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SHORT CONTRIBUTION

Comments on an assumption made in "A problem in the parameterization of large-scale eddies"

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Upon re-reading a previous contribution [Srivatsangam (1976), hereafter referred to as SR], the author realized that an inadvertent assumption had been made by him in that paper. The assumption concerns the calculations presented in Table 2 of that paper.

In SR, the zonal variance of the height of an isobaric surface[1]

$$[z^{*2}] = \sum_{n=1}^{\infty} \tfrac{1}{2} C_n^2 \tag{1}$$

was parameterized by

$$[z^{*2}] = \tfrac{1}{2} \tilde{C}_n^2 \tag{2}$$

Therefore

$$\tilde{C}_n = (\Sigma\, C_n^2)^{1/2} \tag{3}$$

(Hereafter we shall not indicate the summation interval, which should, for the geostrophic, hydrostatic eddies being considered here, include only small zonal wave numbers.)

Equation (2) leads to

$$\{\partial [z^{*2}]/a\,\partial\phi\}^2 = 2[z^{*2}](\partial \tilde{C}_n/a\,\partial\phi)^2 \tag{4}$$

which is eq. (14) in SR.

We substituted $(\partial\tilde{C}_n/a\partial\phi)^2$ for $\Sigma(\partial C_n/a\partial\phi)^2$ in eq. (9a) of SR. The question, then, is

$$(\partial\tilde{C}_n/a\partial\phi)^2 \stackrel{?}{=} \Sigma(\partial C_n/a\partial\phi)^2 \tag{5}$$

In eq. (5) and what follows we place a question mark on unproved equalities.

Let us consider a situation in which no wave numbers over 2 are present. Therefore, $n = 1$ or 2.

Expanding both sides of eq. (5) we obtain

$$(C_1^2 + C_2^2)^{-1} \left\{ C_1^2 \left(\frac{\partial C_1}{a\,\partial\phi}\right)^2 + C_2^2 \left(\frac{\partial C_2}{a\partial\phi}\right)^2 + 2C_1C_2 \frac{\partial C_1}{a\partial\phi}\frac{\partial C_2}{a\partial\phi} \right\} \stackrel{?}{=} \left(\frac{\partial C_1}{a\partial\phi}\right)^2 + \left(\frac{\partial C_2}{a\,\partial\phi}\right)^2 \tag{6}$$

Now

$$(C_1^2 + C_2^2)\left\{\left(\frac{\partial C_1}{a\partial\phi}\right)^2 + \left(\frac{\partial C_2}{a\partial\phi}\right)^2\right\} = C_1^2\left(\frac{\partial C_1}{a\partial\phi}\right)^2 + C_2^2\left(\frac{\partial C_2}{a\partial\phi}\right)^2 + C_1^2\left(\frac{\partial C_2}{a\partial\phi}\right)^2 + C_2^2\left(\frac{\partial C_1}{a\partial\phi}\right)^2 \tag{7}$$

If either side of eq. (7) were substituted within the braces of eq. (6), we obtain an identity, and the assumption of eq. (6) would be valid. Therefore, for eq. (6) to be valid

$$2C_1C_2 \frac{\partial C_1}{a\partial\phi}\frac{\partial C_2}{a\partial\phi} \stackrel{?}{=} C_1^2\left(\frac{\partial C_2}{a\partial\phi}\right)^2 + C_2^2\left(\frac{\partial C_1}{a\,\partial\phi}\right)^2 \tag{8}$$

[1] For a list of symbols, see SR.

should be valid. [Cf. the terms within braces of eq. (6) with the righthand side (r.h.s.) of eq. (7).]

If eq. (8) were correct, eq. (5) will be correct for the case $n = 1, 2$. For the general case, it can be shown that eq. (5) is valid if

$$\left.\begin{aligned}&\sum_n \sum_m C_n C_m \left(\frac{\partial C_n}{a\partial\phi}\right)\left(\frac{\partial C_m}{a\partial\phi}\right)\\ &\quad \overset{?}{=} \sum_n \sum_m C_n^2 \left(\frac{\partial C_m}{a\partial\phi}\right)^2, \quad n \neq m\end{aligned}\right\} \tag{9}$$

Similarly, the substitution of $(\partial\tilde{C}_n/\partial \ln p)^2$ for $\Sigma(\partial C_n/\partial \ln p)^2$ in SR depends on whether

$$\left.\begin{aligned}&\sum_n \sum_m C_n C_m \left(\frac{\partial C_n}{\partial \ln p}\right)\left(\frac{\partial C_m}{\partial \ln p}\right)\\ &\quad \overset{?}{=} \sum_n \sum_m C_n^2 \left(\frac{\partial C_m}{\partial \ln p}\right)^2, \quad n \neq m\end{aligned}\right\} \tag{10}$$

is valid.

It may be readily seen that for any specific values of m and n, eqs. (9) and (10) assume the form of eq. (8).

Now let us replace C_1, C_2, $\partial C_1/a\partial\phi$ and $\partial C_2/a\partial\phi$ in eq. (8) by k, l, m and n, which are four real numbers. We then obtain

$$(kl - mn)^2 \geqslant 0$$

from which,

$$2klmn \leqslant k^2l^2 + m^2n^2 \tag{11}$$

Therefore,

$$2C_1C_2 \frac{\partial C_1}{a\partial\phi} \frac{\partial C_2}{a\partial\phi} \leqslant C_1^2 \left(\frac{\partial C_2}{a\partial\phi}\right)^2 + C_2^2 \left(\frac{\partial C_1}{a\partial\phi}\right)^2 \tag{12}$$

The inequality sign in eq. (12) is likely to prevail, with the consequent usual invalidation of eqs. (8), (9) and (10).

We have used the l.h.s. of eqs. (9) and (10) in SR. Therefore, we may expect the components of zonal variances of zonal velocity u and virtual temperature T *unavailable* for meridional transports to be underestimated by the parametric eqs. (16) and (17) in SR.

This is proved by the data presented in Tables 1 and 2.

It may be noted that although eqs. (16) and (17) in SR underestimate the components of $[u^{*2}]$ and $[T^{*2}]$ unavailable for meridional transport, eqs. (16) and (17) yield better estimates of these components than subtraction of eqs. (11) and (12) of SR from $[u^{*2}]$ and $[T^{*2}]$. Also, the assumption described in this note does not in any way affect the important conclusion made in SR, that the weighted root mean square value of the slopes of atmospheric eddies partitions $[u^{*2}]$ and $[T^{*2}]$ properly in terms of their availability for meridional transport. It invalidates only eqs. (16) and (17) of SR, which, being erroneous, do *not*

Table 1. *Ratio of the zonal variance of zonal wind (u) unavailable for meridional transport to the total zonal variance of u*

Data within parentheses derived through parametric eq. (16) of SR. These are weighted averages for 25 °N to 80° N. The other data are sums of the first 10 zonal harmonics, averaged over 25° N to 75° N

Pressure (mb)	January 1, 1970	January 2, 1970	January 3, 1970
100	0.51 (0.28)	0.53 (0.25)	0.48 (0.22)
200	0.38 (0.18)	0.47 (0.13)	0.40 (0.14)
300	0.39 (0.18)	0.49 (0.14)	0.47 (0.18)
500	0.41 (0.17)	0.45 (0.16)	0.47 (0.18)
700	0.36 (0.18)	0.39 (0.17)	0.40 (0.19)

Table 2. *Ratio of the zonal variance of virtual temperature (T) unavailable for meridional transport to the total zonal variance of T*

Data within parentheses derived through parametric eq. (17) of SR. These are weighted averages for 20° N to 85° N. The other data are sums of the first 10 zonal harmonics, averaged over 20° N to 80° N

Pressure (mb)	January 1, 1970	January 2, 1970	January 3, 1970
200	0.48 (0.12)	0.43 (0.16)	0.47 (0.12)
300	0.37 (0.34)	0.35 (0.32)	0.35 (0.23)
500	0.72 (0.54)	0.70 (0.48)	0.66 (0.44)

parameterize the eddies in terms of the r.m.s. slopes.

No way of overcoming the assumption related here is obvious at the time of writing.

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REFERENCE

Srivatsangam, S. 1976. A problem in the parameterization of large-scale eddies. *Tellus 28*, 193–196.