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SHORT CONTRIBUTION

On the determination of the predictability of a four-level geostrophic model using the effects of grid point numerical errors

By P. J. EVERSON and J. C. PERFECT, *University of Exeter, Department of Mathematics, North Park Road, Exeter, EX4 4QE, England*

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1. Introduction

Numerical models of the atmospheric general circulation can be improved by increasing the resolution of the horizontal and vertical finite difference grid. This sensitivity is not confined solely to the resolution of the grid; the model can be shown to be highly sensitive to the precision of the arithmetic used in the computations. This is demonstrated here by writing the FORTRAN forecast code in double and single precision, with all real-valued constants used in the program suitably defined. Searle & Davies (1975b) described the divergence of the eddy kinetic energy, K', computed from double and single precision forecasts respectively, after 1500 timesteps of $\frac{1}{2}$ hour duration. In the present work, these experiments are extended and an attempt will be made to measure the difference between the two forecasts by the definition of a suitable statistic to quantify the analysis.

The model used in these experiments is described in Searle & Davies (1975a). Following Phillips (1956) all values were calculated as a zonal average and a departure from this average value. Liebmann relaxation techniques were used to calculate the stream function values from the predicted vorticity at each timestep. All the integrations described here were made on the IBM 360/195 at Rutherford High Energy Laboratory. On this machine, single precision arithmetic, using a 24-bit mantissa, is correct to approximately seven significant figures, and double precision arithmetic with a 56-bit mantissa is equivalent to fourteen significant figures.

2. The preliminary forecast

The eddy forecast was initialized from the zonal mean stream function data of the preliminary forecast. At this time, equator to pole temperature contrasts of 54 °C at 750 mb, 44 °C at 500 mb and 16 °C at 250 mb were noted. The computed eddy kinetic energy functions, K', shown in Fig. 1, grew from an initial zero value to 600 KJm^{-2} after 20 days using single precision arithmetic, whereas 21 days of integration were required in double precision to attain the same level. For the first 17 days of the forecast, the K' values differed by less than 50 KJm^{-2}, but after 20 days of integration, the K' curves diverged considerably, and by day 33 of the forecast, the functions were completely out of phase, exhibiting a phase difference of about 11 days until the end of the forecast. The integrations were begun with a timestep of 1 hour, which was automatically reduced to $\frac{1}{2}$ hour at day 20 by the demands of computational stability; thus the forecasts were out of phase after 1100 timesteps, equivalent to 33 days of model time.

The remaining computed energy functions, i.e. zonal mean kinetic energy, zonal mean and eddy available potential energy, and the total energy, which are not illustrated here, displayed very similar characteristics to the K' function. It is interesting to note that between 15 and 20 days of the integration, the total energy of the single precision forecast was about 10% greater than that computed from the double precision version. The major contribution to the total energy is from the zonal mean available potential energy, and this difference in the total energy was reflected in the P

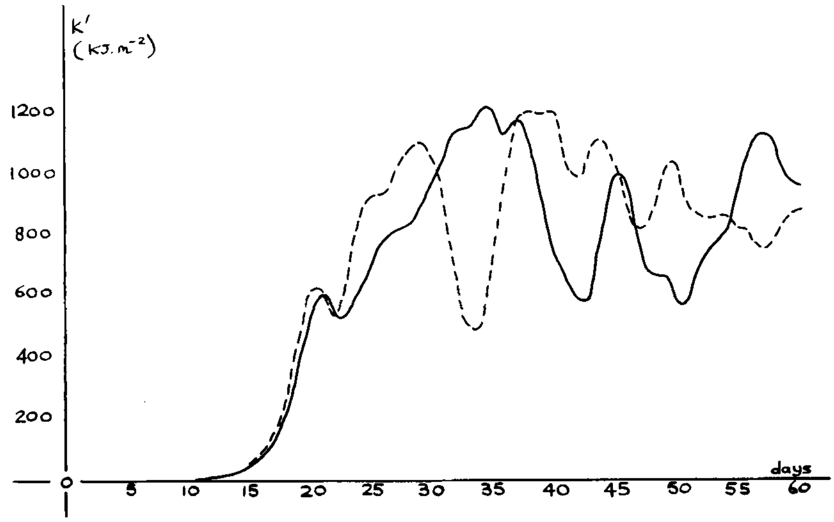

Fig. 1. Eddy kinetic energy function K' for double and single precision computations on the IBM 360/195 computer, in units of KJm^{-2}. —— double precision, ---- single precision.

values. From the energy budget of the model, the increased $\bar{P}$ value reflected a greater transfer of non-adiabatic heat energy to potential energy in the single precision version of the model.

The energy functions can only reflect the general characteristics of the flow throughout the whole domain of the model. To consider the deviation of the two modes of arithmetic in more detail, differences between the synoptic charts of the two forecasts were computed. The actual variations between the two integrations over the first 25 days of the forecast were found to be remarkably small at both 1000 mb and 500 mb levels. The greatest variations were always less than $2\frac{1}{2}\%$ of the largest absolute value of the double precision isobaric field at the appropriate level, and occurred in mid-latitudes where synoptic-scale cyclonic and anti-cyclonic disturbances develop throughout the forecast.

Since the initial grid-point numerical error, as measured by the eddy kinetic energy function, increased at approximately the same rate as the K' function itself, it was impossible to draw any conclusions on the precise nature of this error growth. An important point to investigate is the dependence of these numerical errors upon the number of timesteps performed in the integrations, or whether the only dependence is upon the real-time simulated by the model. A further series of experiments was then designed to investigate this point, avoiding the initialization period of the eddy forecast, which is about 40 days.

The eddy forecast was restarted from zonally averaged and perturbation stream function data at day 49 of a run which was initialized from day 0 in double precision. This day corresponded to a maximum of eddy kinetic energy, and was chosen as the starting point because it was thought that the four-level model used here would have similar properties to the two-level model used by Everson & Davies (1971). They showed that the two-level, quasi-geostrophic model was less predictable when temperature perturbations were introduced at a maximum value of K' than when the same perturbations were introduced at a minimum of this function. In this series of experiments, a measure of the root mean square errors was the statistic used to quantify the error-growth analysis, and used to estimate the predictability of the four-level model.

3. The predictability of the four-level model

Many attempts have been made in recent years to evaluate the limit of deterministic predictability of the atmosphere, i.e. what is the limit beyond which we cannot hope to forecast the global atmospheric state using general circulation models? The particular model equations were integrated for a

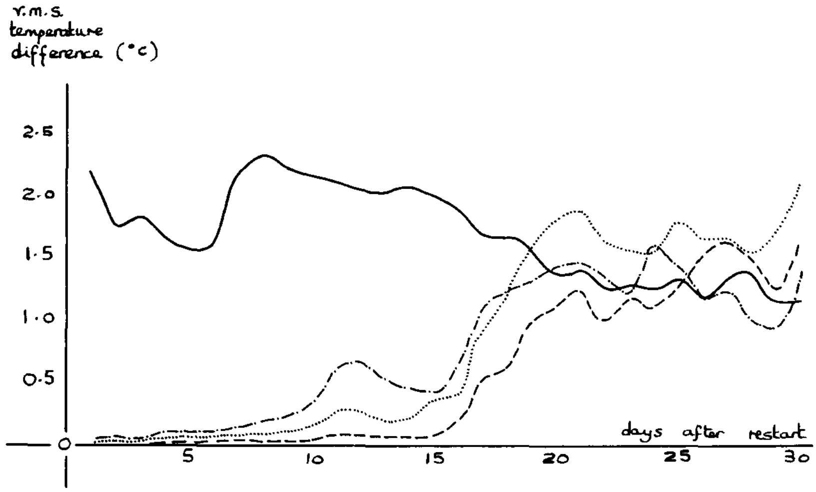

Fig. 2. Predictability analysis for restarted forecasts, compared with the control experiment DC, in deg. C. —— Persistence level calculated from control, ---- experiment D1, -·-·- experiment S2, ······ experiment S3.

long period until a "steady" state was achieved, in the sense that the initial conditions no longer had an influence on the flow characteristics; this is termed the control integration. Small perturbations were then introduced to selected flow variables about 30 days before the end of the controlled experiment, and the integration was continued for a period of 30 days; this is the perturbed integration. The root mean square (r.m.s.) temperature difference between the perturbed and controlled integrations is the usual statistic calculated in order to obtain a measure of the deviation between the two forecasts. This statistic can be used to measure the doubling time of the initial perturbation errors, but on its own cannot determine the absolute limit of predictability. However, defining persistence as the r.m.s. temperature difference between two random atmospheres, in this case two random model atmospheres, then the generally accepted limit of predictability is said to have been attained when the r.m.s. temperature error of the perturbed integration compared with the control reaches the persistence value.

Studies made with the nine-level GFDL general circulation model showed that the practical limit of predictability was attained after 14 days' integration by analysis of the observed flow patterns, although the deterministic limit, measured by the r.m.s. temperature error had not been reached even after 21 days' integration; Smagorinsky et al. (1970). The global, two-level UCLA model showed that the persistence level was attained after 25 days' integration, when an initial r.m.s. perturbation of 0.5 °C was applied to the temperature field (Charney, 1965). Everson & Davies (1971) showed that the limit of deterministic predictability for a two-level, quasi-geostrophic model to be 34 days when a r.m.s. perturbation of 1 °C was introduced at a minimum of eddy kinetic energy, but only 24 days when the same perturbations were introduced at a maximum of K'.

In the evaluation of the limit of deterministic predictability of the four-level model used here, the perturbations introduced to the initial temperature fields were caused only by the different round-off errors characteristic of each mode of arithmetic. A double precision integration with a timestep of $\frac{1}{2}$ hour was considered to be the control experiment (DC), and three perturbed integrations were made; double precision with $\frac{1}{4}$ hour timestep (D1), and two single precision integrations with $\frac{1}{2}$ hour (S2) and $\frac{1}{4}$ hour (S3) timesteps respectively. The use of the different timesteps was designed to determine whether the limit of predictability was dependent upon the real-time simulated by the model, or whether predictability was only a function of the number of timesteps used in the integration. In this experiment, predictability was evaluated by comparing the zonal mean temperature fields at three levels in the model from each of the experiments D1, S2 and S3 with that of the control. Persistence was calculated using zonal mean temperature fields from the control integration at day 45 and day 85. The results obtained are shown in Fig. 2.

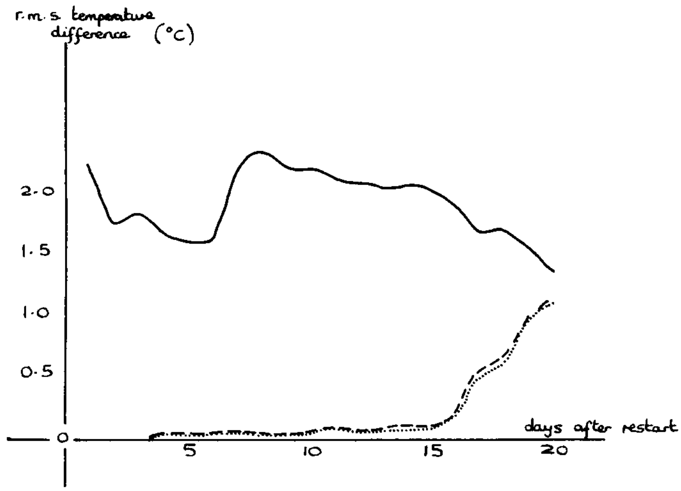

Fig. 3. Predictability analysis for restarted forecasts compared with the control experiment DC with 30-minute timestep. —— Persistence level calculated from DC with 30-minute timestep, ---- integration using 15-minute timestep, · · · · · · integration using 3-minute timestep. Units are deg. C.

An overall persistence level of 1.25 °C was considered to be reasonable for this model, when using zonally averaged temperature data. The r.m.s. temperature deviations for experiments D1 and S3 varied quasi-periodically with slowly increasing mean value for 15 days. After this time, both experiments exhibited a non-linear increase; a doubling time of about 1 day was noted for experiment S3, which reached the persistence level at day 18. Experiment D1 showed a doubling time of approximately 1.5 days, and attained the persistence level after 20 days' integration. Experiment S2 also attained the persistence level at day 18, but the r.m.s. temperature differences in this case had a quasi-periodic variation of much larger amplitude. As in the case of the other single precision integration, the doubling time of the errors at day 15 was approximately 1 day.

4. The effect of a very short timestep

A final experiment was made using the stream function data at day 49; in this integration, double precision arithmetic was used with a timestep of 3 minutes. The computed eddy kinetic energy function for the 20-day period after the restart was seen to resemble more closely the K' function computed from the double precision integration with the 15-minute timestep than the control. Fig. 3 compares the r.m.s temperature errors for both the short timestep double precision forecasts, and it can be seen that there is very little difference between the forecast made with a 3-minute timestep and the 15-minute timestep integration.

5. Conclusions

Williamson & Washington (1973) made a similar series of experiments with the NCAR global circulation model, to compare the effects of using 48-bit mantissa arithmetic with 24-bit mantissa arithmetic. They noted some differences in the results of the experiments but concluded that the errors due to the lower precision were much smaller than typical observational errors. However, the experiments described here have clearly demonstrated the sensitivity of the four-level model to grid-point numerical errors encountered in the numerical integration. These results show the importance of using one mode of arithmetic in parameterization tests for numerical models, so that the effect of the particular parameter values only may be assessed. The limit of deterministic predictability, as defined in the text, for the four-level model was found to be approximately 20 days, and this limit was clearly shown to be independent of the number of timesteps used in the integration.

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