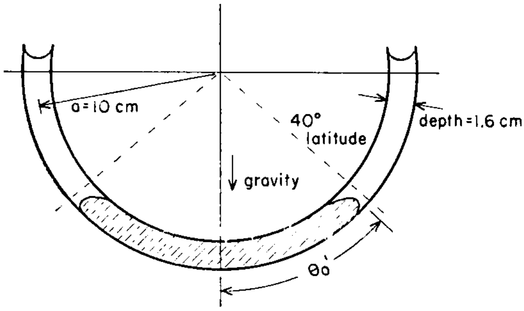

Fig. 1. Cross-section through the hemispherical shell showing the approximate position of the cap of denser liquid.

lower cap of fluid with respect to the upper fluid and the hemispherical bowls. The two-fluid arrangement was adopted in order to help confine the relative motion to the lower cap and to assist in identifying the cap.

The simplest case calculated by ROSSBY (1949) was that of a circular vortex of incompressible fluid of unit depth and radius R rotating rigidly with angular velocity ω in a resting environment. He found for a vortex of infinitesimal size centered near the pole, that the displacement b of the center from the pole would obey approximately an equation of the form[1]

$$\frac{d^2b}{dt^2} = -\left(\frac{1}{4} f_0 \omega \frac{R^2}{a^2}\right) b \qquad (1)$$

where f_0 is the Coriolis parameter at the pole and a is the radius of the earth or sphere. The term on the right is an average force per unit mass on the vortex that results from the variation of the Coriolis parameter. It tends to displace the vortex as a whole either toward or away from the pole along a meridian.

Equation (1) is seen to imply that a cyclonic vortex ($\omega > 0$) will undergo stable oscillations across the pole while an anticyclonic vortex ($\omega < 0$) will experience an exponential flight away.

In the experiment described there are several factors that do not correspond with Rossby's calculation where the Coriolis force makes the only contribution to be calculated:

1) The two-fluid system of lighter above denser liquid introduces a hydrostatic stability not present in the atmospheric vortex.

2) In terms of the equations of motion relative to the rotating bowls the centrifugal force terms do not drop out as they do in the ordinary meteorological equations.

Both of these differences could theoretically be eliminated by setting up a vortex relative to a rotating paraboloidal shell of fluid that is rotating at such a rate that the sum of the horizontal components of the gravity and centrifugal forces vanish. We contemplate carrying this out with apparatus which is being designed at present, even though the problems of generating a more or less isolated vortex will be more difficult to overcome.

Qualitatively in the present instance, we attempted to find whether there was any difference between cyclonic vortices and sufficiently strong anticyclonic ones of the type to be expected from the theory: namely, any tendency for the center of mass of the experimental vortex to be thrown away from the pole beyond some critical anticyclonic rotation value. Such a phenomenon was actually observed over certain ranges of conditions, with some accompanying difficulties of interpretation which are considered at some length below.

2. Theoretical considerations

In the first attempts at making an analysis corresponding to Rossby's for this experimental case the centrifugal term was neglected. The gravity term was estimated as approximately that for a simple pendulum of length a with gravity reduced by the factor $\gamma \equiv \frac{\varrho - \varrho'}{\varrho}$ where ϱ is the density of the lower fluid and ϱ' that of the upper fluid. (1) then becomes

$$\frac{d^2b}{dt^2} \approx -\left(\frac{1}{4}\frac{f_0 R^2}{a^2} - \frac{g}{a}\gamma\right) b \qquad (2)$$

where g is the acceleration of gravity and the other terms have been defined previously.

However, in order to get a reasonable sized experimental vortex it was necessary to carry

[1] Among other approximations to be discussed later, it may immediately be noticed that, since the fluid is frictionless, the assumption of solid rotation within the vortex is incompatible with the vorticity equation: $f + \zeta$ = individual constant, except when the center is located at the pole itself. Thus the real problem being considered in this case is that of a rotating solid disc in a frictionless fluid.

the lower cap up to as far as 40° latitude. This made it obvious that some attempt would have to be made to perform the calculations without restriction to infinitesimal size and also if possible without restriction to infinitesimal displacements of the center from the pole since only rather pronounced ones are visible. At first we tried analytical integration without approximation of ROSSBY's equations (1949, p. 170). These are written for a transformation plane tangent at the pole with projection from the center of the sphere. These approaches were only partially successful and Mr R. R. Long of our laboratory, going further, discovered that an exact integration of these terms could be carried out for the sphere under any of several assumptions as to the velocity field in the vortex. The remaining uncertainties in this calculation are mainly of a kind associated with what Rossby calls the "escape motions" of the environment fluid.

The above mentioned analysis by Mr Long may be put in the following form: consider a circular vortex of incompressible fluid and of unit depth centered at colatitude α of a rotating spherical shell. This vortex is to be imbedded in a layer of incompressible fluid of density ϱ' which temporarily we consider at rest relative to the shell. Choose coordinates as in fig. 2 where

- x, y, z are rectangular coordinates with z in the direction of gravity and also that of the angular velocity $\underline{\Omega}$ of the spherical shell.
- x', y', z' are auxiliary rectangular coordinates as shown with z' passing through the center of the vortex.
- r, Θ, λ are spherical polar coordinates having the usual relation to the x, y, z frame (Θ colatitude).
- r', Θ', λ' are a similar set corresponding to the x', y', z' frame.
- α is the colatitude of the center of the vortex in the r, Θ, λ system.
- Θ_0' is the angular radius of the vortex.

The essential point noticed by Mr Long is that for displacements of the vortex as a whole the constraints are such that the appropriate equations of motion are the torque-angular acceleration equations taken about the proper axis through the center of the sphere. These equations for moments about the origin

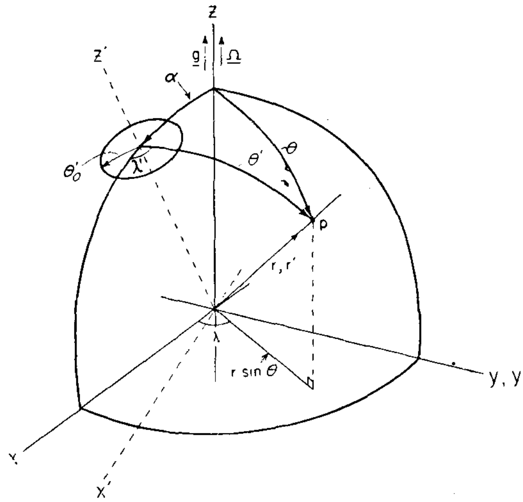

Fig. 2. Schematic diagram of the coordinate systems used. The primed coordinate system is fixed relative to the vortex.

written relative to the system rotating about the z axis with angular velocity $\underline{\Omega}$ are

$$\frac{d}{dt}\iiint \underline{r}\times\varrho\underline{v}\,d\tau = \iint \underline{n}\times p\,\underline{r}\,dA - \\ - \iiint \underline{r}\times(2\underline{\Omega}\times\varrho\underline{v})\,d\tau - \\ - \iiint \underline{r}\times[\underline{\Omega}\times(\underline{\Omega}\times\underline{r})]\,\varrho\,d\tau + \\ + \iiint \underline{r}\times\varrho\underline{g}\,d\tau \qquad (3)$$

where $\underline{r}$ is the radius vector from the origin,
$\underline{v}$ is the relative vector velocity inside the vortex,
$\underline{n}$ is the outward normal at the boundary of the vortex,
p is the pressure,
$\underline{g}$ is the gravity acceleration vector,
ϱ is the density of liquid in the vortex,

and the volume integrals are taken over the entire volume of the vortex while the area integral is over its boundary.

The left-hand term is the total rate of change of moment of momentum of the vortex about the origin and the right-hand terms are the moments of the applied forces neglecting friction. In the simple case we are considering all these terms on the right will be found by symmetry about the xz-plane to give net contributions only to moments about the y-axis. Consequently we may take scalar products with the $\underline{j}$ unit vector along the y-axis and treat only the scalar moments about this axis.

Also it may be noted that all components of Coriolis forces, etc., normal to the shell will make no contribution since, regardless of their magnitude, they have zero moments about the origin and consequently about the y-axis.

At least two of the right-hand terms in (3), the moments of the centrifugal and gravity forces, are functions of the geometrical configuration only and may be simply calculated. For the centrifugal force moment

$$\begin{aligned} m_y, CF &= -\iiint \underline{j}\cdot\underline{r}\times[\underline{\Omega}\times(\underline{\Omega}\times\underline{r})]\,\varrho\, d\tau \\ &= -\iiint \underline{j}\cdot\underline{r}\times\underline{\Omega}\,(\underline{\Omega}\cdot\underline{r})\,\varrho\, d\tau \\ &= \iiint \varrho\,\Omega^2 x\, z\, d\tau \qquad (4) \end{aligned}$$

which transforming first to r, Θ, λ and then to r', Θ', λ' and taking the volume element for unit depth, becomes

$$m_y, CF = \varrho\,\frac{\Omega^2 a^2}{2}\int_0^{\Theta_0'}\int_0^{2\pi}(\sin\Theta'\cos\lambda'\cos\alpha - \cos\Theta'\sin\alpha)\,(\cos\Theta'\cos\alpha - \sin\Theta'\cos\lambda'\sin\alpha)\,a^2\sin\Theta'\,d\lambda'\,d\Theta' \qquad (5)$$

where a is the radius of the spherical shell. This gives

$$m_y, CF = \varrho\,\frac{\Omega^2 a^4 \pi \sin 2\alpha}{4}(\cos\Theta_0'\sin^2\Theta_0') \qquad (6)$$

The gravity force moment may be simply calculated by finding the center of mass of the vortex and calculating the moment of the force considered as acting at that point. The total mass of the spherical cap of unit depth is

$$M = \varrho\, 2\pi a^2\,(1 - \cos\Theta_0') \qquad (7)$$

The center of mass is located on the radius vector from the origin to the center of the vortex at a distance $(a/2)\,(1 + \cos\Theta_0')$ from the origin. Thence

$$m_y, G = -g\,\varrho\,\pi a^3 \sin\alpha\,(1 - \cos^2\Theta_0') \qquad (8)$$

The resultant Coriolis force moment is

$$m_y, C = -\iiint \underline{j}\cdot\underline{r}\times(2\,\underline{\Omega}\times\varrho\,\underline{v})\,d\tau \qquad (9)$$

For the case analogous to Rossby's we take $\underline{v} = \omega\, a\sin\Theta'\,\underline{\lambda}'$ where ω is the relative angular velocity of the vortex (a rigid rotation) and $\underline{\lambda}'$ is the unit vector in the direction of the λ' coordinate. Proceeding as before

$$m_y, C = \int_0^{\Theta_0'}\int_0^{2\pi} 2\,a^2\,\omega\,\Omega\,\varrho\,(\cos\Theta'\cos\alpha - \sin\Theta'\cos\lambda'\sin\alpha)\,(\sin\Theta'\cos\lambda')\,a^2\sin\Theta'\,d\lambda'\,d\Theta' \qquad (10)$$

$$m_y, C = -\frac{2}{3}\pi\sin\alpha\, a^4\,\varrho\,\omega\,\Omega\,[2 - 2\cos\Theta_0' - \cos\Theta_0'\sin^2\Theta_0'] \qquad (11)$$

The most difficult term to calculate is the pressure term over the boundary

$$m_y, p = \iint \underline{j}\cdot(\underline{n}\times p\,\underline{r})\,dA \qquad (12)$$

Only the lateral boundaries enter since both top and bottom give zero contributions to the moment. The principal problem is that of assigning proper conditions outside the vortex. Once these are assigned a straightforward computation from the appropriate Bernoulli equation for the relative frame will give the pressures and consequently the total moment. For the Rossby assumption of an environment at rest relative to the rotating shell a very simple consideration gives this term. In this case if the vortex cap is imagined to be replaced by a cap of the environment (water) density ϱ' at relative rest, the pressures outside will be unchanged and the entire system will be in equilibrium.

Consequently the pressure force moment over the boundary is just sufficient to equilibrate the centrifugal and gravity moments for a cap of density ϱ', these being the only other terms present then. The pressure term therefore contributes terms equal to the negatives of m_y, CF and m_y, G with ϱ replaced by ϱ'. When the entire right-hand side of (3) is divided by the total moment of inertia of the cap, involving the factor ϱ, the gravity and centrifugal terms combined with the pressure terms give the same terms as above except with ϱ replaced by the factor

$$\gamma = \frac{\varrho - \varrho'}{\varrho} \tag{13}$$

If I_v is the total moment of inertia of the vortex about the y-axis

$$I_v = \pi \varrho \frac{a^4}{3} [4 - 4\cos\Theta_0' - \cos\Theta_0' \sin^2\Theta_0']$$

and (3) becomes

$\frac{1}{I_v}\frac{d}{dt} \int\int\int \underline{r} \times \varrho \underline{v}\, d\tau = \Sigma$ moments/unit moment of inertia,

The angular acceleration of the vortex as a whole about the y-axis

$$\frac{d^2\alpha}{dt^2} = \frac{1}{I_v}\frac{d}{dt} \int\int\int \underline{r} \times \varrho \underline{v}\, \tau$$

$$\approx \left\{ 3\gamma \frac{\Omega^2 \cos\alpha \cos\Theta_0' \sin^2\Theta_0'}{4 - 4\cos\Theta_0' - \cos\Theta_0' \sin^2\Theta_0'} \right.$$

$$- 3\gamma \frac{g \sin^2\Theta_0'}{a[4 - 4\cos\Theta_0' - \cos\Theta_0' \sin^2\Theta_0']}$$

$$\left. - 2\omega \frac{\Omega[2 - 2\cos\Theta_0' - \cos\Theta_0' \sin^2\Theta_0']}{4 - 4\cos\Theta_0' - \cos\Theta_0' \sin^2\Theta_0'} \right\} \sin\alpha \tag{14}$$

This is entirely analogous to Rossby's result in equation (1), particularly if multiplied through by a to give a linear measure proportional to α. This equation is exact for evaluating equilibrium conditions (α fixed) but for $\frac{d^2\alpha}{dt^2}, \frac{d\alpha}{dt} \neq 0$ does not allow for such things as the inertial effect of the environment liquid, the Coriolis force for the motion of the cap as a whole, or the Coriolian pressure field associated with the escape motions of the environment. Other factors will be discussed in section *C* in connection with evaluation of the experimental results.

In particular, at the instant of starting from rest under the influence of torques, two independent analyses to be published elsewhere by Mr Long and Mr Phillips of the University of Chicago show that the effective inertia of the vortex in the spherical and in a plane case is twice that implied in equations (1), (2), or (14). This result arises from an additional contribution to the pressure force on the vortex due to the inertial reaction of the environment to the attempted acceleration of the vortex. This pressure torque in the above case equals one-half the resultant of all the others acting on the vortex[1] so that effectively the vortex appears to possess twice the mass relative to the original torques. The result might have been anticipated from the comparison with that for the plane motion around a cylinder given by Lamb ("Hydrodynamics" 6th Edition, p. 77). Part of a numerical difference between ROSSBY's (1948) and DAVIES' (1948) result may be due to this factor since Davies carried out directly a series expansion for the pressure on the boundary. Since we wish to discuss mainly the equilibrium criterion for no motion of the vortex, this particular factor will not affect the conclusions.

It will be noted that in this form the Coriolis term reduces to Rossby's expression (equation [1]) for Θ_0' of the first order of small quantities. Thus, substituting the series expressions for sine and cosine and retaining all terms leading to 4th and 2nd order quantities, respectively in the numerator and denominator, one obtains

$$\frac{3\omega\Omega\, 3/4\, (\Theta_0')^4 + \dots}{3(\Theta_0')^2 + \dots} \approx \frac{\omega\Omega}{2} (\Theta_0')^2$$

$$\approx \frac{\omega f_0}{4} \frac{a^2 (\Theta_0')^2}{a^2}$$

$$\approx \frac{\omega f_0}{4} \frac{R^2}{a^2}$$

Rewrite the equation (14) as

$$\frac{d^2\alpha}{dt^2} \approx \sin\alpha \left\{ -2\omega\,\Omega s_1 - 3\gamma \frac{g}{a} s_2 + \right.$$

$$\left. + 3\gamma\,\Omega^2 s_3 \cos\alpha \right\} \tag{15}$$

where the size factors s_1, s_2, s_3 depend only on Θ_0'.

The transition from a stable to an unstable motion as a whole occurs at equlibrium (at

[1] The one-half factor applies where $\varrho = \varrho'$ and is altered to $\varrho'/(\varrho + \varrho')$ in the case we are actually considering. In the spherical case there is a size restriction on the vortices to which this result is applicable.

least with $d\alpha/dt = 0$ also) where the factor in curly brackets equals zero and independently of the inertial pressures of the environment. This condition for a critical relative angular velocity ω_c of the vortex may be written as

$$\frac{\omega_c}{\gamma\,\Omega} = -\frac{3}{2}\,\frac{g}{a\,\Omega^2}\left(\frac{s_2}{s_1}\right) + \frac{3}{2}\left(\frac{s_3}{s_1}\right)\cos\alpha \quad (16)$$

Thus the boundary of instability for this case is a straight line in the $\omega_c/\gamma\,\Omega$, $g/a\,\Omega^2$ plane (see fig. 14). In this form the rough criterion (2) becomes

$$\frac{\omega_c}{\gamma\Omega} = -\frac{2\,a^2\,g}{R^2\,(\Omega^2\,a)} = \frac{-\,2\,g}{\Theta_0'^2\,(a\,\Omega^2)} \quad (17)$$

Table 1. See equation (16)

	Solid rotation		Linear shear from $\Theta_1' = 28°$	
	slope = $= 3/2\ s_2/s_1$	intercept = $= 3/2\ s_3/s_1$	slope = $= 3/2\ s_2/s_4$	intercept = $3/2\ s_3/s_4$
Θ_0'				
2°	~1830	~1820		
5°	~ 230	~ 225		
10°	65.54	64.45		
15°	29.29	28.30		
20°	16.40	15.41		
25°	10.506	9.522		
30°	7.291	6.314	8.284	7.175
35°	5.352	4.384	8.065	6.606
40°	4.093	3.136	7.751	5.938
45°	3.230	2.284	7.400	5.233
50°	2.610	1.678	7.044	4.528
55°	2.151	1.234	6.694	3.840
60°	1.800	.900	6.362	3.181
65°	1.526	.644	6.055	2.559
70°	1.306	.446	5.778	1.976
75°	1.128	.292	5.537	1.433
80°	1.004	.210	5.341	1.1187
85°	.856	.074	5.205	.4536
90°	.750	0	5.151	0

The factors $\frac{3}{2}\,\frac{s_2}{s_1}$ and $\frac{3}{2}\,\frac{s_3}{s_1}$ are given in table 1. The slopes derived from (17) are almost exactly equal to those in the table but the intercepts of course are zero in (17). This table also gives the corresponding factors $\frac{3}{2}\,\frac{s_2}{s_4}$ and $\frac{3}{2}\,\frac{s_3}{s_4}$ for a case which will be important for the later discussion. In the actual operation of the experiment the fan that generates the vortex motion is usually smaller in radius than the polar cap of liquid. While it is quite efficient in generating velocities approaching solid rotation out to its outer radius there is a more or less rapid decrease in relative velocity out to the interface between the two liquids. In the water itself the relative zonal velocity is generally observed to be quite small so that the boundary condition of relative rest for the top liquid at the interface is probably not far from correct.

An observationally plausible assumption about the vortex velocity field is to take a solid rotation to colatitude Θ_1' (where Θ_1' is approximately the angular radius of the fan) followed by a linear decrease to zero at the boundary colatitude Θ_0'. s_1 in equation (15) and equation (16) is then replaced by

$$s_4 = \frac{1}{4 - 4\cos\Theta_0' - \cos\Theta_0'\sin^2\Theta_0'}$$
$$\Bigg\{\left[2 - 2\cos\Theta_1' - \cos\Theta_1'\sin^2\Theta_1'\right]$$
$$+ \frac{\frac{1}{2}\sin\Theta_1'}{\sin\Theta_1' - \sin\Theta_1'}$$
$$\Big[\sin\Theta_1'\cos\Theta_1'\,(2\sin\Theta_1' - 3\sin\Theta_0')$$
$$+ 3\sin\Theta_0'\,(\Theta_1' - \Theta_0') + \sin^2\Theta_0'\cos\Theta_0'$$
$$+ 4\cos\Theta_1' - 4\cos\Theta_0'\Big]\Bigg\}$$

in the above factors. Θ_1' is taken as 28° for the experimental arrangement and in table 1.

Further discussion of the significance of these relations and the approximations used is given in section 3 in connection with the experimental results.

3. Experimental results

A general view of the apparatus used, slightly modified from its earlier form (FULTZ 1949), is given in fig. 3. A hemispherical part of a 5-liter round-bottom boiling flask is fastened to a vertical hollow shaft. Inside it a half-section of a 3-liter flask is floated giving a space 1.6 cm deep in which liquid is placed. The liquid has the form of a hemispherical shell of mean radius 10 cm. The vertical shaft is connected by V-belt pulleys to a split-phase

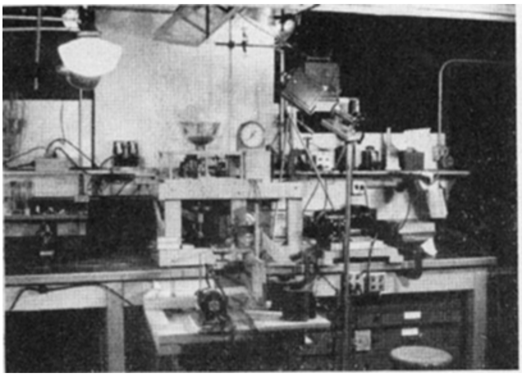

Fig. 3. General view of the apparatus showing the hemispherical bowls in the center, mirror above the bowls, camera to the right rear above, motor driving the bowls to the right rear, Rotoscope to the right forward, and the motor driving the fan shaft in the center front.

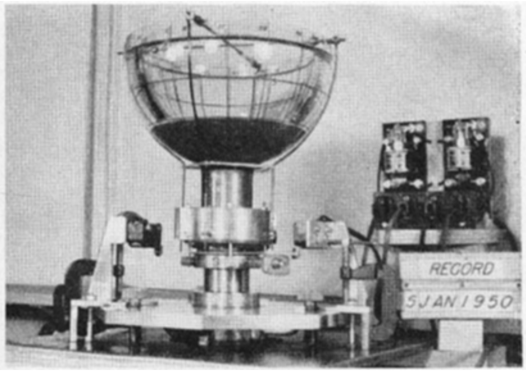

Fig. 4. Close-up view of the hemispherical bowls showing the lower cap of darker fluid reaching about to 42° latitude. Notice the inner bowl which is held down by the cross-rods. The space above the cap to the equator is filled with water. A contact switch for the revolution counter is on the right.

electric motor which rotates the shaft at a variety of constant rates depending on the pulley combination. (Occasionally, it was convenient to use a series-wound motor whose speed could be altered continously by varying the impressed voltages, though in many circumstances this arrangement does not give steady enough angular velocities.)

We used various liquids that were immiscible with water and of slightly greater density to form an isolated cap at the pole. The rest of the hemispherical shell was filled with water as shown in fig. 4. In order to produce relative motion in the polar cap, fans of various forms fixed on a separate shaft concentric with the hollow shaft were used (fig. 5). This inner shaft passes through a packing gland at the top of the outer shaft that prevents liquid from leaking out of the shell. It is driven by pulleys and oval leather belting from a series-wound motor operating off a controlled voltage source. By these means we were able to work in ranges of up to 125 rpm for the outer shaft and glass shells and up to 200 rpm in either direction for the inner shaft and fan.[1]

Some of the data used has been obtained by taking sequence photographs from above along a line of sight parallel to the axis of rotation and at the same point in successive rotations of the hemispheres. However, much the largest mass of numerical observations has been taken visually with the aid of a "Rotoscope" patterned after that designed by D. Thoma (FISCHER 1931) and used in the investigation

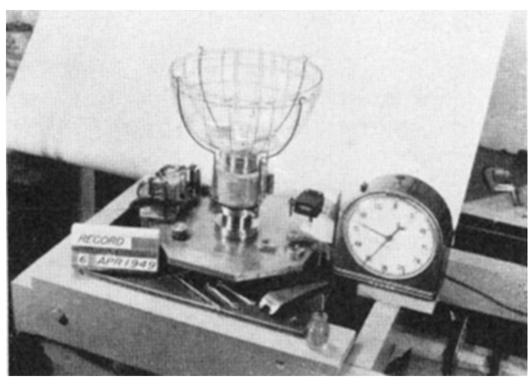

Fig. 5. Close-up of fan inside the outer bowl with inner bowl and liquid removed. The fan shown is that used in most of the earlier investigations and, e. g., for fig. 12. The fan used for fig. 13 was much larger.

[1] Since both the absolute and relative motions will have to be discussed below the following terminology will be used:

"cyclonic absolute" for an absolute rotation in the same direction as the hemispheres,

"anticyclonic absolute" for an absolute rotation in the opposite direction,

"cyclonic and anticyclonic relative" for the same relations of a relative rotation to the absolute motion of the hemispheres.

Cyclonic absolute rotation at a rate less than that of the hemispheres will thus be a relative anticyclonic rotation.

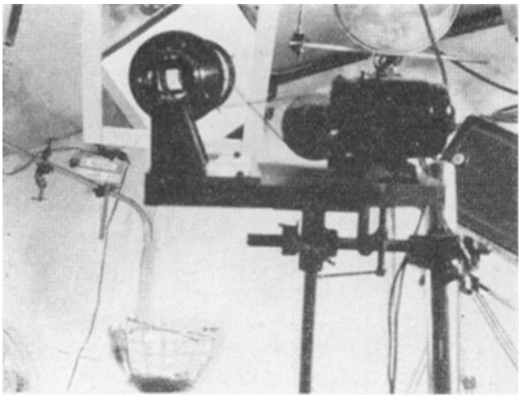

Fig. 6. Detail view of the Rotoscope without magnifying eyepiece. The image of the hemisphere can be seen both in the mirror and in the reversing prism. The variable speed motor is on the right.

of the relative flow in centrifugal pumps. This instrument utilizes the fact that if a mirror, observed at almost glancing incidence, is rotated about the line of sight the virtual image seen in the mirror will rotate uniformly at twice the rate. In the instrument, a Dove reversing prism to give a wider field of view replaces the mirror. Our version (fig. 6) consists of nothing but the prism, a rotatable mounting cylinder, and a small variable speed motor which drives the cylinder by string and pulley or by gears.

For best results the line of sight of the Rotoscope must be along the axis of rotation. This requires a rather careful adjustment but, once it is attained, the apparatus when rotating at any speed can be reduced to apparent rest by rotating the Rotoscope at half the rate in the proper direction. Furthermore, any wave pattern, for example, which is moving around relatively unchanged in shape at a steady rate can similarly be reduced to rest. Thus the relative rotational motion of such a feature can be determined either by reducing the glass shells to a conveniently low apparent rotation and clocking its travel between fixed meridians or by reducing the feature to rest, measuring the required rate of rotation of the Rotoscope, and computing the relative motion, knowing the rate of rotation of the hemispheres. Both of these methods have been extensively used wherever one or the other was convenient. This device has also been invaluable in correcting mistaken impressions gained from direct observations of the shells at moderately high rotations and from the rather widely separated photographs taken only once per revolution.

A number of liquids were tried in the attempt to find some with a favorable value of γ, i. e., with densities close to that of water. It is apparent from either (2) or (16) that, within practical limits, small values of γ will give the better chance of obtaining critical angular velocities and unstable motions of the polar cap as a whole. Several mineral oils and, in particular, castor oil with a γ of about $1/30$ were tried, but were found generally to be too viscous to give any well defined motions. As a result of a suggestion by Dr Gardner of Sinclair Research Laboratories we attempted using a mixture of monochlorobenzene and toluene which turned out to be almost ideal. Both compounds have low solubilities in water and are completely miscible with one another. By mixing in the proper proportions any density between that of toluene (0.86) and that of chlorobenzene (1.1) can be obtained. The mixtures have viscosities which are even lower than that of water and by using a water-insoluble dye (Sudan III) the cap can be made to stand out clearly from the rest of the liquid. If the above liquids are used alone, a relatively rapid contamination of the interface occurs which causes sticking of the lower cap to the glass. The liquids used had to be changed for each experiment and the glass very carefully cleaned until we discovered that a pinch of laboratory detergent in the water would completely prevent the sticking. The water must be arranged to wet the whole glass surface at least once and from then on a surface film prevents any sticking of the organic liquids to the glass.

The experimental procedure varied somewhat depending upon the type of data being taken but in all cases was quite straightforward. For instability point determinations, the bowls were adjusted to the speed desired and the fan then slowly changed from zero relative motion to cyclonic or anticyclonic relative velocities until the phenomenon wanted was reached or until the greatest practicable value was attained. Simultaneous angular velocity determinations for the bowls and the fan were made at any significant points. If the two liquids were then unduly beaten together the fan was returned

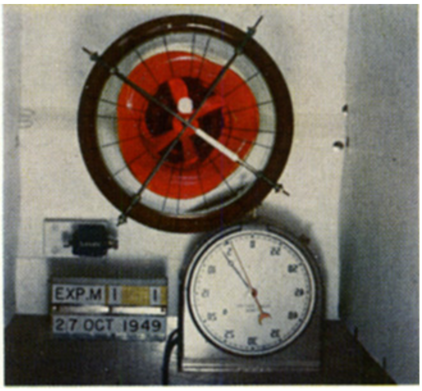

Fig. 7. Three-crest pattern observed when fan is rotating faster than bowl at a moderate rate. The crests progress relatively cyclonically at a rate equal to about .16 of the bowl rate of absolute rotation. (Apparent rotation of bowl counterclockwise, bowl absolute rotation 56.4 rpm, fan rate +30.1 rpm relative cyclonic, $\gamma = 0.096$ with chlorobenzene as lower liquid.)

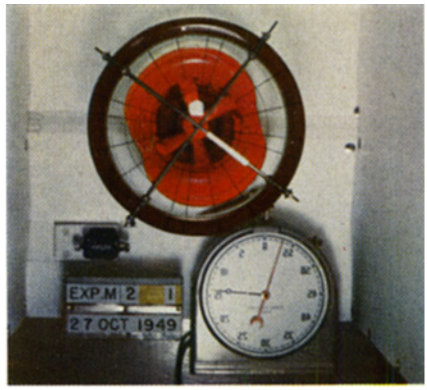

Fig. 8. Two-crest pattern observed when the fan is rotating faster relatively than in fig. 7. The crests progress relatively cyclonically at a rate equal to about .20 of the bowl rate of absolute rotation. (Apparent rotation of bowl counterclockwise, bowl absolute rotation 56.4 rpm, fan rate +41.4 rpm relative cyclonic, $\gamma = 0.096$ with chlorobenzene as lower liquid.)

Fig. 11. Photograph taken one bowl revolution later than fig. 10. The off-center crest has moved clockwise to the top of the figure corresponding to a relative anticyclonic rotation equal to about 0.4 of the absolute bowl rate (conditions as for fig. 10).

to zero relative velocity until settling out had occurred and the experiment was then continued. If necessary, the liquids were changed.

Qualitatively, the results concerning instability of the polar vortex as a whole were clearcut. For cyclonic relative motion of the fan, no indications of asymmetry about the pole or throwing of the center of gravity of the cyclonic vortex away from the pole were observed at any velocity conditions which could be reached with the apparatus. On the other hand, for anticyclonic vortices, at every velocity of the bowl except the lowest ones there was some relatively sharp value of anticyclonic fan speed at which the center of gravity of the polar cap was thrown visibly away from the pole (see figs. 10 and 11). Quantitatively there are difficulties, discussed below, which are still not satisfactorily accounted for.

When the fan type pictured in fig. 5 is used to drive the vortex the phenomena which occur are roughly as follows (the general features are insensitive to the number of fan blades):

A. For relative cyclonic fan motion ($\varrho - \varrho' \sim 0.1 - 0.01$ gm cm^{-3}) as the speed of the fan is increased:

First, no distinct motion of the interface is seen in the Rotoscope though flashes of up and down motion can be seen from the side. Then a distinct and very stable set of 3 crests is seen (fig. 7) which moves around cyclonically relative to the bowls but more slowly than the fan. The rate of this motion increases with the fan speed as appears in fig. 15. At a quite sharp fan speed the number of crests suddenly becomes 2 (fig. 8). It will be noticed that the interface is very smooth and well defined in these figures. As the fan speeds are increased the number of lobes remains 2, their relative rate of travel increases, and ultimately the centrifugal action of the fan tosses the pole free of the lower liquid and draws water down in its place (fig. 9). Throughout, the center of gravity of the denser liquid remains sensibly on the pole. Bubbling of the two liquids is apparent in fig. 9 and ultimately the polar cap is beaten into a froth which, however, remains symmetrical in form about the pole.

B. For relative anticyclonic fan motion, as the speed of the fan is increased from zero relative:

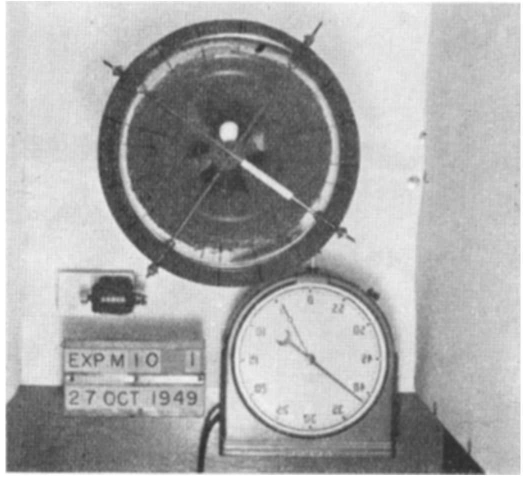

Fig. 9. Two-crest pattern with center partly thrown clear of chlorobenzene and bubbling occurring at high cyclonic fan rates. Note that the pattern is still quite symmetric about the pole. The crests progress relatively cyclonically at a rate equal to about .34 of the bowl rate of absolute rotation. (Apparent rotation of bowl counterclockwise, bowl absolute rotation 56.4 rpm, fan rate +99.6 rpm relative cyclonic, $\gamma = 0.096$ with chlorobenzene as lower liquid.)

First, the same sequence as in the cyclonic case of passage through no discernible motion, four or three crests, and finally two-crest patterns, is seen. These crests move relatively in the direction of the fan but at a lesser rate (fig. 15). Very soon the interface is less smooth and shows much more irregular, short wavelength motion than in the cyclonic case.

Then, in a range of about 5 rpm or less of the fan speed, the two-crest pattern suddenly goes over into a single-crest pattern of more or less regularity. This corresponds to a very distinct displacement of the vortex center of gravity away from the pole (figs. 10 and 11). As the fan speed increases further, the violence of the motions increases and also the displacement of the center of gravity from the pole. Even after the liquids have begun to be beaten together and up to the speed limits of the apparatus the same general feature is present.

The single crest, or center of gravity, moves anticyclonically relative to the bowls at about 20 rpm over the range of speeds seen in fig. 12. This immediately involves dynamical factors which were omitted in the analysis of

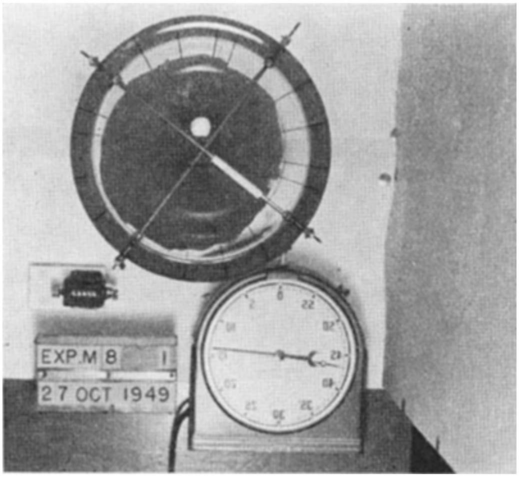

Fig. 10. Cap thrown off-center when fan is run at a high anticyclonic rate. The lower left-hand side is much farther out from the pole than the other side, the amount being even decreased from its actual magnitude by the perspective appearance from above. The fan rate at which this appears for the present bowl rate is about —95 rpm relative anticyclonic. (Apparent rotation of bowl counterclockwise, bowl absolute rotation 56.4 rpm, fan rate —146.2 rpm relative anticyclonic, nominal $\gamma = 0.096$ with chlorobenzene as lower liquid. The effective γ is smaller because of the bubbling which occurs under these conditions.)

the previous section such as effects of the escape motions of the environment. The comparison with the theoretical curve in fig. 12 plus other data brings out several points:

1° The observed fan speed for transition away from the pole is almost constant with bowl speed instead of strongly falling off. Except at the lower end where, as the bowl speed is reduced below about 45 rpm, the critical fan speed takes a very sharp rise and (with $\gamma = 0.10$) for smaller bowl speeds the cap is badly torn up before a well-marked critical throw-off point is reached.

2° The throw-off point of this type seems to be very much less sensitive to the density difference than it was expected to be. In fact, there is almost no real change in the character and location of the curve as the γ-factor is varied from 0.1 to less than 0.01.

3° The critical fan speed curve shows even the wrong slope beyond about 80 rpm bowl speed. This may be connected with a circulation of the escape motion about the cap and is also probably related to the fact that the interface begins to climb rapidly up the surface of the outer bowl at about this bowl speed. This flattens and lengthens the cap edge and makes it depart radically from the two-dimensional assupmtion in the calculations. In addition, the whole outer rim is less efficiently driven by the fan with a consequent decrease of the total unstabilizing Coriolis force. The very strong effect this might have is shown in fig. 14 by how much the stable area is increased under the theoretical critical curve when a linear shear correction is applied.

Qualitatively then, this phenomenon is of the expected order but quantitatively the discrepancies are disturbing, though of course understandable, particularly in view of the motions of the center of mass. We looked around then for possibly some other manifestation of the same effect which might give better quantitative comparisons. A very slight tendency had been noticed for the cap to be very slightly higher on one side than the other at moderate and high rotations. This had been thought to be due to lack of sphericity of the

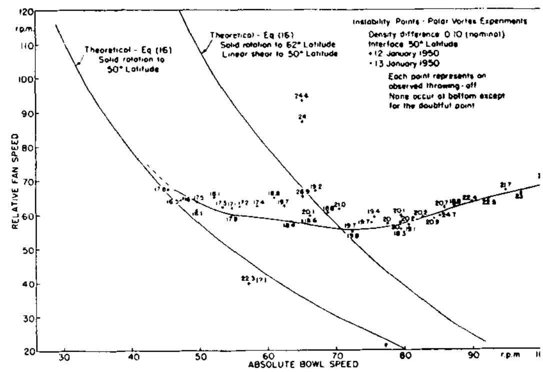

Fig. 12. Curve showing values of relative anticyclonic fan speed where the throwing-off phenomenon of figs. 10 and 11 first occurs. Off-center conditions occur everywhere above the curve except for very turbulent conditions to the left of the end of the indicated curve. It will be noted that figs. 10 and 11 and the curve to the left of 60 rpm bowl speed in this case correspond to absolute anticyclonic fan rates while to the right the fan rate is relative anticyclonic but absolute cyclonic. Note also that the cap radii are different for this figure and for figs. 10 and 11. Theoretical curves from equation (16) for solid rotation and linear shear assumptions for a cap reaching 50° latitude are shown. Since the linear shear correction looks too strong, but corresponds best to the velocity pattern, it appears definitely that one of the escape motion torque factors is involved in this phenomenon.

outer bowl which was very carefully centered in the attempt to combat it.

Meanwhile, it was decided to try fan blades which reached almost to the interface and had leather attachments reaching across almost the full depth of the shell in order to drive the cap as efficiently as possible. With this fan the violent motions associated with the earlier anticyclonic throw-off point had much less chance to develop so that this phenomenon and also the various waves were only poorly developed. However, the tilting seemed to be quite regular. Scales were therefore placed around the outer bowl near the interface edge which enabled its position to be measured to within less than a $^1/_2$ mm. The differences of level between the highest and lowest points of the interface were then observed as functions of bowl and fan speed, the cap being found to remain very closely circular. At any given condition, the high and low points were found to remain at the same longitude indicating that the center of gravity was stationary with respect to the bowl. This of course better fits the elementary analysis of the previous section.

The results of one such determination with $\gamma = 0.10$ is shown in fig. 14 where the figures entered are the approximate maximum difference of level at the edge of the cap in millimeters. The curves drawn are for 1, 5, and 10 mm difference, respectively. The general form of these curves is very encouraging, particularly as to slope, except for two difficulties:

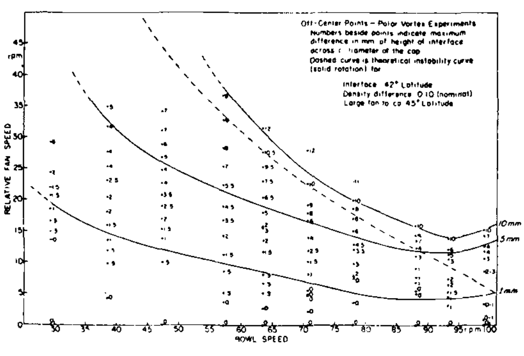

Fig. 13. Curves of equal tilt (in mm difference in level) for throwing-off occurring with large fan. The theoretical curve is calculated from equation (16) for assumed solid rotation in the vortex. The general appearance of these curves is encouraging but a definite reason for the throwing-off to begin before it should is still to be satisfactorily established.

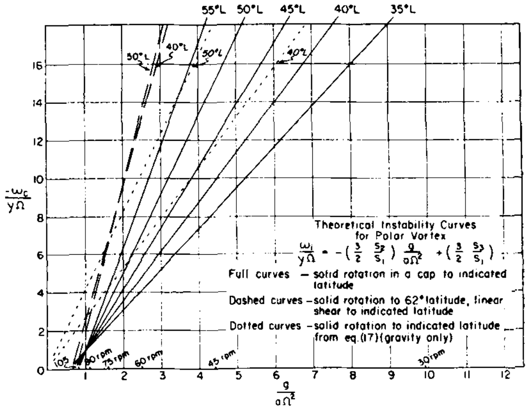

Fig. 14. Theoretical throwing-off curves from equation (16) in the $\frac{\omega_c}{\gamma\Theta}$, $\frac{g}{a\,\Theta^2}$ plane for vortices of various sizes centered near the pole ($\cos \alpha \sim 1$). Throwing-off should occur in the upper left-hand region relative to each line. The solid lines show the critical curves for caps in solid rotation reaching from the pole to the indicated latitudes. The dashed lines show the critical curves for caps with solid rotation to 62° latitude followed by a linear shear to the indicated latitudes. The dotted curve is the critical one from equation (17) (ie omitting centrifugal terms) for a cap extending to 40° latitude.

a) there is little reason to expect the difference of level to be a regular function of fan speed since equation (16) indicates very little dependence of the equilibrium near the pole on α, the colatitude of the vortex center;

b) there is no reason in the analysis so far to expect any tilting at all until the theoretical curve is passed, whereas there are definite tilts very much below it.[1]

Our present conjecture is that this behaviour is caused by a net surface tension torque arising at the interface due to the fact that the two bowls are not completely accurately aligned. It is definitely known that this factor exists and is appreciable because at low density differences ($\gamma \approx 0.001$) the cap can be made to climb all the way up one side of the shell by shifting the inner bowl slightly toward the other side so that the mean curvature of the interface is higher on the side where there is least distance between the bowls. Work is

[1] It will be noticed that the slope turns up beyond about 90 bowl rpm. This appears definitely to be associated with the flattening and rising of the interface edge which begins strongly at about 80 rpm and was mentioned above.

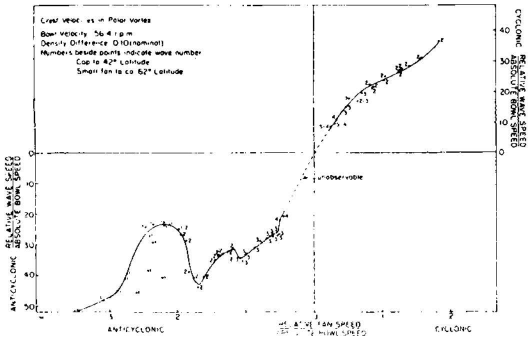

Fig. 15. Relative crest (wave) speeds versus relative fan speeds in non-dimensional measure for a bowl velocity of 56.4 rpm and nominal γ of 0.096. The cap extends to about 42° latitude and is being driven by the small fan of fig. 5. The figures by the observed points indicated the number of crests. Note apparent jump in speed as wave number goes from 3 to 2 on the anticyclonic side. The transition from 2 to 1 crest is identical with the throwing-off phenomenon of figs 10 and 11. The last few points toward the left are uncertain.

underway to track down this effect by varying the interfacial tension between the two liquid phases. This will be done by varying the concentration of a pure detergent in the water. The interfacial tension will be directly measured and it is expected that a close relation between its value and the run of the curves of equal tilt will be found.[1]

We also intend to rotate solid obstacles which approximate to the cap form in the shell of liquid. By making streak photographs of the environment motion it should be possible to form a definite estimate of the type of environment motions going on (especially for the phenomenon of figs. 10 and 11) to check and supplement further theoretical analysis being carried out by Mr Long. This will attempt to go as far as possible with the general case of a moving vortex (disc) and will be published in a succeeding paper by Mr Long.

4. Conclusion

Qualitatively speaking, the results of the previous section can, I think, hardly fail to be related to the effect discussed by Rossby and to be in some measure a confirmation of his prediction that anticyclones can exhibit such an instability. Quantitatively, there will remain little more to ask for in the simple case if the discrepancies of fig. 13 can be definitely related to a surface tension or other effect.

No attempt will be made here to carry the discussion of definite meteorological applications further, as this should wait on the more extensive analyses of environment effects to be undertaken in later work. It should be mentioned, however, that the occurrences in these experiments have relations to so-called "dynamic or inertia instability", shearing instability, and wave motions in zonal currents which should probably be investigated further. The crest systems are very regular and show very well-marked steady features which appear at certain definite speeds. For example, the curve of wave speeds in fig. 15, and other results not included here, show very intriguing breaks at several points where the wave number changes. It would be very feasible to investigate photographically the internal motions associated with these waves and their possible association with these changes. Concerning instability phenomena a relatively unexpected result is that the interface is roughest with the anticyclonic vortex for which the shear across the interface is cyclonic, while it is extremely smooth for the cyclonic vortex where the shear is anticyclonic and might be expected possibly to reach inertia instability values.

One additional rather wild speculation may be of some interest. Call the critical angular velocity, at which the throwing-off of figs. 10 and 11 occurs, ω_k. Then if we accept at face value in a dimensional analysis the experimental independence of γ and of the bowl speed Ω that is exhibited by ω_k, $\frac{\omega_k}{\Omega}$ would be pro-

[1] Some of this work has been completed (July 1950) and has indicated the following: It was discovered that the inner bowl was actually riding too high and was leaving a space of about 2 cm between the bowls at the pole instead of the 1.6 cm observed at the equator. As a result, the surface tension effect seems to be predominantly stabilizing. This may possibly be connected with result 2° above, since the surface tension is relatively independent of the solution used and of the density difference. Almost the only apparent destabilizing effect then left to explain the small early tilts is lack of sphericity of the bowls. These results will be collected and discussed in Mr Long' succeeding contribution. Their importance obviously lies primarily in the problem of estimating how far they adversely affect the geophysical implications to which the experiments may give rise.

portional to $\frac{1}{\Omega}$. The principal parameter remaining would be the gravity parameter $\frac{g}{a\,\Omega^2}$ so that:

$$\frac{\omega_k}{\Omega} \propto \sqrt{\frac{g}{a\,\Omega^2}} \tag{18}$$

For the same sized cap on the earth, taking the experimental anticyclonic $\omega_k = 65$ rpm and $a = 10$ cm, the critical angular velocity for an earth analogue to this throwing-off phenomenon would be 8×10^{-4} sec^{-1}. Unfortunately, or perhaps fortunately, this is far on the absolute anticyclonic side and would not be realizable on a large scale by ordinary atmospheric processes.

I do not seriously suggest that this computation has any real meaning in the present case but offer it as an example of one of the sorts of things that may eventually be possible to do in the way of real meteorological discovery arising from experimental work.

Acknowledgments

Many persons have been involved in the work reported in this paper of whom the first, of course, is Prof. Rossby to whom I am very grateful for support and encouragement. I am principally indebted also to Mr R. R. Long for theoretical discussions and the carrying on of much of the experimentation and to Messrs W. Bohan and G. V. Owens who have done part of the experimental measurements and construction. Much of the computing was done by Mr W. D. Kleis and the drafting was carried out under the direction of Mrs Della B. Friedlander.

REFERENCES

DAVIES, T. V., 1948: Rotatory flow on the surface of the earth — Part I. Cyclostrophic motion. *Phil. Mag.* (7) **39**, pp. 482—491.

FISCHER, K., 1931: Untersuchung der Strömung in einer Zentrifugal-Pumpe. *Mitteilungen des Hydraulischen Inst. d. Tech. Hochsch. München* **4**, pp. 7—27.

FULTZ, D., 1949: A preliminary report on experiments with thermally produced lateral mixing in a rotating hemispherical shell of liquid. *J. Meteor.* **6**, pp. 17—33.

ROSSBY. C.-G., 1948: On displacements and intensity changes of atmospheric vortices. *J. Marine Res.* **7**, pp. 175—187.

ROSSBY, C.-G., 1949: On a mechanism for the release of potential energy in the atmosphere. *J. Meteor.* **6**, pp. 163—180.