On the Influence of the Earth's Orography on the General Character of the Westerlies

By B. BOLIN, University of Stockholm

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Abstract

The upper westerlies in middle latitudes possess, in the mean, a wave-like character which has been explained as a result of thermal contrasts between land and sea. On the other hand, recent theoretical investigations by QUENEY, CHARNEY and ELIASSEN have shown that an obstacle of the dimensions of the Rocky Mountains generates a wave pattern downstream whose scale is comparable to the observed mean waves. In the present paper these theoretical studies are extended, and an attempt is made to discuss, in a general way, the influence of the northern hemisphere mountains on the character of the westerlies. Finally the paper points out some of the climatic consequences of the proposed dynamic-topographic control of the prevailing flow patterns.

I. Introduction

The rapid increase of upper air observations over the northern hemisphere during the last few years has made it possible to construct circumpolar upper air charts which describe the conditions at higher levels in some detail. These maps show that the middle latitude westerlies are concentrated to a rather narrow band ("jet-stream") on which large, wave-like perturbations are superimposed. The latitude of the jet shifts from one day to the next and the waves usually move from west to east. The formation and the location of the jet-stream, as well as the behaviour of the waves, appear to be determined by the distribution of continents and oceans and by the large mountain barriers which are located in middle latitudes (the Rocky Mountains and the large mountains in the interior of Asia in the northern hemisphere and the Andes in the southern hemisphere). This is seen on the daily upper air maps and is made somewhat more obvious by the fact that well-defined perturbations are found also on the *monthly mean* charts for the mid-troposphere. The existence of such deviations from a straight zonal flow on the *mean* maps suggests that the influence of the surface of the earth is of fundamental importance for the character of the westerly flow.

The location of the waves aloft indicates that they are connected, in an intimate manner, with the stationary cyclones and anticyclones which characterize the mean conditions at the surface of the earth in middle latitudes. These stationary high and low pressure areas have been explained as the result of the temperature difference between the air over the continents and the air over the oceans. The thermal influence of the earth has been considered to be of conclusive importance. (See e.g.: V. BJERKNES and collab., 1933, pp. 686—689, SCHERHAG, 1948, pp. 29—30 and 70—71.) The reasoning is as follows: A solenoidal field is established along the coasts. In the equilibrium state the main part of this solenoidal field is balanced by the wind change with height, but in order to compensate for the frictional loss of kinetic energy a direct circulation around the solenoids must occur.

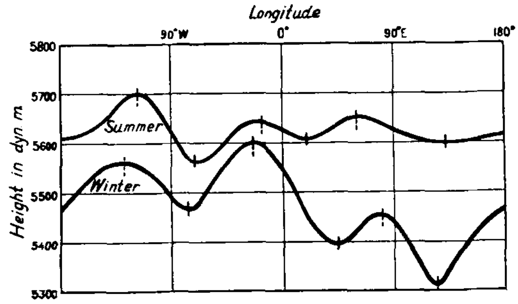

Fig. 1. The mean height of the 500 mb surface as a function of longitude. In summer the figure represents conditions in the latitude belt 45° N—50° N, in winter 35° N—40° N. The profiles have been computed from the hemispherical mean charts published by SCHERHAG (1948).

The deviation from geostrophic flow because of friction must then be consistent with this solenoidal circulation. The picture fitting this requirement is well-known: Anticyclonic circulation at lower levels and cyclonic circulation aloft around regions which are colder than the environment, cyclonic flow in the surface layers and anticyclonic flow aloft around regions which are warmer than their surroundings. This pattern is superimposed upon the general westerlies around the hemisphere. Since the intensity of the zonal current increases with height the resulting pattern at upper levels should have a wave-like structure with troughs over areas of low temperature at the surface of the earth, and with ridges where the air is heated from below.

No doubt this monsoonal circulation is an important process which effects the general character of the westerlies. There are, however, several features of the flow pattern aloft which seem to indicate that the monsoonal circulation is essentially a process in the lower troposphere. We know that the continents act as cold sources and the oceans as warm sources in winter while conditions are reversed in summer. In spite of this fact certain basic features of the wave pattern at upper levels do not change essentially from winter to summer. This is brought out quite strongly in fig. 1, which shows the variation with longitude of the height of the 500-mb surface, in winter for the latitude belt between 35° N and 40° N, in summer for the belt 45° N—50° N. This diagram was constructed with the aid of the mean circumpolar maps which have been published by SCHERHAG. The seasonal displacement of the westerlies has been taken into consideration through the choice of different latitudes for summer and for winter. Even though the intensity of the waves around the hemisphere is much less in summer than in winter the position of the troughs and ridges is almost the same.

Considering now for a moment conditions in the southern hemisphere it is well known that a pronounced trough is located to the east of the Andes Mountains (see fig. 2) in approximately the same relative position as the North American trough east of the Rocky Mountains, in spite of the fact that the thermal conditions in the two cases are quite dissimilar (cf. BOFFI, 1949). Both these troughs occur in winter as well as in summer. These facts point to a *dynamic* effect of the mountains rather than to a thermal influence of the surface of the earth on prevailing zonal air currents.

Indications of the type listed above suggest the importance of exploring, in further detail, the dynamic-orographic control of the prevailing upper level flow patterns. In the follow-

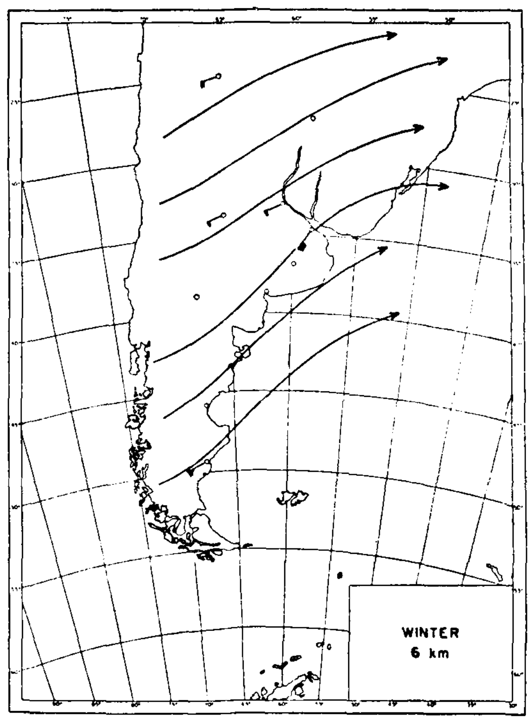

Fig. 2. Mean flow at 6 km in winter over South America. (after BOFFI, 1949).

ng, we shall attempt to extend the analysis of this control as given by CHARNEY AND ELIASSEN (1949), by considering also the north—south extent of the mountain ranges and the width of the zonal current.

II. Basic assumptions

The speed of the westerlies increases with height and has a maximum value at the tropopause in middle latitudes. The greater the kinetic energy of a current, the greater must be the force required to change its direction and character. Thus it seems reasonable to begin by considering the layer of maximum westerlies and then to assume that the flow at other levels (lower troposphere and lower stratosphere) is adjusted in accordance with the conditions prescribed by that level. At lower levels this adjustment must be in agreement with regional differences in heating from below and with the restrictions imposed by ground friction. Thus we treat at first the flow pattern in the layer of maximum westerlies.

The thermal influence of the ground on the atmosphere through radiation and convection decreases with increasing height, primarily because of two facts:

(1) The distance from the heat and cold sources at the surface of the earth increases upward.

(2) The increase of the westerlies with height reduces the time during which an upper level air mass remains above a region of prescribed thermal characteristics.

This reduced influence of the ground on the upper atmosphere is shown by the fact that the annual temperature range of the upper atmosphere is less affected by the distribution of land and sea than the corresponding range below (FLOHN, 1943). The *dynamic* influence on the large scale flow patterns should therefore be relatively more important at higher levels than at the surface of the earth and may even be of greater importance than the *thermal* influences at the upper levels. In order to see to what extent it is possible to explain the wave character of the upper westerlies from a purely dynamic point of view we shall at first disregard completely heating and cooling from below and consider only the effect of the mountain ranges. The degree of similarity between the theoretical results and the observed pattern will show to what extent these assumptions are justified.

When treating the mean flow pattern by the procedure outlined above we have *a priori* made three assumptions, which must be kept in mind. These are:

(1) The variations from day to day of the mean current around the hemisphere are assumed to be small or at least do not represent any considerable deviation from the wave pattern computed from the mean zonal wind.

(2) The effect of a mountain upon the westerlies is assumed to be the same even if the current is not straight zonal, but non-stationary troughs are superimposed upon it.

(3) There exists no critical velocity, at which the zonal motion breaks down into vortices.

We shall return to a discussion of these assumptions later on.

Several theoretical investigations have been carried out in which the stationary flow pattern caused by an infinite mountain range has been computed. QUENEY (1948) has given a summary of the different types (spectrum) of waves which are generated in an unlimited and infinitely broad current striking a mountain range at right angles. In our particular study we are interested only in waves which are of the same order of magnitude as those observed in the mean pattern, that is the long planetary waves. It has been shown by CHARNEY (1948) that other types of waves (gravitational waves, inertia waves) may be eliminated from the very beginning of the analysis by assuming hydrostatic equilibrium and geostrophic balance. Furthermore, CHARNEY AND ELIASSEN (1949) have shown that an equivalent-barotropic atmosphere may be defined whose large-scale motion reasonably well corresponds to the motion of the real barocline atmosphere at about 500 mb. We know from experience that the flow at 500 mb may be considered as representative for the upper troposphere in general. By treating the equivalent-barotropic atmosphere we thus get an idea of the flow in the layer of maximum westerlies. This is the reference level which we have chosen according to the discussion above.

QUENEY took the vertical stability of the atmosphere into account in his discussion of the waves generated by a mountain range. CHARNEY and ELIASSEN, on the other hand, assumed a homogeneous atmosphere with a constant height of the upper boundary and this simplified the computations considerably. In spite of the great difference between these two models of the atmosphere the principle features of the large-scale flow patterns are the same. This suggests that a fixed upper boundary of the homogeneous atmosphere in some measure may be interpreted as being equivalent to a certain degree of internal gravitational stability of the real atmosphere. In our analysis we shall follow CHARNEY and ELIASSEN and consider a homogeneous atmosphere with a fixed upper boundary.

As mentioned in the introduction we shall extend the analysis from one to two dimensions in the horizontal plane. Some calculations have been made by STEWART (1948) for the purpose of determining the flow *around* a mountain with finite extension in the north—south direction. He studied the stationary horizontal waves generated by a cylindrical obstacle. The considerations below are an attempt to combine the results obtained by CHARNEY and ELIASSEN with those obtained by STEWART. Also in this analysis we shall assume that the basic current is infinitely broad. This is, however, a rather serious restriction since we know that the westerlies by no means are broad compared to the north—south extension of the mountains. In sec. IV we shall try to get some idea of the mountain influence on quite narrow currents. The results from III and IV will then be used in the last section for a qualitative discussion of the mountain influence upon the westerlies in middle latitudes in the northern hemisphere.

III. Stationary flow pattern generated by a circular mountain in an infinitely broad westerly current

In accordance with the discussion above we consider a homogeneous, barotropic and incompressible atmosphere of depth D. A straight infinitely broad westerly current of the intensity U (= const) is deformed by a mountain, the height of which is $h = h(x, y)$. The x-axis is directed to the east and the y-axis to the north. For this atmospheric model the motion may be described with the aid of the vorticity equation:

$$\frac{d}{dt}\left(\frac{\zeta + f}{D}\right) = 0, \quad f = 2\Omega \sin\varphi \tag{1}$$

ζ is the vertical component of the relative vorticity for an air column with the depth D, Ω is the angular velocity of the earth and φ is the latitude. The height of the upper boundary of the atmosphere is assumed to be constant and equal to D_0. Thus

$$D + h = D_0 = \text{const}^1 \tag{2}$$

The continuity equation then may be written:

$$-\frac{1}{D}\frac{dD}{dt} = \frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} \tag{3}$$

where $\bar{u}$ is the component of velocity in the x-direction and $\bar{v}$ is the component of velocity in the y-direction. The equations (1) and (3) form a set of differential equations, from which $\bar{u}$ and $\bar{v}$ are determined as functions of x, y and t. In order to solve this system the equations must be linearized. If u and v denote the perturbation velocities in the horizontal plane we thus assume

$$\left.\begin{cases} u \sim v \ll U \\ h \ll D_0 \\ \zeta \ll f_0 \end{cases}\right\} \tag{4}$$

These assumptions evidently imply that the height of the mountain, h, is small compared to D_0. Then the continuity equation is transformed to

$$-\frac{1}{D}\cdot\frac{dD}{dt} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = \frac{U}{D_0}\frac{\partial h}{\partial x} \tag{5}$$

and if stationary conditions are considered the vorticity equation reduces to

$$U\frac{\partial \zeta}{\partial x} + \beta v = -\frac{f_0 U}{D_0}\cdot\frac{\partial h}{\partial x} \tag{6}$$

where

$$\beta = \frac{\partial f}{\partial y} \tag{7}$$

[1] In fact D_0 must be a function of y in order to get the proper pressure field for the basic current. An estimate shows, however, that no considerable error is introduced by treating D_0 as a constant.

We now introduce a new variable Ψ, defined by

$$\begin{cases} u = -\dfrac{\partial \Psi}{\partial y} + \dfrac{U}{D_0} h \\ v = \dfrac{\partial \Psi}{\partial x} \end{cases} \tag{8}$$

The continuity equation (5) is identically satisfied by this definition. If the expression for v is introduced in (6), this can be integrated with respect to x. For $x = \infty$ we assume that $\zeta = 0$, $\Psi = 0$ and $h = 0$. Then (6) may be written

$$\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} + \frac{\beta}{U} \Psi = \frac{U}{D_0} \frac{\partial h}{\partial y} - \frac{f_0}{D_0} h \tag{9}$$

If Ψ and h are assumed to be independent of y, this equation is reduced to the one treated by CHARNEY and ELIASSEN. It is possible to solve this equation quite general, taking the spherical shape of the earth into account. The computations become, however, rather tedious in this case and we shall restrict ourselves to a qualitative discussion, based on an exact computation for the very simple case of perturbations caused by a circular, solitary mountain in an unlimited zonal current.

The solution of the non-homogeneous differential equation (9) is obtained as the sum of the general solution to the corresponding homogeneous equation and of a particular solution to the non-homogeneous one. From (9) is seen that the homogeneous equation is obtained by putting $h = 0$, i.e. these solutions will describe the free perturbation which may exist independently of the orography of the surface of the earth. The particular solution of (9), on the other hand, describes the stationary wave pattern caused by the mountain. It is possible to find a particular solution in a simple manner, if we assume that the perturbation pattern is symmetrical with respect to the y-axis. This may be considered as correct to a first approximation in the vicinity of the mountain, (cf. fig. 4). Because of the assumption that $h = h(r)$ where $r^2 = x^2 + y^2$ we transform (9) into polar coordinates. If a new independent variable ϱ is introduced, we obtain

$$\frac{\partial^2 \Psi}{\partial \varrho^2} + \frac{1}{\varrho} \frac{\partial \Psi}{\partial \varrho} + \frac{1}{\varrho^2} \frac{\partial^2 \Psi}{\partial \vartheta^2} + \Psi = \frac{U}{D_0} \sqrt{\frac{U}{\beta}} \frac{\partial h}{\partial \varrho} \sin \vartheta - \frac{f_0 U}{\beta D_0} h, \tag{10}$$

where

$$\varrho = \sqrt{\frac{\beta}{U}}\, r \tag{11}$$

A particular symmetrical solution of this equation may be written

$$\Psi = P(\varrho) + Q(\varrho) \sin \vartheta \tag{12}$$

thus only considering the zonal harmonics of zero and first order, because of the structure of the function on the right hand side of equ. (10). $P(\varrho)$ and $Q(\varrho)$ are particular solutions of the two equations

$$\begin{cases} \dfrac{d^2 P}{d\varrho^2} + \dfrac{1}{\varrho} \dfrac{dP}{d\varrho} + P = -\dfrac{U}{\beta} \cdot \dfrac{f_0}{D_0} \cdot h \\ \dfrac{d^2 Q}{d\varrho^2} + \dfrac{1}{\varrho} \dfrac{dQ}{d\varrho} + \left(1 - \dfrac{1}{\varrho^2}\right) Q = \dfrac{U}{D_0} \sqrt{\dfrac{U}{\beta}} \cdot \dfrac{dh}{d\varrho} \end{cases} \tag{13}$$

Thus $P(\varrho)$ and $Q(\varrho)$ are given by

$$\begin{cases} P(\varrho) = -\dfrac{\pi U f_0}{2 \beta D_0} \left[Y_0(\varrho) \displaystyle\int_0^\varrho h(\varrho) \varrho^3 J_0(\varrho)\, d\varrho - J_0(\varrho) \int_\infty^\varrho h(\varrho)\, \varrho^3\, Y_0(\varrho)\, d\varrho \right] \\ Q(\varrho) = \dfrac{\pi}{2} \dfrac{U}{D_0} \sqrt{\dfrac{U}{\beta}} \left[Y_1(\varrho) \displaystyle\int_0^\varrho \frac{dh}{d\varrho} \varrho^3 J_1(\varrho)\, d\varrho - J_1(\varrho) \int_\infty^\varrho \frac{dh}{d\varrho} \varrho^3\, Y_1(\varrho)\, d\varrho \right] \end{cases} \tag{14}$$

where $J_0(\varrho)$, $J_1(\varrho)$, $Y_0(\varrho)$ and $Y_1(\varrho)$ are ordinary Bessel functions. Now u and v can be computed from (8) and (12). As, however, $\mathrm{div}_H\, \mathbf{v} \neq 0$ the velocity field cannot be represented by a system of streamlines. A new function Φ is then introduced, defined as the streamfunction for the volume transport of

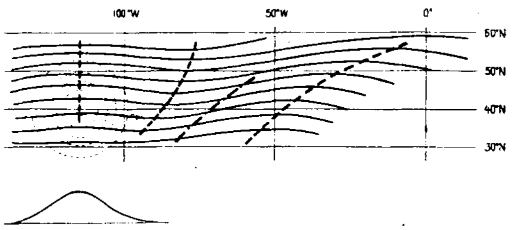

Fig. 3. The theoretical flow pattern generated by a circular mountain in a homogeneous basic current. Dashed lines are troughs and ridges and the dash-dotted line indicates maximum zonal velocity.

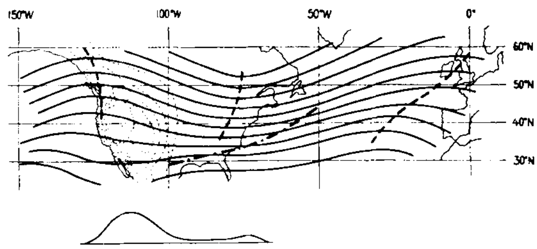

Fig. 4. The observed flow pattern in winter at the 20,000 foot level (after Normal Hemispherical Mean Maps). Streamlines are approximately drawn for every (Δz sin φ) m instead for every Δz m, in order to make the pattern comparable with fig. 3.

the total layer. This is possible as the fluid is assumed to be incompressible. Consequently

$$\begin{cases} D(U+u) = -\dfrac{\partial \Phi}{\partial y} \\ D v \qquad\quad = \dfrac{\partial \Phi}{\partial x} \end{cases} \tag{15}$$

If the same approximations are made as in the linearization of the basic equations, the function Φ is given by

$$\Phi = D_0\,(\Psi - U y) \tag{16}$$

As seen from (15) the velocity vector is tangent to the streamlines for the function Φ. Thus the scalar field Φ may be used to illustrate the deformation of the current around the mountain. We will now first discuss the pattern thus generated by a simple circular mountain, the height of which is given by

$$h = h_0 \cdot e^{-\frac{r^2}{l^2}} \tag{17}$$

l is chosen so that $h = \dfrac{1}{e}\, h_0$ in a distance of 1,000 km from the centre of the mountain. The order of magnitude of the dimensions is the same as for the Rocky Mountains.

A polar coordinate system has been used for the derivation of the perturbation pattern created by this circular mountain. This causes some difficulties for the applicability of the theory to real conditions because of the spherical shape of the earth. Furthermore, β is not a constant but varies with latitude. We assume, however, that the radius of the earth is great compared with the wave-length of the waves and that the variation of β with latitude is small. We may then carry through the computations considering the surface of the earth as plain and β as a constant during the integration, but varying β and taking the smaller circumference of the earth in high latitudes into consideration when constructing the final perturbation pattern. This method cannot be expected to give more than a first crude approximation of the exact solution.

Another difficulty results from the fact that the basic current U is not homogeneous as assumed in the theory. We must then choose a reasonable mean value which may be considered as representative for the actual atmosphere. It is obvious that the appropriate value of U varies with season. According to CHARNEY and ELIASSEN the proper value for U in our equivalent-barotropic atmosphere is about equal to the mean velocity at a level somewhere between 600 and 500 mb in the real atmosphere. We consider winter conditions and choose $U = 20$ m.p.s., which is approximately the mean value at the 500 mb level for a 10 degrees broad band centered around latitude 40° N.

The flow pattern generated by this circular mountain is shown in fig. 3. Some interesting features of this figure shall be pointed out as well as some conclusions drawn from the basic expressions for the perturbations given by (14). This will be useful in the last section of the paper, when discussing the general importance of the orography of the earth.

a) The pattern is rather sensitive to variations of the horizontal scale of the mountain as seen from the expressions for $P(\varrho)$ and $Q(\varrho)$. This depends upon the fact that the integrand contains a factor ϱ^3. An estimate shows that if the horizontal dimensions of the mountains decrease to one half, the values of $P(\varrho)$ and $Q(\varrho)$ are reduced to about one tenth. *Thus only mountains of at least the dimensions of the Rocky Mountains are of importance for the generation of planetary waves in the westerlies.*

b) The computed pattern may be considered to be a combination of two simple pattern, one in principle the same as the pattern QUENEY has derived as being generated by an infinite mountain range because of the flow *over* the mountain, the other similar to the pattern which STEWART has computed as an effect of the flow *around* a circular cylindrical obstacle. Both components are of almost equal importance in the immediate vicinity of the mountain. The STEWART-pattern is, however, completely negligible already one wave-length downstream. *A rather good approximation of real conditions may therefore be expected from a treatment of the problem in one dimension in the horizontal plane. Modifications of considerable importance exist only in the vicinity of the mountain.*

c) The flow pattern varies with the value of the zonal velocity U in two respects. *In the first place* the wave-length is a function of U, as seen from equations (11) and (14): The wave-length is proportional to the squareroot of U. HAURWITZ (1940) and CHARNEY and ELIASSEN have shown that a correction is introduced if the disturbances are assumed to have a finite extension in north—south direction, i.e. if the basic current is limited by fixed boundaries to the north and south. The wave-length is in fact very sensitive to the value of the width of the current. The real westerlies do have a finite meridional extension but it is difficult to choose a value which may be considered as representative for this special purpose since no *fixed* boundaries exist. Thus no assumption of boundaries to the north and south have been introduced. *In the second place* small changes in the shape of the flow pattern in the vicinity of the mountain are found for varying U. For certain values of U the ridge at the mountain is somewhat stronger, for other values it is weaker than for the particular value of U used in the construction of fig. 3. These variations are small and the principal features of the pattern are the same within the atmospheric range of U.

d) The tilt of the trough and ridge lines increases with increasing distance from the mountain.

e) Two regions with intensified zonal flow are found theoretically, one just to the north of the mountain and the other on the south-eastern side of the first trough downstream.

IV. Influence of a mountain range on a very narrow current

It should be kept in mind that the theory advanced in the preceding section is based on the assumption that the width of the basic current is large in comparison with the north—south extent of the mountain. In the case of a very narrow current the mountain influence may have an entirely different character. When the current passes a mountain, horizontal divergence takes place resulting in a generation of anticyclonic vorticity within the current. If the mountain is large enough and the current not too strong, the anticyclonic curvature of the current will be so strong that the current is turned back to its source region. To demonstrate this particular orographic effect we shall disregard the latitude variation of the Coriolis parameter and consider the extreme case of an infinitely narrow current striking a mountain range at right angles, though this last assumption is not essential. Furthermore we disregard the influence of the environment completely. As before we assume that the vertical contraction of the air column is made up for by an equivalent transversal streching (= the velocity, c, of the current remains constant). As the relative vorticity of the undisturbed current is zero and if f_0 denotes the constant value of the Coriolis parameter and D_0 the undisturbed height of the air column, (1) may be written

$$\frac{\zeta + f_0}{D} = \frac{f_0}{D_0} \tag{18}$$

or

$$\zeta = -\frac{f_0}{D_0} h \tag{19}$$

Disregarding changes in the horizontal shear we get

$$\frac{1}{R} = -\frac{f_0}{c D_0} h \tag{20}$$

where R is the radius of curvature of the cur-

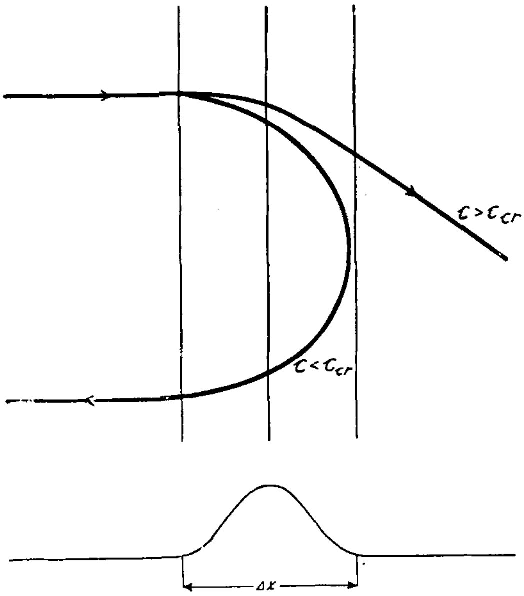

Fig. 5. Deformation of a narrow current because of divergence when striking a mountain range. The upper part of the figure shows the horizontal flow pattern and the lower part a vertical cross-section across the barrier.

rent. Introducing the analytical expression for R (20) becomes

$$\frac{\dfrac{d^2y}{dx^2}}{\left[1+\left(\dfrac{dy}{dx}\right)^2\right]^{3/2}} = -\frac{f_0}{cD_0}h \tag{21}$$

which may be transformed to

$$d\left(\frac{\dfrac{dy}{dx}}{\left[1+\left(\dfrac{dy}{dx}\right)^2\right]^{1/2}}\right) = -\frac{f_0}{cD_0}h\,dx \tag{22}$$

or

$$d\,(\sin\alpha) = -\frac{f_0}{cD_0}h\,dx \tag{23}$$

α being the angle between the deflected jet and the x-axis. As the undisturbed current is assumed to be straight westerly we get

$$\sin\alpha = -\frac{f_0}{cD_0}\int_{x_1}^{x} h\,dx \tag{24}$$

where x_1 is the x-coordinate for the point where the current strikes the mountain (cf. fig. 5). If the current is to cross the mountain, $|\alpha|$ must be less than 90° everywhere. Hence

$$c > \frac{f_0}{D_0}\int_{x_1}^{x} h\,dx \tag{25}$$

We may define a critical velocity

$$c_{cr} = \frac{f_0}{D_0}\int_{x_1}^{x_1+\Delta x} h\,dx \tag{26}$$

where Δx is the width of the mountain. If

$$c < c_{cr} \tag{27}$$

the current is turned back to its source region. Putting $f_0 = 10^{-4}$ sec^{-1}, $D_0 = 10$ km, $\Delta x = 1\,000$ km and assuming a mean height of the mountain range of 2000 m we get $c_{cr} = 20$ m sec^{-1}.

If the variation of the Coriolis parameter with latitude is taken into account c_{cr} will be modified. For a westerly current the critical value will be smaller, for an easterly current greater than given in (26). This is easily seen from a qualitative discussion. It is believed that the effect indicated above may be of importance in the atmosphere even if (26) merely gives the order of magnitude of the critical value. As shown by the numerical example only mountains of the same order of magnitude as the Rocky Mountains are effective.

V. The importance of the orography of the earth for the general character of the westerlies

The discussion in this section must necessarily be rather vague since a very simplified atmosphere has been used for the theoretical considerations on which it is based. Following the general procedure outlined in II we shall at first discuss conditions at upper levels (500 mb), then consider the effect at the surface of the earth and in doing so also take account of the thermal influence from below. The degree of similarity between the theoretical results and the observed pattern will show to what extent the hypothesis of a dynamic-orographic control of the upper level westerlies may be considered as correct.

As seen from III, a as well as from IV we need to consider only those mountains which are of the same order of magnitude as the Rocky Mountains. Hence it follows that *the only mountains which may be of real importance in generating planetary waves in the westerlies are the Rocky Mountains and the mountains in the interior of Asia.*

Let us at first consider the general character of the waves around the northern hemisphere with regard to wave-length and the location of the troughs and ridges. It was pointed out in III that a rather good first approximation may be obtained from a treatment of the problem in one dimension in the horizontal plane as made by CHARNEY and ELIASSEN. It is essential, however, that we consider the belt of maximum westerlies and that the mountains are located in the centre of this current. It then seems reasonable to assume that the relatively weak currents in the environment ajust themselves to the wave pattern in the belt of maximum kinetic energy. Therefore CHARNEY's and ELIASSEN's computations are not quite representative since they considered the latitude belt between 40° N and 50° N, which is to the north of the highest parts of the Asiatic mountains. It is nevertheless of some interest to compare their results with actual conditions. Fig. 6 shows the 500-mb profile around the whole hemisphere computed by CHARNEY and ELIASSEN and the observed one obtained from the hemispherical upper air mean charts for winter, which have been published by SCHERHAG. The agreement between the computed and observed patterns is good, in fact better than CHARNEY and ELIASSEN found themselves using Normal Weather Maps published by U. S. WEATHER BUREAU (1944).

A comparison between winter and summer conditions is of special interest when discussing the relative importance of dynamic-topographic versus thermal factors for the control of stationary waves in the westerlies. The thermal influence from the surface of the earth is quite different in the two seasons and should thus be expected to bring about a rather far-reaching rearrangement of the wave pattern, as was already pointed out in the introduction. From fig. 1 is seen that such a

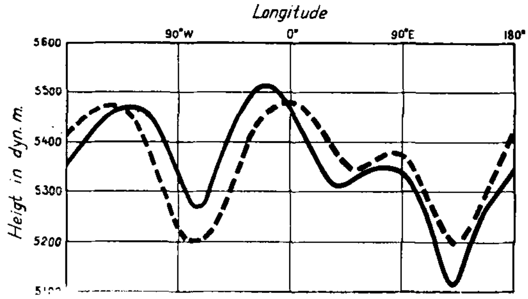

Fig. 6. Computed stationary profile around the hemisphere of the 500 mb surface (dashed curve) according to CHARNEY and ELIASSEN (1949) and observed mean conditions (solid curve) after SCHERHAG (1948). The curves represent winter conditions in the belt between 40° N and 50° N.

change does not occur. On the other hand if one assumes that the large mountains are of importance in generating these waves, the observed similarity between summer and winter patterns is obtained as a necessary consequence. The stationary wave-length is proportional to $\sqrt{U/\beta}$. The zonal flow decreases from winter to summer, but at the same time the belt of maximum westerlies is displaced farther to the north. These two facts counteract each other and the stationary wave-length (measured in degrees longitude) remains almost constant. If we assume that the zonal wind decreases at the rate of 50 per cent and that the meridional displacement is 10° we obtain

$$\frac{L_w}{L_s} \approx 1.15 \tag{28}$$

where L_w and L_s denote the stationary wave-length in winter and summer respectively. Since furthermore the waves are anchored by the mountains, there should be little or no difference in the location of the waves during the two seasons. This points to the correctness of interpreting the waves as generated mainly by orographic influences. As seen from (28) there is a slight decrease of the wave-length from winter to summer which in reality shows up as a *tendency* for a fourth trough to develop in summer off the American westcoast. This is in agreement with the fact that two polar fronts are often observed in the Pacific in summer. These two troughs can hardly be explained by assuming that the thermal influence from below is of conclusive importance.

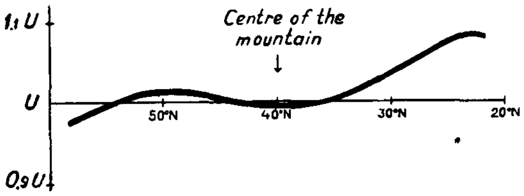

Fig. 7. Theoretically computed profile of the zonal velocity in a cross-section 3,000 km downstream from the circular mountain.

It is impossible to pursue the discussion further without considering the meridional variation of the height of the mountains and thus allow for flow around as well as over them. There is an essential difference between the mountains in America and in the interior of Asia in this respect.

The Rocky Mountains extend all along the American westcoast and thus the current must always pass *over* the range in spite of the fact that the westerlies are displaced northward from winter to summer. Therefore the actual wave pattern should in principle be in accordance with the pattern obtained from the treatment of the problem in one dimension in the horizontal plane also in the vicinity of the mountains. It must be pointed out, however, that the Rocky Mountains are broadest and highest at about 40° N. According to the previous discussion this should result in a tendency for the current to flow around these parts of the mountain range. Some indications of this are also found by a comparison between fig. 3 and 4. The general structure of the mean flow pattern in the vicinity of the Rocky Mountains has several similarities with the pattern

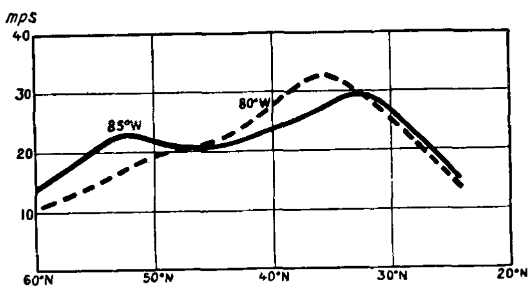

Fig. 8. Zonal wind profile across the westerlies at the 20,000 foot level. The solid curve is computed from Normal Weather Maps along longitude 85° W in January and the dashed curve is obtained from the mean cross-section in winter along 80° W, which has been published by HESS (1948).

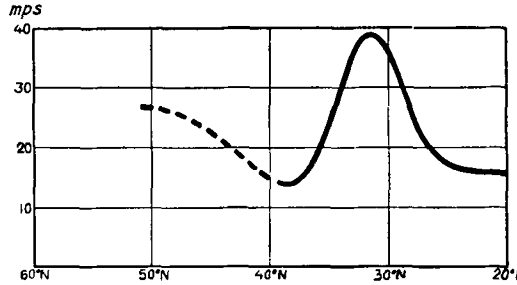

Fig. 9. Zonal wind profile in winter at 76° E across the westerlies at the 20,000 foot level (after CHAUDHURY, 1950).

which we have computed for the flow around a circular mountain. Some indications of the tendency for the splitting of the basic current may also be found by comparing the computed and observed windprofiles across the current some thousand kilometers downstream from the mountains (fig. 7 and 8). However the dominating deformation caused by the Rocky Mountains results from the fact that the westerlies are forced to flow *over* these ranges.

The mountains in the interior of Asia, on the other hand, do not have the same extent in the north-south direction. Hence the westerlies enclose the mountains in winter, while the mean jet is found well to the north of the Tibetean Plateau in summer. In winter the westerlies are split in two branches, one on each side of the mountains (fig. 9), which is in qualitative agreement with the computed effect (fig. 7). Further downstream a *well developed* wave pattern is found. In summer, on the other hand, the jet is affected only by the relatively low mountains further to the north. This may explain the fact that there exists only one mean jet at that time of the year and that the waves in the westerlies further downstream have *small* amplitudes (see fig. 1). The mountains obviously are of considerable importance, but it must be kept in mind that the Himalayas in winter acts as a boundary between cold air to the north and warm air to the south. This certainly also is of great importance for the character of the westerlies in the vicinity of the mountains.[1]

[1] After this manuscript had been completed the writer' attention has been called to a paper by YEH (1950), which is also published in this issue of Tellus. YEH's data support most of the conclusions drawn in the last two paragraphs. Some similar results have also been obtained

The division of the westerlies in two branches has one consequence which is essential when discussing intensity variations of the jet. SUTCLIFFE (1940) and NAMIAS and CLAPP (1949) have pointed out the significance of the confluence mechanism for the intensification of a jet. This mechanism means simply the flowing together of two different air currents, one stream coming from the warmer southerly latitudes, the other from the colder north. As seen from fig. 3 a confluence pattern is generated on the lee side of a mountain in a westerly current. If these two streams joining to the east of the mountain are of different thermal character, there should be an intensification of the current downstream. It is a fact that in the mean the most intense jets are found in the regions to the east of the large mountain areas (cf. NAMIAS and CLAPP). The difference between summer and winter conditions may be explained by the seasonal differences in the confluence mechanism. These variations should thus depend a) upon the extent to which the mountains are capable of splitting the westerlies, b) upon the magnitude of the thermal contrast between the two currents.

Until now we have only discussed the normal upper air charts as representing a mean condition at upper levels. In reality considerable deviations from this normal pattern may persist for periods as long as several weeks. It must be kept in mind, however, that the latitude and speed of the maximum westerlies which have been used above are the normal seasonal values; deviations from these normal values must cause changes in the stationary patterns generated by the mountains. Furthermore the effect discussed in IV may be of importance during low index periods. This may for example be one explanation of the stationary extended troughs, that sometimes are found over the Rocky Mountains.

It must be emphasized that the solutions used in the preceding discussion are not unique; but they represent the particular flow patterns which are the immediate consequence of the mountain barriers. Besides these waves there may exist stationary waves which are not fixed by the orography, and which disappear if the mean pattern is computed over a sufficiently long time. Such waves may influence the character of the mean pattern during periods of a few weeks duration.

If one accepts the hypothesis that the mountains are of primary importance for the generation of the mean stationary long waves in the westerlies, the ridge at the European westcoast and the trough over the White Sea must be considered as to a certain extent being formed in resonance with the waves at the Rocky Mountains (cf. fig. 1). On the whole, the positions of the ridge and trough are in agreement with the computations made in III if one allows for the correction that must be introduced because of the finite width of the current. In addition, it must be pointed out that the mean ridge to the west of Europe to some extent may be the result of the fact that this region appears to be a preferable place for blocking to start (ELLIOTT and SMITH, 1949; REX, 1950). This effect has not been considered in our theoretical treatment. It is probable that blocking may be of some importance for the formation of the upper ridge but it should be noted that the ridge often is found even in the absence of blocking. It thus seems quite probable that the mean ridge to a considerable extent is caused by the topography of the earth further to the west.

Thus we have found that there are several features of the upper flow pattern which indicate that the orography of the earth seems to be of considerable importance for the mean large scale wave pattern around the northern hemisphere. We shall now finally discuss the interrelation between this upper flow pattern and conditions at the surface of the earth. Following the general chain of reasoning outlined in the beginning of the paper we shall try to get an idea of the adjustment in the layer below 500 mb considering the restrictions which are impressed by the flow pattern aloft, the surface friction and the differential heating from below. Assuming then that the flow pattern above a certain height H ($\approx$ 5 km) to a great extent is controlled dynamically it follows that the pressure distribution at this level is known apart from a constant. Adjustment below means a redistribution of mass, so that the pressure distribution at the ground fits the requirements of the monsoonal circulation in the lower part of the troposphere.

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in two theses by BOFFI and CHAUDHURY, which were carried through under the supervision of Dr H. Riehl at the University of Chicago.

This redistribution of mass in turn means a change of the mean temperature of the air column below H and consequently *the mean temperature of this air column is at least in part controlled dynamically from above, in addition to the direct thermodynamic control from below*. This conclusion, if supported by further investigations, is of great importance to dynamic climatology (Bergeron, 1930). It implies that both pressure and temperature at the ground are *in part* controlled by the mean flow pattern aloft, which in turn are intimately related to the orography of the earth all around the hemisphere. At the present time it is difficult to make a qualitative estimate of this influence of mountain systems upon the climate of far distant regions, but qualitatively we are justified in assuming that both the upper and surface high pressure ridge from the Azores to France in winter and the northerly position of the mean storm tracks over the eastern Atlantic resulting in relatively mild winters in northwestern Europe are not exclusively to be looked upon as an effect of the warm ocean but also as a result of the influence of the Rocky Mountains.

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