On the Use of Areas of Rising and Falling Pressure in the Free Atmosphere

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(Original manuscript received 14 July 1949, revised manuscript received 15 June 1950)

Abstract

A method for the determination of areas of rising and falling pressure in the free atmosphere is given, together with a brief discussion of how these pressure variations lead to the growth of isallobaric areas at sea level.

I. Introduction

The upper-air service in Norway was reorganized after the war by SVERRE PETTERSSEN. The current work is now performed along the lines described by S. PETTERSSEN and C. H. B. PRIESTLEY: "Preparation of Upper Air Analysis and Forecast Charts." (Reprinted in Norway in a somewhat modified form as: Værtjenestememorandum nr. 2.) We are making use of the rule of SUTCLIFFE, that the thickness lines approximately move with the speed of the gradient wind, and of the following rules of PETTERSSEN:

"(1) Where the thickness lines climb up the pressure surface the pressure must fall, and the fall increases with the isobaric thermal wind and the slope of the pressure surface.

(2) Where the thickness lines are parallel to the contour lines (i. e. no climb) the pressure remains stationary.

(3) Where the thickness lines descend the pressure surface, the pressure must rise, and the rise increases with the isobaric thermal wind and the slope of the pressure surface.

In applying these rules, due allowance must be made for the non-conservation of the thickness in regions where there is indication of subsidence or ascending motion, and also in regions where the air is being appreciably heated, or cooled, by the underlying surface. In particular, in outbreaks of polar air the non-adiabatic heating will be appreciable in the lower troposphere, and the change of the thickness from 1,000 to 700 mb. is often not higher than 50 per cent of that which would be expected of sheer advection."

In the following paragraphs we shall study such weather situations where the upper-air charts show a distinct grid between contour lines and thickness lines, as in the rule of PETTERSSEN, but we shall consider the problem from a different point of view.

II. Variations in the height of the upper-air surfaces of pressure

We shall try to judge these variations in height from the movement of the thickness lines and from the barometric tendencies at sea level. As the tendencies are valid for three hours, we must determine the movement of the thickness lines for the same interval. At the weather service in Oslo only the 700 and 500 mb. upper-air charts are drawn, and only twice a day, at 4^h and 16^h G. M. T.

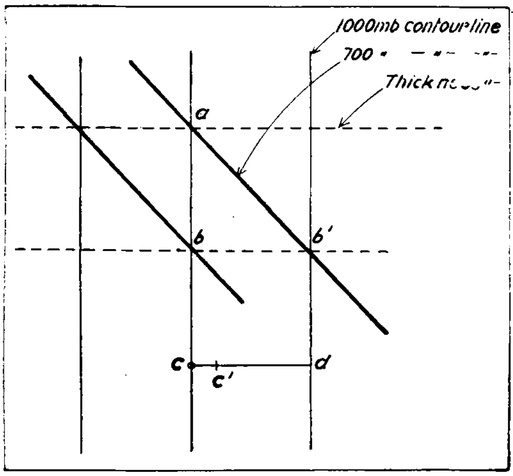

Fig. 1. Intersecting contour lines and thickness lines. Further explanation in the text.

We shall first study the variation in the height of the 700 mb. surface by means of some examples. In order to simplify our discussion, we assume that the contour lines and the thickness lines are straight, fig. 1. As a first illustration we assume that the barometric tendency has been zero at the ground during the last three hours. We may then, with sufficient accuracy, say that the 1,000 mb. surface has been stationary during this interval of time. With the help of the "gradient ruler" we can determine how far the thickness line through point a, fig. 1, has moved in three hours. Let us say it has reached the position of the foregoing thickness line through point b; then the 700 mb. surface has sunk 40 m at point b during three hours (assuming that the thickness lines are drawn at intervals of 40 m and that cold air is moving forward).

If a point a in the 1,000 mb. surface has moved to b, the corresponding point in the 700 mb. surface has moved to b'. A column of air through point a, originally vertical, cannot remain vertical throughout the movement. But, during a short interval of time we may, as a first approximation, think of the columns as remaining vertical during the movement, at least as far as the single layers between the chosen pressure surfaces are concerned.

As a second illustration we assume that the surface barometric tendency has been + 1.0 mb. over the area in question. See fig. 1. Then the 1,000 mb. contour line abc has moved 1/5 of the distance between c and d, i. e. c has moved to c' (approximately). At the point b the 1,000 mb. surface consequently has risen 8 meters. Now let the thickness line through point a reach b in 3 hours. The fall of the 700 mb. surface at b will then be $40 - 8 = 32$ m. If the barometric tendency had been + 5 mb. instead of 1, in the same manner we would have found, that there had been no change in the height of the 700 mb. surface. Further if the tendency had been greater than 5 mb., we would have found a rise instead of a fall of the 700 mb. surface, assuming the speed of the thickness lines has been the same in all cases.

In a similar manner we can determine the change of height when warm air is streaming forwards.

If we possessed absolutely reliable charts, we could determine the rise or fall of the 700 mb. surface over any chosen point at sea level by the method described above. In the same manner we could later determine the rise and fall of the 500 mb. surface relative to the 700 mb. surface and hence to the sea level (or 1,000 mb. surface). With current charts we can only hope to determine the main features of the isallobaric areas in the free atmosphere, and only during great changes in the weather situation.

When the rising and falling areas have been determined — as far as possible — we can take into account accelerations in the upper air, which we have put aside up to now. We can thus in a qualitative way decide if a decrease or increase or no change in the sea level isallobaric areas is to be expected. In order to get a better understanding of such accelerations we shall study them with the help of some simple computations.

III. Movement of a particle of air in a variable field of pressure.

In the following discussion we can only make use of point dynamic relationships and can take into account only horizontal movements. In order to make the calculations as simple as possible we shall consider only straight, parallel and equidistant isobars and isallobars. In general the isobars and isallobars will intersect, and we shall take the x-axis in the direction of the isallobars, see fig. 2.

G is the gradient force with components G_x and G_y. The change in the isallobaric gradient in t seconds is $G_1\, t$. In fig. 2 this vector is chosen vertical to the x-axis. The specific volume α and the Coriolis parameter $\lambda = 2\,\omega \sin \varphi$ are regarded as constants. The horizontal components of the wind velocity are u and v. Regarding a unit mass, we have the following equations of motion:

$$\frac{d^2x}{dt^2} - \lambda \frac{dy}{dt} = \alpha\, G_x \qquad (1)$$

$$\frac{d^2y}{dt^2} + \lambda \frac{dx}{dt} = \alpha\, (G_y - G_1 t) \qquad (2)$$

Putting $u = u_0$ and $v = v_0$ by $t = t_0$ we have the solutions:

$$u = \left[\left(u_0 - \frac{\alpha G_y}{\lambda}\right) \cos \lambda t + \left(v_0 + \frac{\alpha G_x}{\lambda}\right) \sin \lambda t + \frac{\alpha G_y}{\lambda} \right] + \left[\frac{\alpha G_1}{\lambda^2} \sin \lambda t - \frac{\alpha G_1}{\lambda} t \right] \qquad (3)$$

$$v = \left[-\left(u_0 - \frac{\alpha G_y}{\lambda}\right) \sin \lambda t + \left(v_0 + \frac{\alpha G_x}{\lambda}\right) \cos \lambda t - \frac{\alpha G_x}{\lambda} \right] + \left[\frac{\alpha G_1}{\lambda^2} \cos \lambda t - \frac{\alpha G_1}{\lambda^2} \right] \qquad (4)$$

$$x = \left[\left(\frac{u_0}{\lambda} - \frac{\alpha G_y}{\lambda^2}\right) \sin \lambda t - \left(\frac{v_0}{\lambda} + \frac{\alpha G_x}{\lambda^2}\right) \cos \lambda t + \frac{\alpha G_y}{\lambda} \right] + \left[-\frac{\alpha G_1}{\lambda^3} \cos \lambda t - \frac{1}{2} \frac{\alpha G_1}{\lambda} t^2 + C_3 \right] \qquad (5)$$

$$y = \left[\left(\frac{u_0}{\lambda} - \frac{\alpha G_y}{\lambda^2}\right) \cos \lambda t + \left(\frac{v_0}{\lambda} + \frac{\alpha G_x}{\lambda^2}\right) \sin \lambda t - \frac{\alpha G_x}{\lambda} t \right] + \left[\frac{\alpha G_1}{\lambda^3} \sin \lambda t - \frac{\alpha G_1}{\lambda^2} t + C_4 \right] \qquad (6)$$

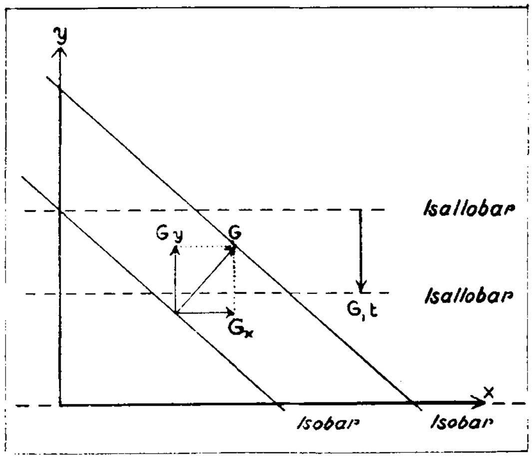

Fig. 2. Coordinate system with the x-axis along the isallobars.

In these equations the first square bracket shows the influence of the original field of pressure, and the second square bracket gives the influence of the isallobars. The last term in the first square bracket represents the geostrophic wind, the two first terms giving the oscillations caused by deviations from the geostrophic wind at $t = 0$. Generally we do not know these deviations, we must use our few direct, wind observations to correct the pressure gradient, when drawing the contour lines. Since the geostrophic wind itself cannot contribute to the development of rising or falling areas, we must limit our research to the influence of the isallobars, i. e. we may only make use of the second square bracket in each of the equations (3) to (6). In the two last equations we can determine C_3 and C_4 by putting x and $y = 0$, and so we get the following expressions:

$$u = \frac{\alpha G_1}{\lambda^2} \sin \lambda t - \frac{\alpha G_1}{\lambda} t \qquad (7)$$

$$v = \frac{\alpha G_1}{\lambda^2} \cos \lambda t - \frac{\alpha G_1}{\lambda^2} \qquad (8)$$

$$x = -\frac{\alpha G_1}{\lambda^3} \cos \lambda t - \frac{1}{2} \frac{\alpha G_1}{\lambda} t^2 + \frac{\alpha G_1}{\lambda^3} \qquad (9)$$

$$y = \frac{\alpha G_1}{\lambda^3} \sin \lambda t - \frac{\alpha G_1}{\lambda^2} t \qquad (10)$$

We cannot expect exact results with so many simplifications, but practice seems to indicate a marked influence of the isallobars, when large changes in the field of pressure take place.

Table 1

$\lambda \cdot t$ in degrees	t	cm/sec.		km	
		u	v	x	y
5°	14.5 min	0.025	0.35		
10°	29 »	0.06	1.1	0.0003	0.006
15°	44 »	0.3	2.5	0.0017	0.022
20°	58 »	0.5	4.4	0.0044	0.051
25°	1 hour 13 »	1.0	6.8	0.011	0.099
30°	1 » 27 »	1.7	9.7	0.023	0.172
35°	1 » 41 »	2.7	13.1	0.048	0.271
40°	1 » 56 »	4.0	17.0	0.071	0.401
45°	2 » 11 »	5.7	21.3	0.113	0.568
50°	2 » 25 »	7.7	25.9	0.171	0.774
55°	2 » 40 »	10.2	31.0	0.249	1.022
60°	2 » 55 »	13.2	36.3	0.351	1.315
65°	3 » 9 »	16.6	41.9	0.480	1.656
70°	3 » 24 »	20.5	47.8	0.641	2.047
75°	3 » 38 »	24.9	53.8	0.839	2.491
80°	3 » 53 »	29.9	60.0	1.078	2.987
85°	4 » 7 »	35.4	66.3	1.362	3.538
90°	4 » 22 »	41.4	72.6	1.691	4.144
95°	4 » 36 »	48	79	2.100	4.806
100°	4 » 51 »	55	85	2.596	5.521
105°	5 » 5 »	63	91	3.018	6.293
110°	5 » 20 »	71	97	3.639	7.116
115°	5 » 35 »	80	103	4.338	7.992
120°	5 » 49 »	89	109	4.966	8.918

In order to compute u, v, x and y we will put $\alpha = 784$ cm³, i. e. air at 0° C and 1,000 mb., $\lambda = 1 \cdot 10^{-4}$ and $G_1 = 1$ mb./1,000 km in 3 hours, i. e. the steepness of the pressure surface has changed 8 m/1,000 km in 3 hours. The result is found in table 1 and figs. 3 and 4. Since u, v, x and y are proportional to G_1, we have only to multiply by a factor to find the corresponding result for other values of G_1, and further since αG_1 is approximately constant in the atmosphere, we may also use sea-level calculations for any surface in the upper air.

We shall use these computations to form an estimate of the influence of the isallobaric areas, assuming that the isallobars can be drawn with great accuracy for a short interval of time.

In a previous paper it is shown that we can use the length of the isobars to determine the mean pressure gradient and the mean

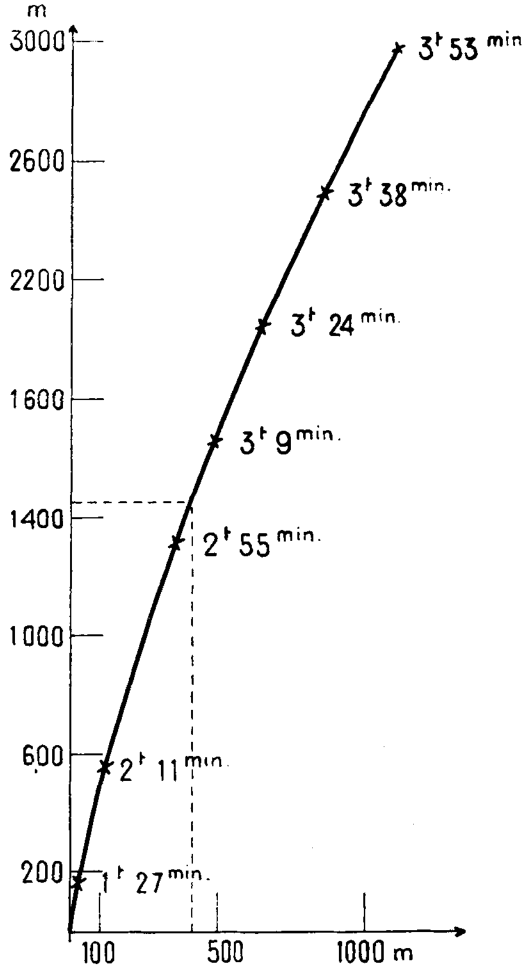

Fig. 3. Path of a particle moving under the influence of straight, parallel isallobars.

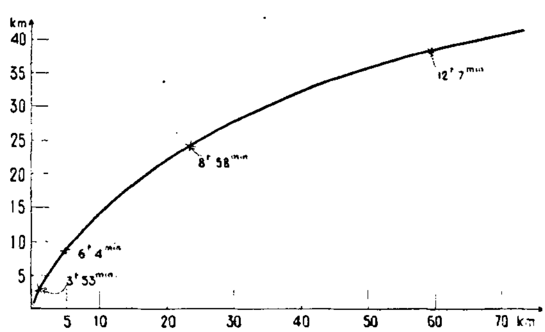

Fig. 4. Same path as in fig. 3, but for a longer interval of time.

geostrophic mass transport in a horizontal unit layer. If we divide the intermediate space between two neighbouring isobars into squares — "isobaric squares" — the kinetic energy is the same for each square, because the wind speed is inversely proportional to the distance between adjacent isobars. Since this last condition holds for isallobars too — equations (7) and (8) — we deduce that $v \cdot \Delta s$ (and $u \cdot \Delta s$) is constant for each isallobaric square, where Δs is the length of a side. Counting the number of squares we may thus calculate the horizontal flow across the isallobars and therefrom determine the horizontal divergence. (We may also calculate the wind circulation around an isallobar, and the mean mass transport produced by the isallobars; the mean isallobaric gradient may be determined by measuring the length of the isallobars. Here only the divergence shall concern us.) In order to get an acceptable approximation, we must use a time interval so short that an arbitrary particle of air can be supposed to stay within an isallobaric square during this interval. If the geostrophic wind is strong at $t = 0$, we will have to choose a time interval considerably shorter than 3 hours. At present we have only vague ideas of the isallobaric field and, therefore, cannot yet make use of a method based on the isallobaric squares in attempting a computation of the mass transport across the isallobars. We must content ourselves with a more qualitative determination. More accurate charts in the future will surely lead to a more accurate determination of this mass transport.

Now let the schematical fig. 5 represent a

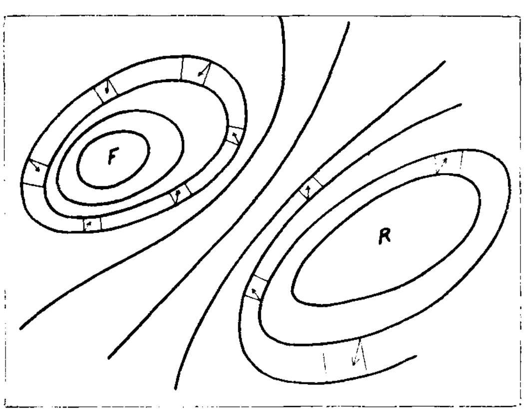

Fig. 5. Rising and falling area in an upper-air pressure surface, schematical.

rising and falling area in the — say — 500 mb. surface. From the rising area air will flow out in every direction and into the falling area air will flow in from every direction. We should consequently expect to find a falling area at sea level under the area of a rising upper-air surface, and a rising area at sea level under the area of a falling upper-air surface. The in- or outflow depends not only upon the isallobaric gradient, but also very much upon the time interval chosen for the isallobars, as is shown by table 1. For instance: Small tendencies at sea level coincident with strong isallobaric areas in the upper air indicate that the flow across the isallobars in the upper air has gone on only for a short time, and that an increase in this flow is therefore to be expected, which will in turn lead to a strengthening of the sea-level isallobaric areas.

IV. Further remarks on thickness lines and isallobars

The rising or falling of the upper-air surfaces depends upon the sea level tendencies and upon the movement of the thickness lines. To estimate the movement of the thickness lines is, however, no easy task, as we cannot make use of the rule of SUTCLIFFE in a servile way. Fig. 6 shows the 700 mb. chart of 4/11 1948, 04^h G. M. T. The two thickness lines *AB* and *CD* are lying approximately motionless at the intersections. The speed of the gradient wind north-east of Iceland is indicated by the arrow, but it is not likely, that a corresponding displacement of the thickness line *AB* will take place in this region. The cold stream of air is relatively narrow, and we cannot expect that the cold air will move within the contour lines as in solid tubes. It will try to sink and spread out along the ground, and we therefore should expect a retardation in the speed of the thickness lines. But how great this retardation will be one can only guess. In the stream under consideration (near the arrow) I should probably have said at least 30 %. Retardation resulting from warming at the sea-surface is also to be taken into account. PETTERSSEN and PRIESTLEY (l. c.) have pointed out that we may expect a retardation up to 50 %, if cold air moves across a warm sea-surface. In cold arctic or polar air moving from Labrador towards the southeast the thickness lines at

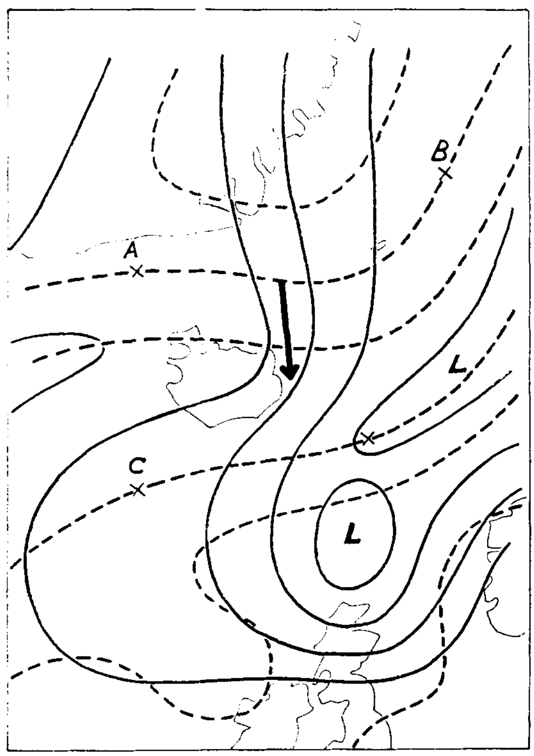

Fig. 6. 700 mb chart of Nov. 4, 1948, 04h G.M.T.

times appear to be retarded even more than 50 %. In the Norwegian Sea I should estimate that 25 % would be a more reasonable value, but accurate estimates cannot be made at present owing to the scarcity of observations both in space and in time. The wind speed and the difference between air and sea temperature must play a rôle too. Cooling from the ground is of lesser importance than warming. A cooling from sea level will not effect the height of the 700 mb. surface in any marked way within the short interval of 3 hours.

The heat of condensation may also effect the height of the upper-air surfaces in a very pronounced way, when we consider great areas of precipitation such as in warm-frontal rain areas. If the rainfall amounts to 1 mm an hour and the condensation takes place in a vertical column between 2,000 and 6,000 m, this column will expand about 30 m in 3 hours. We should expect therefore a rapid rising of the upper-air surfaces over and near any large area of heavy precipitation and that outflow from this area would consequently produce a strong falling area at sea level. In fact the most marked falling areas at sea level seem to be closely associated with areas of heavy rain. If we therefore find it likely that a rain area will increase, we may also conclude that an increase of the falling area at sea level will probably take place. Such a development must at present be forecast by indirect methods. The upper air humidity observations are too scarce to permit drawing of any conclusions.

If the thickness lines and the contour lines are running parallel, no great changes can take place in the height of the upper-air surfaces. The changes are then determined by the sea level tendencies, but in such cases the sea level tendencies are apt to be small too. Distinct grids between thickness and contour lines may be found in the vicinity of strong cols, where cold and warm air masses are drawing together. Fig. 7 shows a schematical example of such a col. The thick, even lines represent 1,000 mb. contours, the broken lines are lines of equal thickness between this surface and the 700 mb. surface. These lines have been drawn arbitrarily in the figure and afterwards the 700 mb. lines have been constructed in the usual way. Cold air is streaming southwards across the Norwegian Sea, warm air is streaming northwards in the Mid-Atlantic. This movement will lead to an accumulation of thickness lines near the col and gradually create a front there.

A vertical plane through the straight line

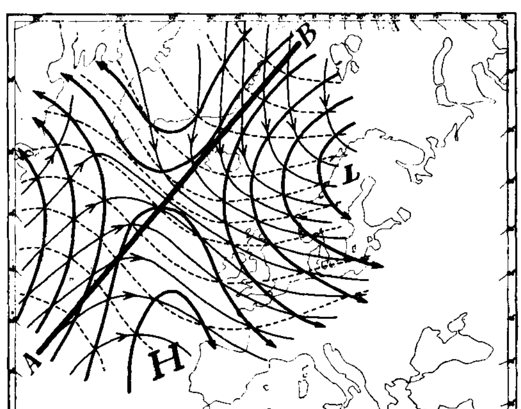

Fig. 7. Schematical illustration of two pressure surfaces near a col.

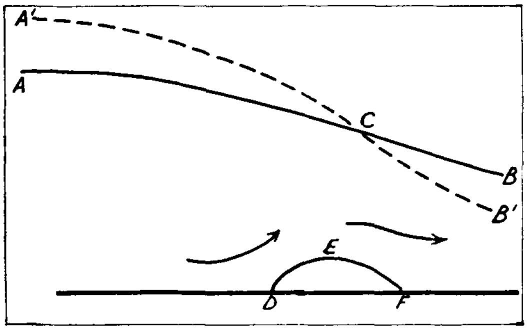

Fig. 8. Vertical section through the line AB in fig. 7.

AB, fig. 7, will intersect the pressure surfaces. In fig. 8 *ACB* represents its intersection with the 700 mb. surface, and *DEF* with a pressure surface near the ground. Let us assume — in order to simplify the case — that the sea-level tendencies are small enough to be neglected. The 700 mb. surface will then rise, whenever the thickness lines are moving northwards in the warm air, whereas it will fall over the cold air moving southwards. The line *ACB*, fig. 8, will gradually change into the line *A'CB'*. *E* represents the col near the ground, *C* the col at the 700 mb. level. The isallobaric areas, which are created in the 700 mb. surface, will therefore cause a mass transport across the isallobars. The movement will be at first slow but will steadily increase in speed. See table 1.

In the same manner we may study the 500 mb. surface and determine how this surface will rise or sink in relation to the 700 mb. surface. In most cases the two surfaces will move qualitatively in the same way, if great air masses are involved in the process.

As soon as outflow from the rising areas in the warm air is perceptible, a falling area will develop below, at sea level, and as the outflow in the upper-air is accelerated, the falling area at sea level will increase. Similarly we should expect an increase in flow into the upper area over the cold air and at sea level an increasing rising area.

The utility of upper-air charts is therefore evident in cases with zero tendencies at sea level, and with distinct grids between thickness lines and contour lines. In these cases the development of isallobaric areas at sea level may be foreseen or at least suspected when

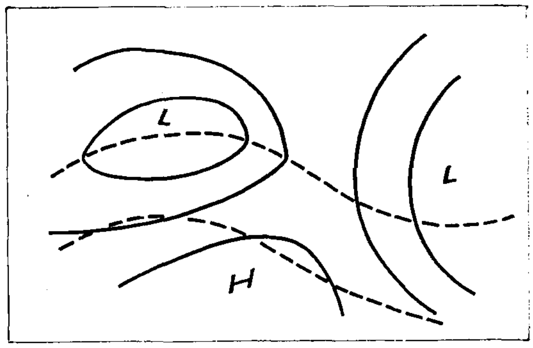

Fig. 9a. Schematical illustration of "open waves" on the contour lines near a col at the beginning of a cyclone cycle.

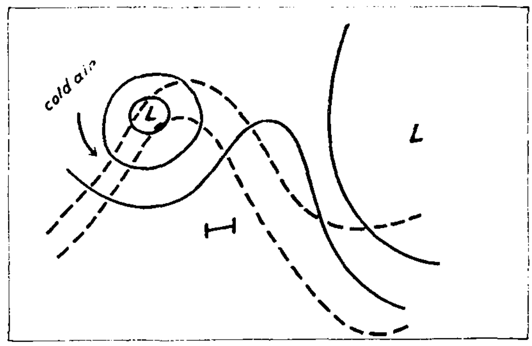

Fig. 9b. Same waves as in fig. 9a, but in a later phase.

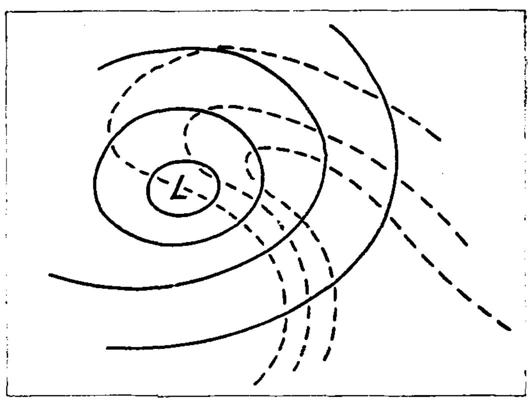

Fig. 9c. Illustrating the form of the thickness lines of a great central cyclone.

the sea-level chart alone would give no sure indications.

In some situations the thickness lines may initially have the shape of broad, open "waves", see fig. 7. As the warm air moves northwards and the cold air southwards, the amplitude of these "waves" will increase. Fig. 9 a shows a thickness line at the beginning of the process and fig. 9 b in a later phase. During phase *b* we must imagine that there is a well developed falling area at sea level in the warm-air tongue. This falling area will accelerate the air in the ground layers, and a flow into the falling area will take place there. This inflow will partially counteract the building of the falling area, but the magnitude of this reaction is not easy to estimate. In fig. 9 b it is suggested that cold air is being drawn into the falling area from the northwest. If this cold air is intersected by many thickness lines, a sinking of the upper-air surfaces is likely to take place during the movement. This may cause an increased outflow from the warm-air tongue towards the west. We may then have an increasing sea-level rising area in the cold air west of the tongue, a continuation of the falling area in the tongue, but a decrease in the sea-level rising area east of the tongue, for the reason that more air is now moving westwards from the upper-air area in the warm tongue. We cannot expect that the process will always follow the same scheme, each case must be studied individually. The warm tongue will at any rate have a tendency gradually to be longer and narrower and to disappear towards the end of the process (together with the dying cyclone).

The falling area at sea level in the warm tongue will try to build a cyclone there or to move an old cyclone in that direction. As we at the same time will have a rising area at sea level in the cold air east of the col, we may point to a practical rule given in a previous paper: A deepening cyclone will generally have a pronounced rising area east of the cyclone. This rule was developed at a time when no upper-air charts were available. By means of such charts we now have a better understanding of what takes place.

A cyclone may ultimately develop into a great central cyclone. In this case the warm, upper-air tongue may take the shape suggested in fig. 9 c; the warm air has a tendency to circle around a central cyclone. A small dying cyclone may, on the other hand, be carried away by the main upper-air stream and the thickness lines will then take the shape of a relatively narrow warm tongue above the occlusion. This warm tongue seems to disappear simultaneously with the disapperance of the rain area.

The building, development and death of a cyclone may require many days, and we can generally make a forecast for only a part of the process, at any given time. We may make use of the rising and falling of the upper-air pressure surfaces to get a qualitative view of the process and to forecast an increase, a decrease or no-change in the sea-level isallobaric areas. Since the method is based on the movement of the thickness lines, it is generally very difficult to make forecasts in this way for more than 12 hours at a time. This is easily understood from a look at fig. 7. North of a line Tromsø—Jan Mayen we have generally no observations and should be forced to draw conclusions from the extrapolated thickness lines in this region. We have similar difficulties in the Atlantic Ocean. In some cases it is, however, possible to suggest the pattern of development over periods even longer than one day, as we have experience with the sea-level charts to assist us.

Example

Conclusions, drawn from upper-air charts, will generally confirm what may be already expected from the sea-level chart together with an application of indirect aerology. We may sometimes find cases however, where the upper-air charts furnish us with new information not given by the sea level chart. An example from my own experience will be given below.

On the afternoon of the 7th Oct. 1948 the 13-hour sea-level chart had to be used instead of the 16-hour chart, because new ship observations were lacking from the Atlantic, where the development was of most importance. The sea-level chart, fig. 10 a, led to the following conclusions:

The small low near Iceland had not deepened much during its existence in the last 15 hours, and it was supposed to be of minor importance. The great low in the Mid-Atlantic had existed for several days. It had been moving slowly from the west, and its falling areas had apparently weakened. We then probably

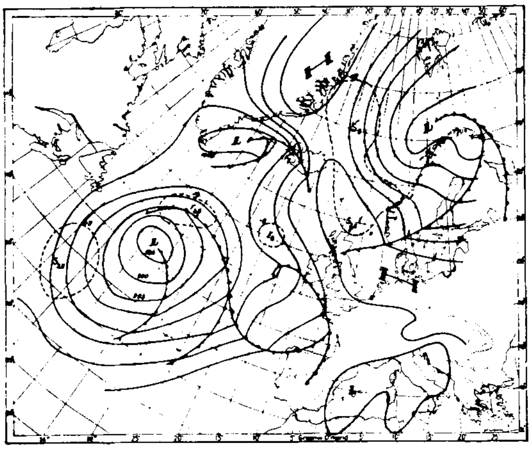

Fig. 10a. Sea-level chart of Oct. 7, 1948, 13h G.M.T.

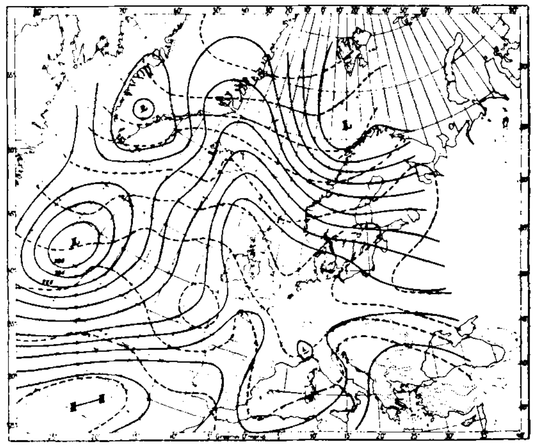

Fig. 10b. 700 mb chart of Oct. 7, 1948, 16h G.M.T.

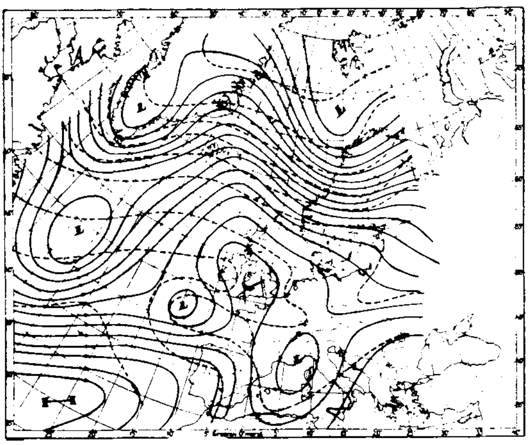

Fig. 10c. 500 mb chart of Oct. 7, 1948, 16h G.M.T.

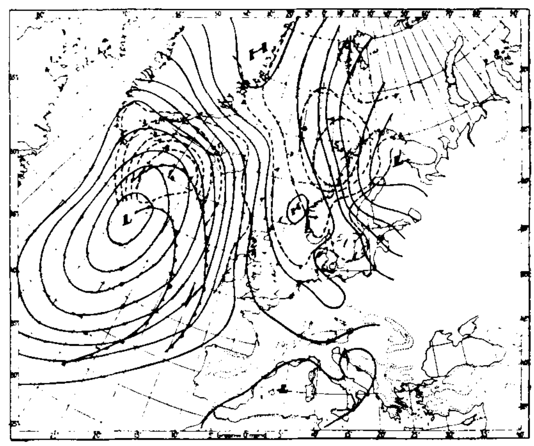

Fig. 10d. Sea-level chart of Oct. 8, 1948, 07h G.M.T.

would expect a slow-moving central cyclone, and the falling areas would therefore have a tendency to circle around the center and to gradually decrease in size and intensity. The rising area east of Iceland was suspiciously strong, but it was supposed to diminish as the central cyclone entered its dying phase.

The upper-air charts, fig. 10 b and 10 c, pointed to quite a different conclusions as to the cyclone in the Mid-Atlantic. Northwest, west and southwest of Iceland we had a good grid between contour lines and thickness lines, and because the falling areas at sea level were mostly small, we should expect a pronounced rising of the upper-air surface in this region, which would soon lead to an increase in the falling areas at sea level. The rising area from Jan Mayen to Scotland would probably not diminish, but move eastwards, because there was a distinct grid between contour lines and thickness lines over the Norwegian Sea and only little change in the sea-level pressure over northern Norway. — Fig. 10 d shows the weather situation at 07^h G. M. T. 8/10 1948. We see a strong falling area southwest of Iceland and a strong rising area near the Lofoten Isles.

It must be emphasized that clear-cut cases

— such as that shown in figs. 10 a to fig. 10 d — may be comparatively seldom seen, but in my own experience as a weather forecaster I have found the above-mentioned ideas to be of practical use. It is necessary however to learn their limitations and proper application through practice.

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