Variation in the Earth's Angular Velocity Resulting from Fluctuations in Atmospheric and Oceanic Circulation[1]

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Abstract

Fluctuations in the circulation of the atmosphere are associated with very small anomalies in the angular velocity of the earth. The seasonal component of these anomalies has been computed from weather maps, and is found to agree, with respect to magnitude and phase, with anomalies first reported by STOKYO in 1936 on the basis of astronomic observations. The effects of fluctuations in the oceanic circulation, and of shifting of air and water masses, have been estimated to account for not more than 15 per cent of the observed effect.

The detection of shorter period fluctuations is presently limited by the accuracy of astronomic time determination. The developments of an atomic clock and of the photographic zenith tube open the possibility of determining anomalies persisting for only two weeks, and of providing a new tool for obtaining an index of *integrated* atmospheric circulation.

Introduction

The problem is well stated by STARR (1948):

> "Although it is true that the combined system composed of the earth and the atmosphere cannot alter its total absolute angular momentum except for extremely slow secular changes resulting from tidal action as was pointed out by Darwin and others, still there is no reason to expect that the partition of angular momentum in the composite system should remain constant when seasonal and other short time-intervals are considered. Because of the great contrast in the moments of inertia of the two components, short-period anomalies of this kind represent major anomalies in the behavior of the wind system, but, on the other hand, imply practically undetectible inequalities in the rate of the earth's rotation."

WIDGER (1949) affirms that "there is little hope" of detecting these minute inequalities. Actually such inequalities have been reported, although not interpreted, by NICOLAS STOYKO, a French astronomer, in 1936, 1937, and 1949. Similar observations have been made by SCHEIBE and ADELSBERGER in Germany and by FINCH in England.

In a recent paper VAN DEN DUNGEN et al. (1949) have ascribed the fluctuations observed by STOYKO to seasonal shifts in air mass. Their computations (for the northern hemisphere only) account for 20 per cent of the observed effect, and the discrepancy is laid to uncer-

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tainties in the various assumptions. We have recomputed this effect, taking into account the corresponding shift of air mass in the southern hemisphere (which largely compensates for the shift in the northern hemisphere), the local pressure variations rather than those reduced to sea level, and the tendency of the ocean water to redistribute itself under the influence of changing atmospheric pressure. Each of these factors substantially reduces the effect, and the computed anomalies in the earth's rotation resulting from the shift in air masses amounts to less than 2 per cent of the observed effect. The fluctuations in the angular momentum of the atmospheric circulation is, however, of the right magnitude to account for the observed effect.

Conservation of angular momentum

Neglecting secular changes resulting from tidal friction,

$$I_e\Omega_e + I_a(\Omega_e + \omega_a) + I_o(\Omega_e + \omega_o) = \text{constant}, \quad (1)$$

where I_e, I_a and I_o are the moments of inertia of the solid earth, atmosphere and ocean, respectively, Ω_e the earth's absolute angular velocity, ω_a and ω_o the components along the earth's axis of the *relative* angular velocities of atmosphere and ocean. Differentiation gives

$$-\frac{\Delta\,\Omega_e}{\Omega_e} = \frac{\Delta\,(I_a\omega_a)}{I\,\Omega_e} + \frac{\Delta\,(I_o\omega_o)}{I\,\Omega_e} + \frac{\Delta\,I}{I}, \quad (2)$$

where $I = I_e + I_a + I_o$

Angular momentum of the atmospheric circulation

The relative angular momentum of the atmosphere between the levels h_1 and h_2, the latitudes φ_1 and φ_2, and the longitudes Θ_1 and Θ_2, is given by

$$I_a\,\omega_a = R^3 \int_{h_1}^{h_2}\int_{\varphi_1}^{\varphi_2}\int_{\Theta_1}^{\Theta_2} \varrho u \cos^2\varphi\, dh\, d\varphi\, d\Theta. \quad (3)$$

where R is the radius of the earth, ϱ the density and u the zonal wind. The largest portion of the angular momentum is contained in the "jet stream" and surrounding westerlies, which extend approximately from 25° to 60° latitudes. The values in Table 1 have been computed from weather maps by numerical integration along vertical sections from the surface to 17 km. For the northern hemisphere[3] sections along 100° E and 80° W longitudes were used. It is assumed that those sections are representative of conditions throughout 90° of longitude to either side. Only one section is available for the southern hemisphere (LOEWE and RADOK, 1950), but it must be realized that this section is far from typical for other longitudes. The easterlies have been taken into account in the northern hemisphere, but not in the southern hemisphere where the belt of easterlies is narrow and their effect comparatively small.

According to Table 1 the seasonal variation in the southern hemisphere amounts to less than one third the seasonal variation in the northern hemisphere. The reason for this is twofold: (*a*) the jet stream in the southern hemisphere during the southern winter is narrower than the northern jet stream in the northern winter; (*b*) the westerlies in the southern hemisphere during the southern summer are very much stronger than the westerlies in the northern hemisphere during the northern summer. The seasonal variation in the southern hemisphere may be even smaller than indicated in Table 1, since our calculations are based on geostrophic winds, and these are almost twice the observed zonal winds at Townsville, Brisbane and Sydney during the southern winter. No wind observations are available for the summer season.

Table 1. Relative angular momentum of the atmosphere in units of 10^{32} g cm² sec⁻¹. The northern hemisphere is divided in two parts, centering at stated longitudes.

	Northern Hemisphere			Southern Hemisphere
	100° E	80° W	Total	150° E
$I_a\,\omega_a$, January	6.8	5.9	12.7	6.9
$I_a\,\omega_a$, July	0.2	−0.1	0.1	10.7
$\Delta\,(I_a\,\omega_a)$	6.6	6.0	12.6	−3.8

Setting $\Delta\,(I_a\omega_a) = 9 \times 10^{32}$ g cm² sec⁻¹, $I \approx I_e = 0.8 \times 10^{45}$ g cm², $\Omega_e = 0.73 \times 10^{-4}$ sec⁻¹, gives a relative seasonal range of

[3] We are indebted to Dr YALE MINTZ of the Meteorology Department at the University of California in Los Angeles for making available the mean monthly cross section on which our calculations for the northern hemisphere are based.

$$\frac{\Delta(I_a\omega_a)}{I\Omega_e} = 1.6 \times 10^{-8},$$

or about one part in sixty million.

Angular momentum of oceanic circulation

To estimate fluctuations in the angular momentum of the ocean circulation we have computed numerically the angular momentum for two of the largest ocean current systems (SVERDRUP et al., 1946, pp. 614, 710):

Antarctic Circumpolar Current:
1.0×10^{32} g cm^2 sec^{-1}

Equatorial Easterly Currents, all oceans:
0.5×10^{32} g cm^2 sec^{-1}.

The angular momentum of the entire zonal circulation of the oceans is perhaps 3 to 4×10^{32} g cm^2 sec^{-1}, and the range in fluctuations probably not more than 20 per cent of this value, or

$$\frac{\Delta(I_o\omega_o)}{I\Omega_e} \approx 0.15 \times 10^{-8}.$$

It may seem surprising that the ocean circulation should contribute only one eighth of that of the atmosphere, especially since the mass of the atmosphere corresponds to only 10 meters of water compared to a mean ocean depth of 4,000 meters. However the mean zonal velocity of the ocean water is probably less than 2 cm/sec, whereas the mean zonal wind speed in the westerlies (weighted for density stratification) is 20 m/sec. Thus the ratio in masses of 400 : 1 is more than offset by a velocity ratio of 1 : 1,000. An additional factor is the larger *percentage fluctuations* of the atmospheric circulation about its mean value.

Moment of inertia

The third term on the right side of equation (2) includes changes in the moment of inertia of the atmosphere, the ocean and the earth. Changes in the ocean are due partly to the effect of wind stress (to be considered later), partly to the effect of atmospheric pressure on the sea surface in redistributing the waters of the ocean. We shall consider two extreme cases: the case of immovable water, as if the entire ocean were frozen, and the case for which complete equilibrium has been achieved. For seasonal variations equilibrium is probably nearly attained. Very short period fluctuations fall into the first category.

For the case of immovable ocean water we have estimated ΔI_a according to the equation

$$\Delta I_a = \frac{R^4}{g}\int_{-\pi}^{\pi}\int_{0}^{2\pi} \Delta p \cos^3\varphi \, d\varphi \, d\Theta, \qquad (4)$$

where Δ again represents January values minus July values. The mass of air above a unit area of the earth's surface was set equal to p/g, and p determined from normal monthly weather maps (U. S. Weather Bureau, 1946, and BRUNT, 1939, pp. 8—9). In equation (4) p is the *local* pressure, and not the corresponding value reduced to sea level, as it is given on climatic charts.[4] Accordingly it was necessary to remove the sea level correction from the p values on the monthly weather charts (JEFFREYS, 1929, pp. 239—244). Values of Δp for pressures reduced to sea level, and for local pressure variations, are shown in Table 2. The quantities $[\Delta p]$ in Table 2 represent averages around the stated latitude circles. In general the local pressure variations are smaller than corresponding sea level pressure variations, and in high latitude the two variations are opposite in sign. Consequently the seasonal changes in the moments of inertia are substantially reduced when allowance is made for the elevation of the meteorological stations. In the northern hemisphere the reduction is by a factor of 2.7, in the southern hemisphere by only 6 per cent. For both hemispheres $\Delta I_a = -0.83 \times 10^{36}$ g cm^2 compared to a value of *plus* 1.8×10^{36} g cm^2 obtained by VAN DEN DUNGEN et al. (1949).

The value of ΔI_a is even further reduced if the ocean water is permitted to move to attain equilibrium. We adapt a method given by JEFFREYS (1929). Let Δm designate the change (January minus July) per unit surface area of the *combined* mass of air and water. Conservation of mass requires that

$$\iint_E \Delta m \, ds = \iint_o \Delta m_o \, ds + \iint_L \Delta m_L \, ds = 0,$$

[4] The error resulting from the use of sea level pressure has already been pointed out by Angot. See Annuaire de la Société Météorologique de France, T. 35, 1887.

Table 2. Computation of ΔI_a (pressure in mb).

	Latitude	Sea level pressure		Local pressure		Local pressure	
		$[\Delta p]$	$\int_0^{2\pi} \Delta p \cos^3 \varphi \, d\Theta$	$[\Delta p]$	$\int_0^{2\pi} \Delta p \cos^3 \varphi \, d\Theta$	$\int_L \Delta p \cos^3 \varphi \, d\Theta$	$\int_L \Delta p \cos \varphi \, d\Theta$
Northern Hemisphere	80—85	1.1	0.0	— 0.7	0.0	0.0	— 1.7
	75—80	1.3	0.1	— 1.1	— 0.1	— 0.2	— 4.2
	70—75	0.7	0.1	— 1.7	— 0.3	— 0.3	— 3.3
	65—70	2.6	0.9	0.2	0.1	0.6	3.8
	60—65	3.4	2.1	— 2.2	— 1.4	— 0.3	— 1.5
	55—60	3.5	3.5	0.0	0.0	2.2	7.2
	50—55	4.1	5.9	— 2.2	— 3.2	0.9	2.5
	45—50	4.4	8.6	— 1.9	— 3.7	2.7	6.8
	40—45	4.9	12.3	— 0.5	— 1.3	4.4	8.0
	35—40	5.5	17.2	— 0.1	— 0.3	3.1	5.0
	30—35	6.3	23.8	0.1	0.2	— 0.4	— 0.6
	25—30	6.9	30.0	0.5	2.2	— 1.7	— 2.1
	20—25	6.4	31.7	5.7	28.3	18.7	22.0
	15—20	4.7	25.5	4.0	21.9	9.0	9.9
	10—15	2.3	13.4	2.2	12.6	3.7	3.8
	5—10	1.4	8.4	1.2	7.3	— 0.4	— 0.4
	0—5	1.4	8.5	1.4	8.7	1.1	1.1
	$\sum$		192.0		71.0	43.1	56.3
	ΔI_a in 10^{36} g cm^2		2.81		1.04	0.63	
Southern Hemisphere	0—10	— 2.7	— 17.1	— 2.7	— 16.8	— 5.8	— 5.8
	10—20	— 4.4	— 25.0	— 4.1	— 23.1	— 7.4	— 7.9
	20—30	— 5.4	— 24.6	— 4.8	— 22.1	— 6.0	— 7.4
	30—40	— 1.6	— 5.5	— 1.6	— 5.4	— 0.4	— 0.6
	40—50	2.0	4.5	2.0	4.5	0.1	0.1
	50—60	1.3	1.5	1.3	1.5	0.0	0.0
	60—70	— 4.0	— 1.9	— 4.0	— 1.9	0.0	0.0
	70—80	— 2.1	— 0.2	— 2.1	— 0.2	— 0.1	— 2.7
	$\sum$		— 68.3		— 63.5	— 19.6	— 24.3
	ΔI_a in 10^{36} g cm^2		— 2.00		— 1.87	— 0.57	
Both hemispheres ΔI_a			0.81		— 0.83	0.06 — 0.08 (equ. 6) — 0.02	

where the subscripts refer to integration over the entire surface of the earth, over the ocean areas, and over the land areas, respectively. The ocean water will move in such a manner that Δm_o has the same value over the entire ocean area. It follows that

$$\int\int_o \Delta m_o \, ds = \Delta m_o \int\int_o ds = \Delta m_o S_o$$

$$= -\int\int_L \Delta m_L \, ds = -\overline{\Delta m_L} \int\int_L ds = -\overline{\Delta m_L} \, S_L$$

or

$$\Delta m_o = -\frac{S_L}{S_o} \overline{\Delta m_L}, \tag{5}$$

where $\overline{\Delta m_L}$ represents a mean value of Δm_L over that portion of the earth's surface which is covered by land.

Changes in the moment of inertia are composed of those occurring over the ocean and those occurring over land:

$$\Delta I_a = R^2 \Delta m_o \iint_o \cos^2 \varphi \, ds +$$

$$+ R^2 \iint_L \Delta m_L \cos^2 \varphi \, ds =$$

$$= - R^2 \frac{S_L}{S_o} \frac{\overline{\Delta p_L}}{g} \iint_o \cos^2 \varphi \, ds +$$

$$+ \frac{R^4}{g} \int_{-\pi}^{\pi} \int_L \Delta p_L \cos^3 \varphi \, d\varphi \, d\Theta, \qquad (6)$$

where $\Delta p = g \Delta m$, and

$$\overline{\Delta p_L} = g \overline{\Delta m_L} = \frac{R^2}{S_L} \int_{-\pi}^{\pi} \int_L \Delta p_L \cos \varphi \, d\varphi \, d\Theta \qquad (7)$$

is the mean (local) pressure variation over all land. The computation is summarized in Table 2. For the earth, $S_L/S_o = 0.419$, $S_E = S_L + S_o = 5.10 \times 10^{18}$ cm^2, $\iint_o \cos^2 \varphi ds = 0.49 \, S_E$, $\overline{\Delta P_L} = 0.183$ mb $= 183$ dynes cm^{-2}, and the first term in (6) reduces to $- 4.32 \times 10^{32} \, \overline{\Delta P_L} = - 0.08 \times 10^{36}$ g cm^2, compared to $+ 0.06 \times 10^{36}$ g cm^2 for the second term. In view of the uncertainty of the calculations, this high degree of cancellation is largely fortuitous, and the final value of ΔI_a may be as large as 0.25×10^{36} g cm^2 and of either sign. We shall adopt this larger value. For comparison, the moment of inertia of a one meter snow cover, or 20 cm water cover, over the entire United States is also 0.25×10^{36} g cm^2. It would be desirable to recompute Table 2 using the original data for the uncorrected local pressure.

A larger effect is related to the shifting of ocean currents, and the distribution of mass associated with the currents, by the variable winds. This effect could best be computed from seasonal changes in the topography of the sea surface as obtained either from standard oceanographic observations of density versus depth, or from tide gauges. Unfortunately there are very few ocean areas from which repeated oceanographic observations have shed light on seasonal variations. Many tide gauge records show seasonal variations in sea level (Δz) of the order of 20 cm, but almost all these variations are associated with coastal currents and other local phenomena.

In order to obtain a rough estimate of the sea level effect on a world-wide basis we have noted a relationship (MUNK, 1950) between the mean annual sea level in the Atlantic from 50° N to 60° S, and the corresponding distribution of atmospheric pressure over the same area. Both the sea level and the pressure reach maxima in 30° N and 30° S and drop sharply towards higher latitudes. The "dynamic sea level" is therefore opposite in phase from the "hydrostatic sea level" previously considered. To obtain a numerical relationship we must first subtract the hydrostatic effect and write

$$\frac{\varrho_w \, g \, z_w + p_{\text{atm}}}{p_{\text{atm}}} = \lambda,$$

where ϱ_w is the density of water, z_w the deviation of sea level from its mean, and λ a proportionality factor which varies from 2.5 near the equator to about 7.5 at 60°, and can be represented by $\lambda = 2.5 + 5 \varphi^2$, where φ is the latitude in radians.

We now assume that this relationship $\lambda(\varphi)$ can be applied to seasonal variations over all ocean areas, and thus compute the dynamic sea level effect from the known seasonal pressure variations according to

$$\Delta I_o = \frac{R^4}{g} \int_{-\pi}^{\pi} \int_o \lambda \left(\Delta p_o - \overline{\Delta p_o} \right) \cos^3 \varphi \, d\varphi \, d\Theta, \qquad (8)$$

where $\overline{\Delta p_o} = - (S_L/S_o) \, \overline{\Delta p_L} = - 0.77$ mb is the mean pressure variation over all oceans. At any latitude

$$\int_o \lambda \Delta p_o \cos^3 \varphi \, d\Theta = \lambda \Big\{ \int_o^{2\pi} \Delta p \cos^3 \varphi \, d\Theta - \int_L \Delta p \cos^3 \varphi \, d\Theta \Big\},$$

and the first term in (8) can be obtained by subtracting Column 6 from Column 5 in Table 2, and multiplying by appropriate values of λ. The second term,

$$- \frac{R^4}{g} \overline{\Delta p_o} \iint_o \lambda(\varphi) \cos^3 \varphi \, d\varphi \, d\Theta$$

has been evaluated numerically and equals 1.3×10^{36} g cm^2 over the northern hemisphere and 1.8×10^{36} g cm^2 over the southern hemisphere. The results are

ΔI_o, northern hemisphere $\quad 1.4 \times 10^{36}$ g cm^2
ΔI_o, southern hemisphere $\quad -2.2 \times 10^{36}$ g cm^2
ΔI_o, both hemispheres $\quad -0.8 \times 10^{36}$ g cm^2.

The extrema in pressure changes and in the computed dynamic sea level changes are summarized in Table 3.

Table 3. Extrema in pressure changes ($\Delta p_o - \overline{\Delta p}$) and sea level changes Δz

Latitude	λ	$\Delta p - \overline{\Delta p_o}$ mb	Δz cm
67° N	9	+ 3	+ 27
57° N	7	− 5	− 35
20° N	3	+ 4	+ 12
20° S	3	− 3	− 9
50° S	6	+ 3	+ 18
70° S	10	− 3	− 30

The computed changes in sea level are of the same magnitude as those obtained from tide gauge records.

There remain the changes ΔI_e in the inertia of the solid earth and biosphere. To the first of these have been ascribed rather abrupt changes in $\Delta\Omega_e/\Omega_e$ as large as 4.0×10^{-8}, which appear to occur every two or three decades. Whatever the cause of these intermittent changes, it is unlikely that *seasonal* changes in the moment of inertia of the solid earth are of sufficient magnitude to compare with the effects discussed above[5]. The effects of vegetation, related to the rise of sap from the ground into the woody part of trees, of the production of leaves, and of other factors, have been estimated by JEFFREYS (1929) to have a seasonal range of 2 g cm^{-2} in the temperate zones, corresponding to a change in the moment of inertia by 0.15×10^{36} g cm^2 for the northern hemisphere. Compared to these values, any direct effect of human activity is negligible. If all American automobiles were driven from Alaska to Mexico, the change in the moment of inertia would amount to only 10^{31} g cm^2.

The combined changes of inertia of atmosphere, ocean and earth probably do not exceed 1.00×10^{36} g cm^2. This leads to a value of $\Delta I/I = 0.12 \times 10^{-8}$.

[5] Dr Mintz has pointed out the interesting possibility that seasonal variations in the solar tides may have an important effect.

Summary of seasonal fluctuations

The interrelationships between the various atmospheric and oceanic terms appearing on the right side of equation (2) is sketched in fig. 1. The magnitudes are as follows:

$\Delta(I_a\omega_a)/I\Omega_e \ldots\ldots\ldots\ldots + 1.6 \times 10^{-8}$
$\Delta(I_o\omega_o)/I\Omega_e \ldots\ldots\ldots\ldots 0.15 \times 10^{-8}$
(plus or minus)
$\Delta I/I \ldots\ldots\ldots\ldots\ldots\ldots 0.12 \times 10^{-8}$
(plus or minus)

The term on the left side of equation (2), $-\Delta\Omega_e/\Omega_e$, has been determined by STOYKO (1936, 1937, 1949) and FINCH (1950) from astronomic observations:

STOYKO
1934—1935 ...... 2.93×10^{-8}
1935—1936 ...... 0.76×10^{-8}
1936—1937 ...... 1.59×10^{-8}
1946—1947 ...... 2.40×10^{-8}

FINCH
1943—1947 ...... 2.0×10^{-8}
1948—1949 ...... 2.3×10^{-8}

Here $\Delta\Omega_e$ represents January rotation minus the preceeding July rotation.

The observations for 1934—1935 can be compared to those by SCHEIBE and ADELSBERGER (1936). These give 1.4×10^{-8} or

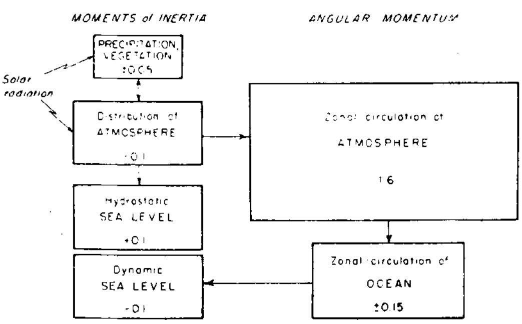

Fig. 1. Schematic presentation of the relationship between factors influencing the earth's angular velocity. The numbers give an indication of the seasonal effect of each factor, expressed in parts per hundred million of the mean rate of rotation, or roughly in milliseconds per day. (+) represents excess of earth's rotation in July over that in January. (±) represents magnitude without indication of the direction of the effect. Sea level changes are composed of those induced by the hydrostatic pressure changes, and those related to the oceanic circulation, which in turn is caused by the winds.

2.5×10^{-8}, depending on whether the difference January minus July or January minus August are formed. For the preceeding year the seasonal variation appears to be small and obscured by the scatter of data.

The meteorologic and astronomic observations agree with regard to both magnitude and sign. The astronomic observations reveal a difference in rotation between March and August which is, on the average, 80 per cent larger than the difference between January and July. The meteorologic implication of this observation is not at all in disagreement with synoptic experience.

Accuracy of determining earth's angular velocity

The detection of short period anomalies in the atmospheric circulation is likely to be of greater practical interest in meteorology than the seasonal fluctuations. Whether the earth's rotation can be determined with sufficient accuracy to make this possible depends largely on two factors: the accuracy of time determination from the zenith passage of stars, and the precision of the clocks with which the astronomic time is to be compared.

By using a "bank" of *quartz crystal clocks*, and "beating" the clocks against one another, it has been possible to achieve a relative accuracy of one part in a hundred million. The term relative is used, because it is found that the frequency characteristics of the crystals undergo sudden small changes from time to time, and the clocks must be compared to some independent time standard, i.e., the mean annual rate of the earth's rotation. In 1898, and again in 1920, the mean annual rate of rotation changed rather abruptly, and during those years a determination of the seasonal components of the earth's angular velocity would have led to erroneous results.

The development of *atomic* clocks promises to provide an absolute time standard independent of the earth's rotation. In the case of the *ammonia* clock now being developed by LYONS (1949) a constancy of one part in 20 million has been achieved, and an accuracy of one part in 100 million is hoped for. However, in another development carried on by the Bureau of Standards with the assistance of Professor P. KUSCH of Columbia University, quantum transitions in beams of atoms such as *cesium* will be used to establish frequency and time standards. The potential accuracy is increased (Bureau of Standards, 1949) "by a factor of 10 to 100 or more", and "an ultimate accuracy of 1 part in 10 billion may be reached."

A quite different limitation is imposed by the accuracy to which the "earth clock" and the atomic clock can be *read* in order to measure the cumulative difference between the rates of the two clocks. In practice only the accuracy in reading the earth clock need be considered. The present accuracy of astronomic time determination is about 0.02 seconds. According to the Astronomer Royal (SPENCER JONES, 1945) the proposed installation at the Royal Observatory of 18 quartz crystal standard clocks in conjunction with the photographic zenith tube will reduce the probable error in time determination to 0.002 or 0.003 seconds. Dr. SOLLENBERGER at the U. S. Naval Observatory estimates (personal communication) that the error can ultimately be reduced to 0.001 or 0.002 seconds.

We may interpret these values in terms of the probable accuracy to which the earth's angular velocity can be determined. Suppose we wish to measure the mean angular velocity of the earth during a period of N days, starting with $t = -{}^1/_2 N$ and ending with $t = +{}^1/_2 N$. Once each day we determine the difference x between the astronomic time y and the clock time z. The slope m of the line $x = f(t)$ equals

$$m = \frac{\Delta \Omega_e}{\Omega_e} = \frac{\sum_{t=-N/2}^{N/2} x_i t_i}{\sum_{t=-N/2}^{N/2} t_i^2}$$

by the methods of least squares, with the understanding that the mean value of x has been subtracted from the individual values.

The probable error m' of m equals the geometric sum of the probable errors of each term in the series:

$$m'^2 = \frac{\sum_{t=-N/2}^{N/2} (x_i' t_i)^2}{\left(\sum_{t=-N/2}^{N/2} t_i^2\right)^2}. \tag{9}$$

The probable error x_i' of $x_i = y_i - z_i$ equals the geometric sum of the probable errors of y_i and z_i:

$$x_i'^2 = y_i'^2 + z_i'^2.$$

Here y_i' is the error in astronomic time determination, which remains constant throughout the time interval N. The probable clock error may be set as $z_i' = \omega' t_i$, where ω' is the constant probable error in the *rate* of the clock relative to its mean rate. Substituting

$$x_i'^2 = y'^2 + \omega'^2 t_i^2$$

into Equation (9) leads to

$$2\,m'^2 = \frac{y'^2}{F} + \frac{f\omega'^2}{F^2}, \qquad (10)$$

where

$$F(N) = 1^2 + 2^2 + \dots (N/2)^2 = \frac{N^3}{24} + \frac{N^2}{8} + \frac{N}{12},$$

$$f(N) = 1^4 + 2^4 + \dots (N/2)^4 = \frac{N^5}{160} + \frac{N^4}{32} + \frac{N^3}{24} - \frac{N}{60}.$$

Both N and y' are given in days.

The uncertainties in the astronomic time determination and in the rate of the clock contribute equally to the uncertainty in the determination of the earth's rotation if

$$\frac{y'^2}{\omega'^2} = \frac{f}{F} \approx \frac{3\,N^2}{20}.$$

The present accuracy of $y' = 0.02$ seconds $\approx 2 \times 10^{-7}$ days, $\omega' = 10^{-8}$, gives $N \approx 52$ days. For periods much longer than 2 months the error is due chiefly to the error in the rate of the clock. For much shorter periods the error is due chiefly to the error in astronomic time determination, and equals

$$m' = \frac{y'}{\sqrt{2\,F}} \frac{\sqrt{12}\,y'}{N^{3/2}},$$

where the approximation holds for all but very small values of N. The chief limitation at present is imposed by the astronomic time determination rather than by the clock.

Conclusions

In fig. 2 the probable error in the determination of the anomalies in the earth's rotation has been plotted against the length of time over which the average has been taken. The lines correspond to the four possible combinations between astronomic time determination (p.e. of present method 0.02 sec, of photographic zenith telescope 0.002 sec) and standard time determination (p.e. of crystal clocks $1:10^8$, of cesium clock $1:10^9$). According to fig. 2 the probable error of monthly averages now is 5×10^{-9}. The probable error of Stoyko's measurements, judged externally from the consistency of his observations, is somewhat smaller, and our values for the accuracy of astronomic time determination and the zenith tube can be regarded as conservative.

The shaded band in fig. 2 is an estimate of the desirable accuracy. For example, the measurement of seasonal variations of two

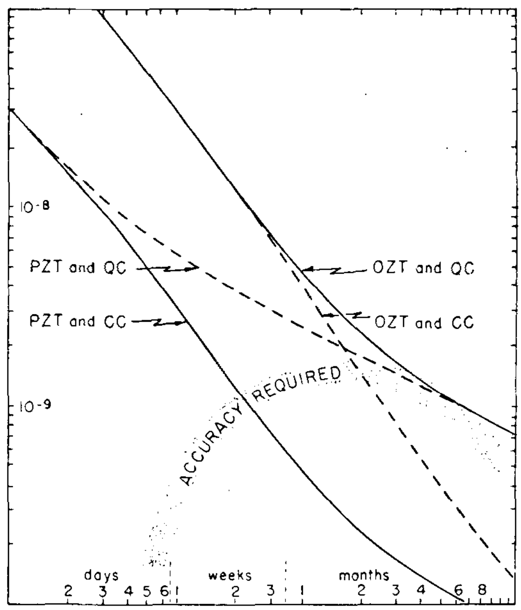

Fig. 2. Probable fractional errors in determining the earth's angular velocity as a function of the length of time over which the measurement of the velocity are averaged. The four lines correspond to the possible combinations of the ordinary zenith tube now used (OZT, $\pm$ .02 sec.), and the proposed photographic zenith telescope (PZT, $\pm$ .002 sec.), with the quartz clock (QC, $\pm 10^{-8}$), and the proposed cesium clock (CC, $\pm 10^{-9}$). The shaded band is an estimate of the largest probable error permissible for determining fluctuations in the earth's rotation of given duration.

parts per hundred million requires monthly (or at least quarterly) values accurate to, say, two parts per billion. The present accuracy is therefore too small for a reliable measurement of these variations, but not too small for their detection.

In order to study fluctuations of annual means insofar as they are related to meteorological conditions it will be necessary to improve the accuracy of the clock, and furthermore to refer to an absolute time standard. Both requirements are fulfilled by the cesium clock.

Anomalies persisting for short periods of time will be relatively small, and the required accuracy correspondingly high. Since the "half life" of atmospheric circulation is only a week we may expect anomalies persisting for a week to be as large as perhaps 25 per cent of the monthly anomalies. The required accuracy can then be set at one part in two billion. According to fig. 2 the development of the photographic zenith tube and of the cesium clock should enable us to measure anomalies persisting for two weeks or more. By relaxing our requirements and assumptions these values can be improved by a factor of two. Any further improvement will depend largely upon our ability to correct the astronomic time determination during zenith passage for the refraction of light from a distant star by the atmosphere directly over the telescope. Thus the ultimate limitation on measuring atmospheric circulation by this method is imposed by the atmosphere itself. It is hoped that the great interest attached to an index of *integrated* circulation as derived from anomalies in the earth's rotation might provide an additional incentive to improve present methods of time determination.

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