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On a differentially heated saltwater-ice system

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ABSTRACT

A previous theoretical study by Welander (1977) on a freshwater-ice oscillator is extended to the saltwater case. The system comprises a deep high-salinity, warm water reservoir (fixed temperature and salinity), and a shallow constant-depth mixed layer with temperature, salinity, and ice thickness varying in time. This layer is cooled by contact with overlaying air kept at a fixed, subzero temperature. The main new feature is the variation of the heat flux through the water interface with the stratification. This variation causes two new phenomena of interest: (a) "false" thermal adjustments to existing ice-covered steady states, and (b) multi-periodic self-sustained oscillations including both ice-covered and ice-free regimes. The usefulness of basic model studies of this type in connection with climate research is pointed out.

1. Introduction

In an earlier paper by one of the authors (Welander, 1977), a theoretical study is made of a system comprising a well-mixed body of freshwater heated internally at a constant rate, while cooled from the top by contact with air, or other gas, kept at fixed, subzero temperature. Two steady state solutions may be found, in a certain range of parameters, one with ice present and one with no ice present. In the former case the water is warmer than in the latter, which is explained by the insulation effect of the ice. One may choose the parameters such that the water is too warm in the ice-covered case to sustain ice, too cold in the ice-free case to stay open. No steady state is then physically possible, and the system must change in time forever. It is predicted that this change takes the form of a self-sustained, single-periodic oscillation, with ice forming and melting in an infinite succession.

The phenomena was eventually demonstrated in a laboratory experiment set up in the Department of Oceanography (Welander, 1977).

It seems of interest to extend this study to the case where the water contains salt. One motivation is the possible application of such a model to the Arctic Sea, which is a natural saltwater-ice system with an internal heat source: warm, high-salinity Atlantic water is advected in under the mixed layer, and at least part of this heat is transferred vertically to the overlaying atmosphere. Another motivation is the interest in the behavior of simple thermal oscillators: adding the salt, the system has a new degree of freedom and new phenomena may result.

To make the model as simple as possible, a two-layer approximation is used. Heat and salt are provided from a lower reservoir, characterized by a mixed temperature T^* and S^*. Above this is a well-mixed layer of constant depth D, with temperature $T(t)$ and salinity $S(t)$. The mixed layer may be covered by ice, with a thickness $\delta(t)$. Cooling is provided by contact with an upper well-mixed layer of air, or other gas, kept at a fixed, subzero temperature T_0. A schematic picture of the system is given in Fig. 1.

As in the fresh water case the heat transfer between water and ice, water and air, and ice and air, will be assumed proportional to the temperature difference, with constant exchange coefficients. Modelling the transfer between the lower reservoir and the mixed layer, it seems, however, necessary to consider the effect of the variable stratification. When the density difference is large, and with the stable sign, as may occur in the ice-melting regime, the fluxes of heat and salt from below are suppressed. In the case where the density difference is low, as may occur in the freezing regime, when the mixed layer salinity is increased, a larger transfer is expected. In the case where the sign of the difference gets reversed (static instability) the effect will, of course, be dramatic with sudden increases in both the fluxes of heat and salt. As will be seen, the saltwater model actually reduces to the freshwater previously investigated in the case where the thermal exchange coefficient between the water layer is a constant. Thus, what new phenomena will be found will hang critically on the stratification control of the heat flux. As in the freshwater case, the ice is assumed to be thin (temperature adjustment in the ice fast compared to the overall changes of the system), and only its latent heat transformation is included in the heat budget. Further, the effect of salinity on the freezing point is neglected, as well as the effect of temperature on the density. The effects of some of these approximations are discussed in the appendix.

Section 2 gives the basic equations in dimensional and non-dimensional forms, Section 3 discusses the steady state solutions, and the stability of these solutions. In Section 4, some numerical examples are given for the non linear regime.

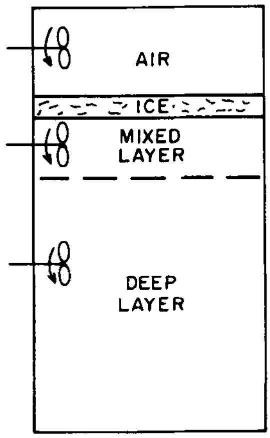

Fig. 1. Schematic picture of the system. The air layer has a fixed, sub-zero temperature, the deep layer fixed temperature and salinity. All layers are assumed well mixed.

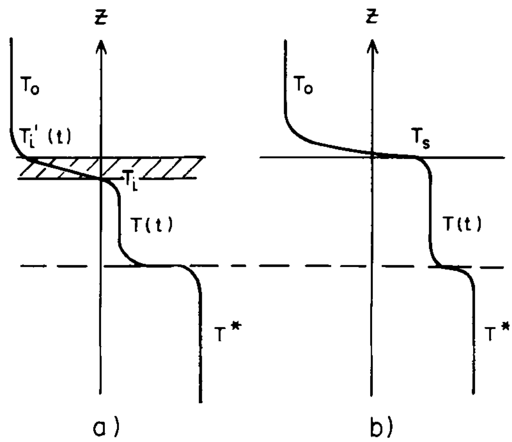

Fig. 2. Schematic picture of the vertical temperature and salinity profiles through the system. The variations are concentrated in thin boundary layers, the temperature is assumed to vary linearly through the ice. Case (a) with ice, case (b) without ice.

2. Model equations

Consistent with the assumptions described in the previous section we can record the following equations for the temperature T, salinity S, and ice thickness δ (see Figs. 1, 2a, 2b):

$$D\rho c\dot{T} = \begin{cases} k_T(T^* - T) - k(T - T_i), \\ \quad \delta > 0, \text{ or } \delta = 0, T < 0 & \text{(1a)} \\ k_T(T^* - T) - k_a(T - T_0), \\ \quad \delta = 0, T \geq 0 & \text{(1b)} \end{cases}$$

$$D\rho\dot{S} = k_S(S^* - S) + \rho_i S\dot{\delta} \qquad (2)$$

$$\rho_i L\dot{\delta} = \begin{cases} k_i(T_i' - T_0) - k(T - T_i), \\ \quad \delta > 0, \text{ or } \delta = 0, T < 0 & \text{(3a)} \\ 0, \\ \quad \delta = 0, T \geq 0 & \text{(3b)} \end{cases}$$

Here, D is the depth of the upper water layer, assumed to be constant in the thin-ice approximation, ρ and ρ_i are the densities of water and ice, c and L the constants of specific heat and latent heat per unit mass for water, T_i' and T_i, the temperatures at the upper and lower boundaries of the ice, respectively. The exchange coefficients k, k_a, k_i are constants, while k_T, k_S depend on the salinity difference:

$$k_T = C_T f(\xi), \quad k_S = C_S f(\xi), \quad \xi = \frac{S - S^*}{S^*} \qquad (4)$$

Here, C_T, C_S are constants, and the function $F(\xi)$ is assumed to be normalized, $f(0) = 1$. Generally it

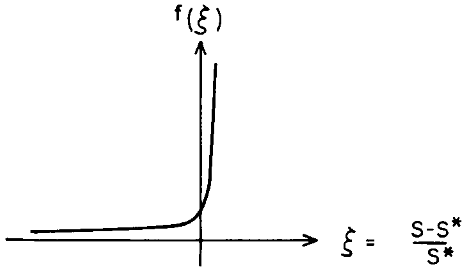

Fig. 3. Schematic picture of the variation of the exchange coefficient, characterizing the heat and salt transfer between the deep and mixed layer, as a function of the stratification. The temperature effect in the stratification is neglected in the present case.

has to be determined experimentally. The function should increase with ξ, and show a rapid increase as ξ passes zero (Fig. 3). If molecular effects are unimportant, the same function should be applicable to both heat and salt transfer.

To close the problem, the temperatures T_i and T_i' must be calculated. The temperature T_i is the melting temperature of ice, which is assumed constant. For convenience we set $T_i = 0$. The temperature T_i' is found from equating the heat flux through the ice and from the ice to the air: $k_i(T_i' - T_0) = \kappa_i(T_i - T_i')/\delta$, assuming a linear temperature profile in the ice according to the thin-ice approximation. This gives (with $T_i = 0$):

$$T_i' = \frac{k_i T_0}{k_i + \kappa_i/\delta} \tag{5}$$

Starting from given initial values, say T^0, S^0, δ^0, at $t = 0$, the equations determine the time developments of T, S, δ. The equations are nonlinear, since k_T, k_S depend on the salinity, and since the equations change form from the ice regime to the no-ice regime. This change in regimes works as follows: If $\delta > 0$, the "ice equations" (1a, 2, 3a) are used. If the value $\delta = 0$ is hit (which only can occur when $T > 0$), a switch is made to the "no-ice equations" (1b, 2, 3b). However, if later $T < 0$, it is assumed that an infinitesimal ice sheet is instantaneously formed on the surface, and the switch is made back to the "ice-equations". In the real world, an instantaneous ice formation does not occur, but hopefully the calculation will be valid in cases where the time for ice formation is very short compared to the other characteristic times of the system. It should also be pointed out that the use of the temperature T to decide on the onset of freezing involves an approximation. The proper parameter should be the water surface temperature T_S (Fig. 2b). Because the large temperature change occurs in the air boundary layer, in the air rather than in the water, T_S can normally be replaced by T.

A convenient non-dimensional form for the system is found by applying the transformation (the same notation is used for the new variables):

$$T \to T^* \cdot T, \quad S \to S^* \cdot S, \quad \delta \to \frac{\rho}{\rho_i} D \cdot \delta, \quad t \to \tau \cdot t,$$

where $\tau = D\rho L / T^* k$ is a characteristic time related to the ice growth. With k_t, k_s given by (4) with $T_i = 0$ and T_i' given by (5), the equations become

$$\dot{T} = \begin{cases} Cf(S-1)\cdot(1-T) - KT, & \\ \quad \delta > 0, \text{ or } \delta = 0, T < 0 & \text{(6a)} \\ Cf(S-1)\cdot(1-T) - K_a(T - T_0), & \\ \quad \delta = 0, T \geq 0 & \text{(6b)} \end{cases}$$

$$\dot{S} = Cf(S-1)(1-S) + S\dot{\delta} \tag{7}$$

$$\dot{\delta} = \begin{cases} \dfrac{a}{\delta + b} - T, & \delta > 0, \delta = 0, T < 0 \quad \text{(8a)} \\ = 0, & \delta = 0, T \geq 0 \quad \text{(8b)} \end{cases}$$

where

$$C = \frac{L}{T^* k}, \quad K = \frac{k}{D\rho c} \cdot \tau, \quad K_a = \frac{k_a}{D\rho c} \cdot \tau, \quad a = \frac{\kappa_i(-T_0) T^* \tau \rho_i}{D^2 \rho^2 L}, \quad b = \frac{\kappa_i \rho_i}{k_i \rho D}.$$

Neglecting molecular effects, the same exchange constant C should apply in the equations for temperature and salinity changes.

It should be noted that when $f(\xi)$ is constant ($f = 1$) the governing equations for temperature and ice-thickness are independent of the salinity. Their forms are the same as in the freshwater case with an internal heat source studied earlier by Welander (1977).

3. Steady states

The non-dimensional steady state equations, obtained by setting $\dot{T} = \dot{S} = \dot{\delta} = 0$, are, for the cases of ice and no-ice:

$Cf(S-1)\cdot(1-S)=0, \quad Cf(S-1)\cdot(1-T)-KT=0, \quad a/\delta+b-T=0 \quad \delta>0$

$Cf(S-1)\cdot(1-T)-K_a(T-T_o)=0,$
$Cf(S-1)\cdot(1-S)=0 \quad \delta=0$

Since $f\neq 0$, the steady state values are (note $f(0)=1$):

$$\bar{T}=\frac{C}{K+C}, \quad \bar{S}=1,$$

$$\bar{\delta}=\frac{a}{\bar{T}}-b=a\left(\frac{K}{C}+1\right)-b$$

$$\delta>0, \text{ or } \delta=0, \bar{T}<0 \tag{9a}$$

$$\bar{T}=\frac{C+K_aT_0}{C+K_a}, \quad \bar{S}=1 \quad \delta=0, \bar{T}\geq 0 \tag{9b}$$

To be consistent, the solution (9a) must give $\bar{\delta}>0$, and the solution (9b) must give $\bar{T}>0$. As in the freshwater case there may exist two steady states, one with ice and one without ice. The condition for this to occur is $a(K/C+1)>b$, and $C>K_a(-T_0)$. One or both of these steady states may obviously be non-existing, in the latter case the system must change forever. This case required that $aK/b-a<C<K_a(-T_0)$, which can be satisfied.

To find the stability of the steady state for infinitesimal disturbances we set $T=\bar{T}+\theta$, $S=1+\xi$, $\delta=\bar{\delta}+\eta$, and look at the linearized equations for θ, ξ, η. Noting that $f=1+\xi\cdot f'(0)$ and that $a/(\bar{\delta}+b)^2=\bar{T}^2/a$ the perturbation equations for the ice-covered case become

$$\dot{\theta}=-(K+C)\theta+Cf'(0)(1-\bar{T})\xi$$

$$\dot{\xi}=-C\xi+\dot{\eta}$$

$$\dot{\eta}=-\bar{T}^2/a\cdot\eta-\theta$$

With solutions proportional to $e^{\sigma t}$ the characteristic equation for σ is

$$\sigma^3+\left[2C+K+\frac{\bar{T}^2}{a}\right]\sigma^2+\left[(2C+K)\frac{\bar{T}^2}{a}+C(C+K)+Cf'(0)(1-\bar{T})\right]\sigma+C(C+K)\frac{\bar{T}^2}{a}=0 \tag{10}$$

A third order equation $\sigma^3+a_0\sigma^2+a_1\sigma+a_2=0$ has negative real parts for all roots if $a_0, a_1, a_2>0$, and $a_0a_1>a_2$ (Wylie, 1966). With all constants C, K, a positive, $f'(0)>0$, $\bar{T}<1$, these conditions are found to be satisfied. Thus, the ice-covered steady state is always stable.

In the ice-free steady state the corresponding perturbation equations are $\dot{\theta}=-K_a\theta$, $\dot{\xi}=-C\xi$, which obviously give stability, with an exponential decay. However, if we consider a perturbation in the form of an added infinitesimal ice sheet, an instability may result. If $\bar{T}<a/b$, equation 8(a) predicts a finite growth rate of ice. Ultimately, the ice may melt, and the ice-free steady state reached (such an example is shown by Welander (1977)), but formally this is a case of linear instability.

While the adjustment to a steady ice-covered state is exponential-like in the freshwater case, the adjustments to the same state in the salt water case may be oscillatory. This oscillation, physically, is a consequence of the coupling of freezing-melting, via the salinity change in the mixed layer, to the exchange with the deep layer. If the ice grows by cooling from the top, the salinity in the mixed layer increases, thereby increasing the heat flux from the deep layer. Because of the lag between heat flux and mixed layer temperature, there is an overshooting in the ice before the raised temperature starts melting. Similar arguments give a delayed restoring force at the other end of the oscillation.

The oscillatory solution occurs when the σ-equation has a pair of complex roots. Using previous notation, this condition is $4a_0^3a_2-a_0^2a_1^2-18a_0a_1a_2+4a_1^3+27a_2^2>0$. When $f'(0)=0$, corresponding to the freshwater case, this expression is always negative. On the other hand, for sufficiently large positive $f'(0)$ the expression must be positive, and oscillations will occur.

4. Nonlinear time developments

For the nonlinear case a solution in closed form has not been found. In analogy with the freshwater case, which is described by two equations for the form $\dot{T}=F(T,\delta)$, $\dot{\delta}=G(T,\delta)$, one may try to understand the solution from looking at the direction field of the trajectories in the T–S–δ–space. However, it turns out difficult to judge three-dimensional trajectory fields. Existence or non-

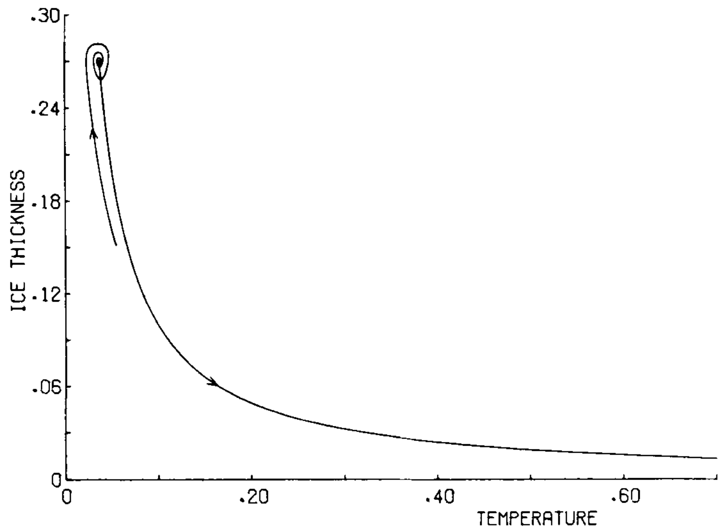

Fig. 4. A numerically calculated example of time-development of the system in the T-δ-plane, showing an adjustment towards a "false steady state" (the spiral part). The true steady state is at $T = 0.91$, $\delta = 0.01$. Parameter values: $C = 1, K_a = 50, K = 0.1, a = 0.01, b = 0.01, f(\xi) = \exp 200\ \xi$.

existence of limit cycles cannot be proved easily.[1] Usually one must try to judge the nature of the solutions from looking at a number of numerically calculated cases.

Some special examples have been calculated, integrating the system of differential equations (6, 7, 8) step-by-step forward in time from given initial conditions by a fifth order Runge–Kutta scheme using the CDC-6400 computer at the Unviersity of Washington. The calculations were carried out in cases where steady state solutions exist, as well as in the cases without any steady states. Comparisons were made with freshwater cases, obtained by setting $S^* = 0$, or by assuming K_T constant.

At this state, it has not been possible to explore the solution as dependent on the model parameters in detail. In the numerical calculations some interesting phenomena have, however, been seen, and these deserve a description. The first phenomenon is the "false adjustment", seen when the upper layer initially has a low salinity, and is ice-covered. The upper layer then adjusts relatively fast toward a nearly steady, cold state. Since the

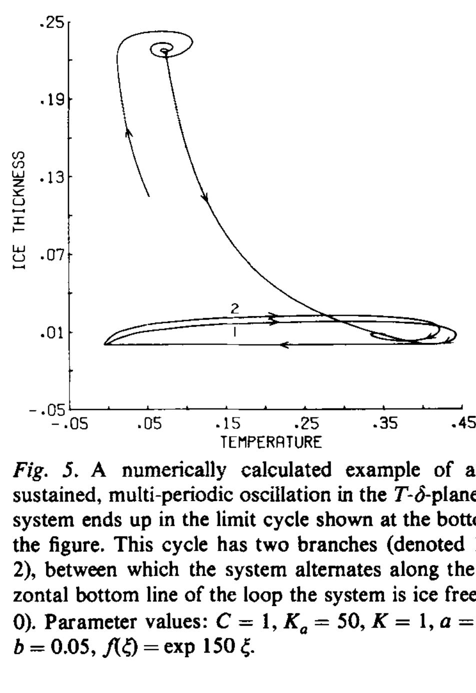

Fig. 5. A numerically calculated example of a self-sustained, multi-periodic oscillation in the T-δ-plane. The system ends up in the limit cycle shown at the bottom of the figure. This cycle has two branches (denoted 1 and 2), between which the system alternates along the horizontal bottom line of the loop the system is ice free ($\delta = 0$). Parameter values: $C = 1$, $K_a = 50$, $K = 1$, $a = 0.02$, $b = 0.05$, $f(\xi) = \exp 150\ \xi$.

stratification is large, only a small amount of heat and salt is added from below. However, gradually the salinity in the upper layer increases, and with the decreased stratification the flux of heat and salt into the upper layer accelerates. The strong non-

[1] For a trajectory field in a plane, described by a second order system of equations, a limit cycle must exist in the absence of stable steady states, if the variables remain bounded. If all steady states are stable, no limit cycle can exist. Such theorems do not carry over to the trajectory field in three dimensions, described by a third order system of equations.

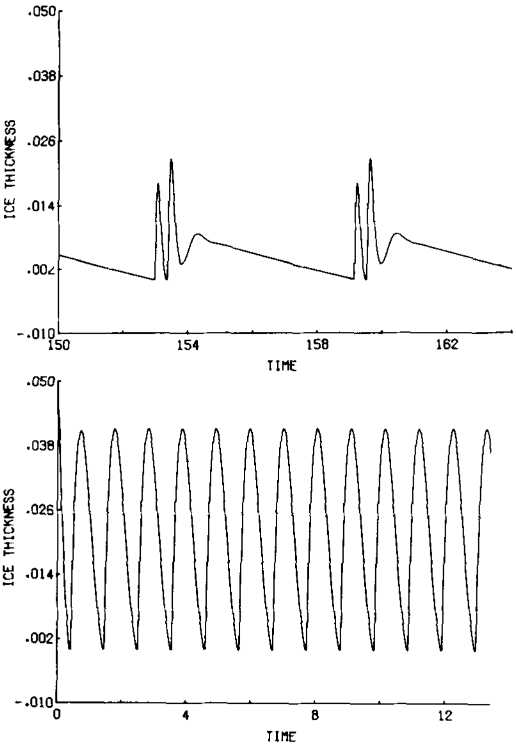

Fig. 6. Time plot of the ice thickness for the limit shown in Fig. 5 (case a). The case (b) shows the corresponding ice thickness for the freshwater system. Parameter values for the saltwater system are the same as in Fig. 5, the freshwater case is obtained by setting $f(\xi) \equiv 1$.

linearity actually causes the system to flip over suddenly from the "false" to the "true" adjustment. An example is given in Fig. 4.

A second phenomena of interest is the multi-periodic, self-sustained oscillation in the case where no steady state exists. An example is given in Fig. 5. A time plot of one of the variables (ice-thickness) is given in Fig. 6a; this can be contrasted with the regular oscillations seen in the corresponding freshwater case (Fig. 6b).

5. Possible applications

The possibility of directly applying this model to natural systems such as the Arctic Sea seems remote. In the real system the radiation heat flux at the surface, the existence of leads in the ice, the change in the mixed layer salinity by evaporation-precipitation, river supply and the inflow through the Bering Strait, are factors which probably must be included in a quantitative theory.[1]

The eventual usefulness of this and similar model studies rather lies in the possibility of detecting and exploring phenomena of new general types, which may occur in coupled air–water–ice systems. There is a definite need for developing a basic geophysical fluid dynamics for such systems. Learning about the nature of the simple systems, we will obtain a background knowledge helping us understand better the results of the more complex numerical models of the global atmosphere-ocean-ice system which are now being tried out, and, ultimately, also understand the real system better.

6. Appendix

On the effect of certain model approximations. The following specific approximations have been considered:

(a) Replacing the variable depth of the upper water layer by a constant value D.
(b) Neglecting the temperature effect on the stratification.
(c) Neglecting the salinity effect on the freezing point.

(a) In this case, the depth D in the equations (1, 2) is replaced by $D - \delta\rho_i/\rho$, the true water depth after correcting for the ice cover. Since D only appears as a factor before $\dot{T}$ and $\dot{S}$ in these equations, it is immediately clear that the steady state values are unchanged. The perturbation equations and the σ-equation (10) determining the linear stability for an ice-covered steady state have unchanged forms, with slightly changed coefficient values; the stability is clearly not effected.

The "thin-ice approximation" also involves the assumption that the conductive heat flux through the ice is determined by the temperatures T_i, T_i' at its two boundaries (linear temperature profile).

[1] Still it may be of interest to consider some consequences of the present simple mode for parameter values representative for the real Arctic Sea. With an assumed vertical heat flux added from below of the order of 10^{21} cal/year over an Arctic Sea are of 10^7 km^2, at present stratification and mean ice-thickness, one finds that the characteristic time for an eventual self-sustained oscillation associated with the ice insulation effect, roughly estimated, is of the order of a few decades to one century. This shows that the oscillation period is short relative to periods characterizing the major climate changes.

When δ becomes comparable to D this cannot generally hold true. Since the transient heat flux through the ice is determined by the heat conduction equation, a second order partial differential equation, the problem now becomes much more complex; no attempt is made to discuss it here.

(b) We write $\rho = \rho(S, T)$. The coefficients K_T, K_S now depend on $S - S^*$, and $T - T^*$ in the non-dimensional equations (6) and (7). $Cf(S - 1)$ is replaced by $Cg(S - 1, T - 1)$. The non-dimensional perturbation equations for θ, ξ, η given previously are changed to

$$\dot\theta = -\left[K + C - C\frac{\partial g}{\partial T}(1 - \bar T)\right]\theta + C\frac{\partial g}{\partial S}(1 - \bar T)\xi$$

$$\dot\xi = -C\xi + \dot\eta$$

$$\dot\eta = -\frac{\bar T^2}{a}\cdot\eta - \theta$$

The equation (11) for σ is replaced by

$$\sigma^3 + \left[2C + K + \frac{\bar T^2}{a} - C\frac{\partial g}{\partial T}(1 - \bar T)\right]\sigma^2 + \left\{\left[2C + K - C\frac{\partial g}{\partial T}(1 - \bar T)\right]\frac{\bar T^2}{a} + C\left[C + K - C\frac{\partial g}{\partial T}(1 - \bar T)\right] + C\frac{\partial g}{\partial S}(1 - \bar T)\right\}\cdot\sigma + C\left[C + K - C\frac{\partial g}{\partial T}(1 - \bar T)\right]\frac{\bar T^2}{a} = 0$$

Here $\partial g/\partial T$, $\partial g/\partial S$ are evaluated at $S = 1$, $T = \bar T$. Since $\partial g/\partial T < 0$ (positive thermal expansion coefficient), all coefficients in the σ-equation are positive. It can also be shown that the second condition for stability ($a_1 a_0 > a_2$ for an equation $\sigma^3 + a_0\sigma^2 + a_1\sigma + a_2 = 0$) is satisfied.

(c) In this case we set $T_i = T_i(S)$ rather than $T_i = 0$. The non-dimensionalized equations become:

$$\dot T = Cf'(S - 1)\cdot(1 - T) - k(T - T_i)$$
$$\dot S = Cf'(S - 1)\cdot(1 - S) + S\dot\delta$$

$$\dot\delta = \frac{a'[T_i - T_0]}{\delta + b} - (T - T_i) \text{ where } a' = \frac{a}{(-T_0)}.$$

The steady state conditions are now

$$\bar T = \frac{C + K\bar T_i}{C + K}$$

$$\bar S = 1$$

$$\bar\delta + b = \frac{a'[\bar T_i - T_0]}{(\bar T - \bar T_i)} = \frac{a'[\bar T_i - T_0](C + K)}{C(1 - \bar T_i)} > 0$$

$\bar\delta + b > 0$ because if $\bar T_i$ were less than T_0, then the ice would melt unless $\bar T < \bar T_i$ also.

The new perturbation equations for θ, ξ, η become

$$\dot\theta = -(K + C)\theta + \left[Cf'(0)(1 - \bar T) + K\frac{dT_i}{dS}\right]\xi$$

$$\dot\xi = -C\xi + \dot\eta$$

$$\dot\eta = -\left[\frac{\bar T - \bar T_i}{\bar\delta + b}\right]\cdot\eta - \theta + T_i'\left[\frac{a'}{\bar\delta + b} + 1\right]\xi$$

where $T_i' = \dfrac{dT_i}{dS}$.

The σ-equation takes the form

$$\sigma^3 + \left[2C + K + \left(\frac{\bar T - T_i}{\bar\delta + b}\right) - T_i'\left(\frac{a'}{\bar\delta + b} + 1\right)\right]\sigma^2 + \left[\left(\frac{\bar T - \bar T_i}{\bar\delta + b}\right)\cdot(2C + K) + C(C + K) + Cf'(1 - \bar T) - T_i'(K + C)\left(\frac{a'}{\bar\delta + b} + 1\right)\right]\sigma + C(C + K)\left(\frac{\bar T - \bar T_i}{\bar\delta + b}\right) = 0$$

Here T_l, T'_l are evaluated at $S = 1$. Since T_l and T'_l are both negative and

$$(\bar{T} - \bar{T}_l) = \frac{C(1 - \bar{T}_l)}{C + K} > 0,$$

all the coefficients in the σ-equation are postive. It can also be shown that the condition $a_0 a_1 > a_2$ is satisfied. Thus, we have stability.

7. Acknowledgements

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REFERENCES

Welander, P. 1977 Thermal oscillations in a fluid heated from below and cooled to freezing from above. *Dynamics of Atmosphere and Oceans 1*, 215–223.

Wylie, C. R., Jr. 1966 *Advanced Engineering Mathematics*. McGraw Hill, New York.

О ДИФФЕРЕНЦИАЛЬНО ПОДОГРЕВАЕМОЙ СИСТЕМЕ СОЛЁНАЯ ВОДА-ЛЁД

Предыдущее теоретическое исследование Веландера (1977) колебательной системы пресная вода—лёд расширено на случай солёной воды. Система состоит из резервуара тёплой и сильно солёной воды (с фиксированными температурой и солёностью) и мелкого перемешанного слоя постоянной глубины с температурой, солёностью и слоем льда, меняющимися со временем. Этот слой охлаждается из-за контакта с вышележащим слоем воздуха, поддерживаемым при фиксированной температуре ниже нуля. Главной новой особенностью является изменение со стратификацией потока тепла через поверхность раздела воды. Это изменение вызывает два новых интересных явления: а) "фальшивое" термическое приспособление к существующим стационарным состояниям с ледовым покровом, и б) многопериодные самоподдерживающиеся колебания, включающие режимы как с ледовым покровом, так и свободные ото льда. Отмечается полезность модельных исследований этого рода в связи с изучением климата.