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A note on the downstream variation of the lower trade-wind circulation

By YOSHIMITSU OGURA *Laboratory for Atmospheric Research, University of Illinois, Urbana, Illinois 61801, U.S.A.*

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ABSTRACT

Mak's (1976) model for the macro-structure of the undisturbed lower trade-wind circulation is modified by taking into account the potential temperature distribution in the layer above the trade-wind inversion. The downstream variations of temperature, wind and the height of the inversion base predicted by the modified model are found to agree reasonably well with those for the BOMEX region during an undisturbed period.

1. Introduction

In a recent article, Mak (1976) presented an interesting introduction to a model study of the downstream variation of the lower trade-wind circulation in undisturbed weather conditions. He concluded his study by stating that "the theoretical expression [for the upward slope of the inversion base] is comparable to but overestimates the slope by a factor of two or three ... Thus, even guided by the basic reasoning of this analysis, it is still difficult to theoretically predict the downstream variation of the trade-wind layer with good accuracy."

The purpose of this note is to show that, with a slight modification, Mak's model gives better agreement with observation than Mak indicated. This modification is made by matching the solution of the potential temperature deviation to the prescribed potential temperature in the layer above the inversion. The result indicates that the height of the inversion base increases with the downstream distance in proportion to the rate of diabatic heating in the sub-inversion layer, and in inverse proportion to the horizontal wind speed and the potential temperature lapse rate in the layer above the inversion.

2. Layer-averaged model

The basic equations Mak employed to investigate the downstream variation of the two-dimensional trade-wind circulation are written in dimensional form as

$$-\frac{\partial \delta p}{\partial s}+\frac{\partial \tau}{\partial z}=0 \tag{1}$$

$$\frac{\partial \delta p}{\partial z}+g\delta\rho=0 \tag{2}$$

$$\frac{\delta\rho}{\rho_0}+\frac{\delta\theta}{\theta_0}=0 \tag{3}$$

$$\frac{\partial u}{\partial s}+\frac{\partial w}{\partial z}=0 \tag{4}$$

$$u\frac{\partial \delta\theta}{\partial s}=\frac{Q}{c_p} \tag{5}$$

$$u_i\frac{dh}{ds}-w_i=[\varepsilon-(\overline{w'\theta'})_i]/\Delta\theta \tag{6}$$

All notations and symbols are the same as those defined by Mak. In particular, ε denotes the rate of cooling by evaporation from liquid drops in the inversion layer, $\Delta\theta$ is the jump of θ across the inversion layer and the subscript i denotes the quantities at the inversion level. Following Mak, we consider the variables averaged over the entire depth of the sub-inversion layer denoted by the symbol [ ]. Assuming $[Q]$ is independent of s and eliminating

δp, $\delta\rho$ and w from eqs. (1)–(6), we derive the following three equations:

$$\frac{dh[u]}{ds} = W \tag{7}$$

$$\frac{g}{\theta_0}\frac{[Q]}{c_p}\frac{h}{[u]} = G \tag{8}$$

$$\frac{dh[\delta\theta]}{ds} = \frac{h[Q]}{[u]c_p} \tag{9}$$

where the following abbreviations have been used:

$$W = [\varepsilon - (\overline{w'\theta'})_i]/\Delta\theta \tag{10}$$

$$G = \frac{1}{\rho_0}\left(\frac{\partial\tau}{\partial z}\right)_i - \frac{1}{\rho_0}\left(\frac{\partial\tau}{\partial z}\right)_0 \tag{11}$$

Elimination of $[u]$ from eqs. (7) and (8) gives

$$\frac{d}{ds}\left(\frac{h^2}{G}\right) = \frac{\theta_0 c_p W}{g[Q]} \tag{12}$$

Mak assumed that both G and W are constants with respect to s and determined h, $[u]$ and $[\delta\theta]$ from eqs. (12), (7) and (9) respectively. However, his solutions were obtained without referring to the atmospheric conditions in the layer above the inversion. Because the atmospheric conditions in that layer are determined by physical processes which are not considered here, it would be reasonable to prescribe these conditions and match the solutions for the sub-inversion layer to the prescribed conditions at the top of the inversion. Furthermore, we can imagine intuitively that the rate of entrainment of air from the layer above the inversion into the sub-inversion layer will increase the inversion height depending upon the static stability in the layer above the inversion.

We shall first define the total potential temperature averaged over the entire sub-inversion layer by

$$[\theta] = [\theta]_* + [\delta\theta] \tag{13}$$

where at a certain upstream location, characterized by $h = h_*$, we have $[\delta\theta] = 0$, and $[\theta] = [\theta]_*$. Let us also denote the total potential temperature above

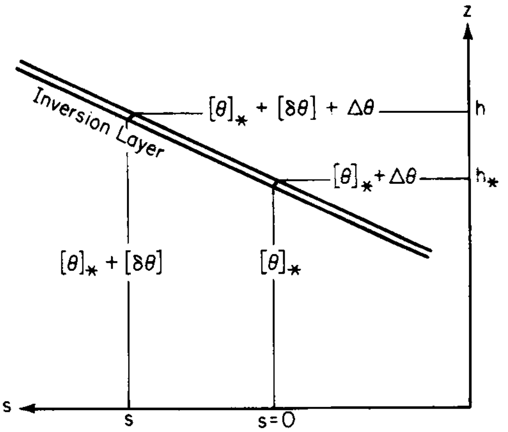

Fig. 1. Schematic distribution of potential temperature in the layers below and above the trade-wind inversion.

the inversion layer by $\theta_u(z)$, and specify it as (Fig. 1)

$$\theta_u(z) = [\theta]_* + \Delta\theta + \beta\,(z - h_*) \tag{14}$$

for $z \geq h \geq h_*$. We could use a more generalized form for $\theta_u(z)$ by including the s-variation of θ_u. However, the above expression gives a good first approximation to the observed field and eq. (14) is sufficient for our present discussion. It follows from eq. (14) that at the top of the inversion at any downstream location, we have

$$\theta_u(h) = [\theta] + \Delta\theta = [\theta]_* + [\delta\theta] + \Delta\theta \tag{15}$$

From eqs. (14) and (15), we obtain

$$[\delta\theta] = \beta\,(h - h_*) \tag{16}$$

We now have four equations [eqs. (7), (8), (9), and (16)] for five variables: h, $[u]$, $[\delta\theta]$, G, and W, provided that $[Q]$ is prescribed. Consequently, one of the five variables must be prescribed to determine the remaining four variables as a function of s.

First we assume that G is a constant with respect to s. Then, by eliminating $[\delta\theta]$ and $[u]$ from eqs. (8), (9), and (16), we can write down h as a function of s, as

$$h = \frac{h_*}{2}\left[1 + \left(1 + \frac{4\theta_0 G s}{g\beta h^2}\right)^{1/2}\right]$$

or, by making use of eq. (8), as

$$h = \frac{h_*}{2}\left[1 + \left(1 + \frac{4[Q]s}{c_p\beta h_*[u]_*}\right)^{1/2}\right] \tag{17}$$

The downstream variations of $|\delta\theta|$, $|u|$, and W are then determined from eqs. (16), (9) and (7) respectively.

Eq. (17) indicates that the characteristic horizontal length scale L in this model is given by $L = \beta h_*|u|_* c_p/4|Q|$. Mak estimated $|Q|/c_p = 0.9\ \mathrm{K\ day^{-1}}$ using the BOMEX data. A typical value of $|u|$ is $7\ \mathrm{m\ sec^{-1}}$ according to Fig. 2 of Holland & Rasmusson (1973), and Figure 4 of Augstein et al. (1973), and $h_* = 1.5$ km. We estimate $\beta = 4\ \mathrm{K\ km^{-1}}$ from Mak's Figure 4 which shows the vertical variations of potential temperature observed aboard the *Discoverer* and mt. *Mitchell* in the layer above the inversion during the BOMEX Phase III period. With these values, we obtain $L = 720$ km. For s much smaller than L, the rates of increase of h, $|\delta\theta|$ and $|u|$ are then found to be given by

$$\left(\frac{dh}{ds}\right)_* = \frac{|Q|}{c_p\beta|u|_*} \tag{18}$$

$$\left(\frac{d|\delta\theta|}{ds}\right)_* = \frac{|Q|}{c_p|u|_*} \tag{19}$$

$$\left(\frac{d|u|}{ds}\right)_* = \frac{|Q|}{c_p\beta h_*} \tag{20}$$

Secondly, we may consider a model where W is a constant but G varies with s. In this case, we derive the equation for h by eliminating $|u|$ and $|\delta\theta|$ from eqs. (7), (8), and (16) as

$$\frac{dh(h-h_*)}{ds} = \frac{|Q|h^2}{c_p\beta(Ws+|u|_*h_*)} \tag{21}$$

The solution of this non-linear equation is readily obtained as

$$2\ln\frac{h}{h_*} + \left(\frac{h_*}{h} - 1\right) = \frac{|Q|}{c_p\beta}\ln\left(1 + \frac{Ws}{|u|_*h_*}\right) \tag{22}$$

Mak estimated that $W \sim 0.7\ \mathrm{cm\ sec^{-1}}$. The characteristic horizontal length scale in the second model is then given by $L = |u|_*h_*/W \simeq 1500$ km. It is interesting to note that for $s \ll L$, dh/ds and $d|\delta\theta|/ds$ in the second model are identical with those in the first model. However, $d|u|/ds$ is different and is given by

$$\left(\frac{d|u|}{ds}\right)_* = \frac{1}{h_*}\left(W - \frac{|Q|}{c_p\beta}\right) \tag{23}$$

It should be also noted that G in the second model decreases in the downstream direction approximately 16% over a distance of 1000 km, while $|u|$ increases about 41% over the same distance. This downstream decrease of G has not been observed.

We may also look at our four basic equations from a different viewpoint. That is, $|u|$ is regarded as a prescribed function of s and the other four variables, h, $|\delta\theta|$, G, and W, are unknown functions to be determined by the model. In this case, we seek to determine the large-scale thermodynamic structure (h and $|\delta\theta|$) and the small scale processes (G and W) in response to the imposed large-scale subsidence. The equation for h is derived by eliminating $|\delta\theta|$ from eqs. (9) and (16), as

$$\frac{dh(h-h_*)}{ds} = \frac{h|Q|}{c_p\beta|u|} \tag{24}$$

The solution of this equation is

$$2(h-h_*) - h_*\ln\frac{h}{h_*} = \frac{|Q|}{c_p\beta}\int_0^s \frac{ds}{|u|} \tag{25}$$

It can be shown that, for small s, $|dh/ds|_*$ and $|d|\delta\theta|/ds|_*$ are again identical with eqs. (18) and (19) respectively. dG/ds can be either positive or negative, depending upon the relative magnitudes of $|Q|/c_p$ and $d|u|/ds$.

3. Comparison with observations and discussions

We shall now compare our prediction equations (18)–(20) with observations. Mak observed that the increases of the height of the inversion base in the downstream direction over a distance of 500 km over the BOMEX area is approximately 150 m. Mak's Fig. 3 shows that the corresponding increase of potential temperature is approximately 0.7 K when averaged over the layer from the surface to the height of 1.5 km. In other words, $dh/ds \sim 3\times10^{-4}$ and $d|\delta\theta|/ds \sim 1.4\times10^{-8}\ \mathrm{K\ cm^{-1}}$ over the BOMEX area. When we assign the values $|Q|/c_p = 0.9\ \mathrm{K\ day^{-1}}$, $|u|_* = 7\ \mathrm{m\ sec^{-1}}$, $h_* = 1.5$ km and $\beta = 4\ \mathrm{K\ km^{-1}}$ referred to earlier, eqs. (18) and (19) give us $(dh/ds)_* = 3.7\times10^{-4}$ and $(d|\delta\theta|/ds)_* = 1.5\times10^{-8}\ \mathrm{K\ cm^{-1}}$. Thus, the predicted values agree very well with observations.

Furthermore, eq. (20) gives $(d[u]/ds)_* = 1.7 \times 10^{-6}\ \mathrm{sec}^{-1}$. If $d[u]/ds$ in this two-dimensional model is interpreted as the horizontal velocity divergence, this value is also comparable with the observed value over the BOMEX area.

At this point, it is of some interest to compare our result eq. (18) with Mak's result eq. (32). By substituting eqs. (18) and (19) into eq. (7), W at $s = 0$ is given by

$$W_* = \frac{2[Q]}{c_p\beta} \tag{26}$$

or,

$$\left(\frac{dh}{ds}\right)_* = \frac{W_*}{2[u]_*} \tag{27}$$

On the other hand, Mak's (32) is written with our notations as

$$\left(\frac{dh}{ds}\right)_* = WG/2\,\frac{gh_*[Q]}{\theta_0 c_p} \tag{28}$$

By substituting G in eq. (8) into eq. (28), eq. (27) is recovered. It will be now clear why Mak overestimated $(dh/ds)_*$ by a factor of two or three. First of all, Mak's W is a prescribed quantity to be evaluated from directly or indirectly estimated values of ε, $\overline{(w'\theta')}_i$ and $\Delta\theta$ in eq. (10). He estimated $W \sim 0.7$ cm sec^{-1}. On the other hand, W estimated from eq. (26) is 0.52 cm sec^{-1}. Secondly, Mak assigned the values $G \sim 1.3 \times 10^{-2}$ cm sec^{-2} and $[Q]/c_p = 0.9$ K day^{-1}. He took this value of G from Holland and Rasmusson's (1973) estimate with the assumption that $(\partial\tau/\partial z)_i = 0$. However, within the framework of Mak's model, G and $[Q]/c_p$ are mutually related by eq. (8), and consequently should not be assigned independently, as Mak did. With $[Q]/c_p = 0.9$ K day^{-1} and $[u]_* = 5$ m sec^{-1} (as Mak assumed), eq. (8) gives $G = 1.0 \times 10^{-2}$ cm sec^{-2}. With $[u]_* = 7$ m sec^{-1} (as assumed in this article), G is further reduced to be 0.75×10^{-2} cm sec^{-2}. Thus, Mak's overestimate might stem from the use of large W and G, aside from not making use of the information above the inversion layer.

A quantitative description of the downstream variation of the Pacific trades was given by Malkus (1956), although the number of samplings of observations was much less than the BOMEX case. Her Fig. 1 indicates that, (i) the increase of the height of the inversion base is approximately 1 km over a distance of 2500 km, giving $dh/ds \sim 4 \times 10^{-4}$, (ii) the corresponding increase of potential temperature is about 6 K at the surface, decreasing slowly with height in most of the layer and rapidly decreasing to zero near the inversion base, (iii) the horizontal velocity at the upstream location is about 5 m sec^{-1} and the velocity also increases in the downstream direction, giving the horizontal divergence of about 2–3 $\times$ 10^{-7} sec^{-1}. If we take $d[\delta\theta]/ds \sim$ 5 K/2500 km $\sim 2 \times 10^{-8}$ K cm^{-1} and $[u] \sim$ 5 m sec^{-1}, eq. (19) gives $[Q]/c_p = 0.86$ K day^{-1}. With this value of $[Q]/c_p$ and $\beta = 4$ K km^{-1}, eq. (18) predicts $(dh/ds)_* = 5 \times 10^{-4}$, very close to the observed value referred to above. However, eq. (20) predicts $(d[u]/ds)_* \sim 2.5 \times 10^{-6}$ sec^{-1}, one order of magnitude larger than the observed value. The observed value of $d[u]/ds$ may not be typical to the eastern Pacific trade or the eastern Pacific trade may correspond to the third model in the previous section with u being nearly constant.

In summary, even though the result of the above analysis is gratifying in that the predicted downstream variations eqs. (18)–(20) agree fairly well with observations, the present work should be regarded at best as an introduction to the complex problem of the downstream variation of the lower trade-wind circulation. The discussion in the preceding paragraphs calls for the more precise estimate of the magnitude of parameters involved. We have ignored the vertical variation of Q and considered only $[Q]$ averaged over the entire sub-inversion layer. However, the recent estimate of Q by Nitta and Esbensen (1974) and Soong and Ogura (1976) using BOMEX data indicates that Q varies considerably with height, Q being negative in the lower portion of the sub-inversion layer, mainly due to the radiative cooling and positive in the upper portion because of the condensation heating.

We have also assumed that $[Q]$ is independent of the downstream distance. Our basic equations (7)–(9) permit horizontally homogeneous solutions, $h = h_*$, $[\delta\theta] = 0$, $d[u]/ds = W_*/h_*$, if $[Q] = 0$. This is why all rates of the downstream variations in eqs. (18)–(20) are proportional to $[Q]$. This is a reflection of the fact that the rates of the downstream variation depend upon the degree to which the dynamic and thermodynamic structures in the lower trade at an upstream location are locally balanced. We should have a more rigorous analysis on how h_* is determined, how the atmosphere is balanced locally and how the downstream variation of $|Q|$ affects the slope of the inversion.

More fundamentally, the hierarchy of the problem of the lower trade-wind circulation should be clearly recognized, as stressed in the earlier works (Malkus, 1956; Riehl & Malkus, 1957). On one hand, the maintenance of the dynamic and thermodynamic structures of the circulation are controlled by much smaller scale processes (heating and cooling by trade-wind cumuli and eddy stresses in the boundary layer). On the other hand, the lower trade-wind circulation, being a downward branch of the Hadley circulation, is controlled by the larger circulation. A modeling study, presented in this note, represents nothing but a low-level hierarchy of the problem in that some of the small scale processes and some of the features of the larger-scale circulation are prescribed.

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REFERENCES

Augstein, E., Riehl, H., Ostapoff, F. & Wagner, V. 1973. Mass and energy transports in the undisturbed Atlantic trade-wind flow. *Mon. Wea. Rev. 101*, 101–111.

Holland, J. Z. & Rasmusson, E. M. 1973. Measurements of the atmospheric mass, energy and momentum budgets over a 500-kilometer square of tropical ocean. *Mon. Wea. Rev. 101*, 44–55.

Mak, M.-K. 1976. A model study of the downstream variation of the lower trade-wind circulation. *Tellus, 28*, 97–107.

Malkus, J. S. 1956. On the maintenance of the trade winds. *Tellus, 8*, 335–350.

Nitta, T. & Esbensen, S. 1974. Heat and moisture budget analysis using BOMEX data. *Mon. Wea. Rev. 102*, 17–28.

Riehl, H. & Malkus, J. S. 1957. On the heat balance and maintenance of circulation in the trades. *Q. J. Roy. Meteor. Soc. 83*, 21–29.

Soong, S. T. & Ogura, Y. 1976. A determination of the trade-wind cumuli population using BOMEX data and an axisymmetric cloud model. *J. Atmos. Sci. 33*, 992–1007.

ЗАМЕЧАНИЕ О ПОДВЕТРЕННЫХ ВАРИАЦИЯХ ПАССАТНОЙ ЦИРКУЛЯЦИИ В НИЖНЕЙ ТРОПОСФЕРЕ

Модель Мака (1976) макроструктуры невозмущенной пассатной циркуляции модифицируется путем учета распределения потенциальной температуры в слое над пассатной инверсией. Предсказываемые модифицированной моделью подветренные вариации температуры, скорости ветра и высоты основания инверсии достаточно хорошо согласуются с соответствующими величинами, измеренными во время БОМЭКСа в течение невозмущенного периода.