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Atmospheric waves in the Mediterranean area on the basis of zonal harmonic analysis

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ABSTRACT

Characteristic features of the atmospheric waves with wavelengths smaller than 3000 km in the Mediterranean area are evaluated on the basis of zonal harmonic analysis of synoptic weather charts. The mean and extreme values of the amplitudes of the waves with wavenumbers from 1 to 5 are presented for the 1000, 850, 700 and 500 mb levels. The spectral distribution of the kinetic energy for the eastward and the northward components of the geostrophic wind for the same levels are computed, as well as the speed of the waves with different wavelengths.

1. Introduction

A number of authors have been studying the long atmospheric waves on the basis of zonal harmonic analysis (Eliasen, 1958; Deland, 1964; Julian et al., 1970; Kao & Kuczek, 1973). These studies are made by hemispherical weather charts where the atmospheric motion is presented as a zonal flow upon which more or less marked disturbances are superimposed. The disturbances, however, represent again a superposition of components with horizontal scales in a very wide range. The aforementioned studies contain quantitative analyses of the components with the largest scales (wavelengths larger than about 3000 km), performed by a decomposition into waves with different wavelengths in the zonal direction.

In this study the same method has been applied in the Mediterranean area in order to outline certain characteristic features related to the

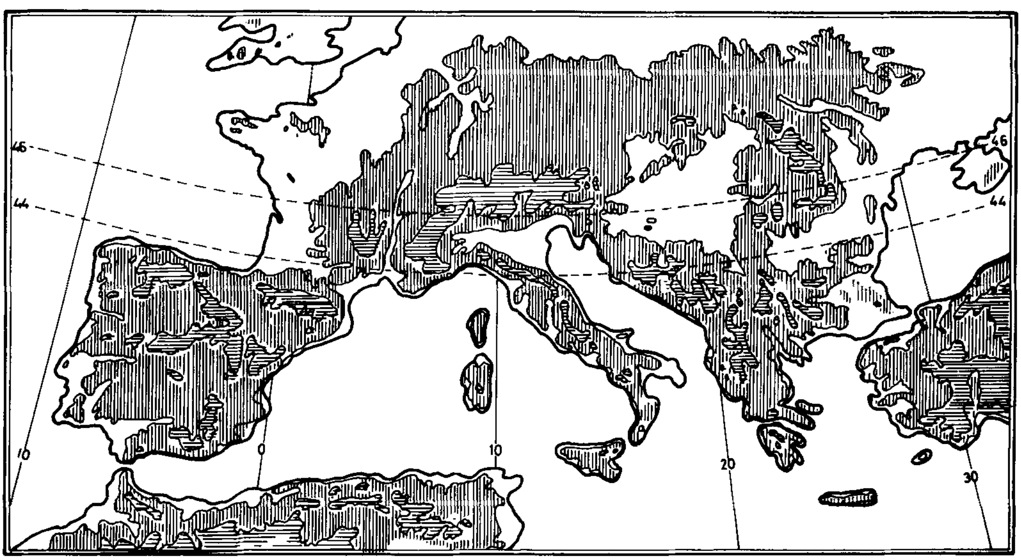

Fig. 1. Area in which the wave spectra is considered.

wavelengths below 3000 km, and to study the distribution of kinetic energy of such waves. The main philosophy is that very pronounced mountain barriers, such as the Alps and Carpathians, modify the large-scale atmospheric waves and create small-scale atmospheric waves. This explains the existence of several cyclogenetic and anti-cyclogenetic areas in the Mediterranean region (Radinović, 1965; Godev, 1972). The analysis in this paper has been carried out twice a day for the periods of the first half of January, April, July and October 1975, on the basis of the 1000, 850, 700 and 500 mb charts. The heights z of these pressure surfaces have been taken at the latitudes 44° and 46° N for each two degrees of longitude, starting from 6° W to 30° E. These latitudes cross the mountain ranges of Pyrenees, Alps, Dinarids and part of the Carpathians, as well as the cyclogenetic areas of the Gulf of Biscay, the Gulf of Genoa, the northern Adriatic, the Panonian Lowland, the Walachian Plains and the Black Sea (Fig. 1). The small-scale high-pressure systems over these mountains and frequent cyclone developments in the aforementioned cyclogenetic areas contribute to the short-wave part of the spectrum of the atmospheric waves.

2. The spectral distribution of wave amplitudes

The previously described longitudes in the length of 36° represent $\frac{1}{10}$ of the latitude circle around the globe. This part of the latitude circle has been taken as a unit of length. Then, by a zonal harmonic analysis, the height z can be expressed as

$$z(\phi, \lambda) = \bar{z}(\phi) + \sum_m [a_m(\phi) \cos m\lambda + b_m(\phi) \sin m\lambda]$$

where $m = 1, \ldots 5$, and $\bar{z}$ is the zonal mean value of z. Values of a_m and b_m are computed by approximating the integrals along the latitudes 44° and 46° N by the corresponding sums of 18 values of the heights. The amplitudes for the wave-numbers from 1 to 5 have been calculated by the expression

$$A_m = \sqrt{a_m^2 + b_m^2}$$

Tables 1 to 4 show the mean and extreme values of the amplitudes in different periods of the year. From these tables the following conclusions follow:

(1) The amplitudes increase from July to January. Further, it can be noted that the ratio between the mean values of A_m for these months is decreasing with an increase of the height and wavenumber, whereas the ratio between the mean values for all five wavenumbers, as well as for all four levels, for these two months is 0.48.

(2) The decrease of the amplitude is nearly proportional to the decrease of the wavelength.

(3) The mean amplitude for $m = 1$ taken for all four months is at 1000 mb three times and at 500 mb about five times larger than the mean amplitude for the same wavelengths and levels obtained by Eliasen (1958). This conclusion seems to be very important because it shows that the waves with the wavelengths of about 3000 km have approximately the same

Table 1. *Mean and extreme values of A_m (in m) for the period 1 to 15 January 1975*

		1000 mb			850 mb			700 mb			500 mb		
ϕ	m	Mean	Min.	Max.	Mean	Min.	Max.	Mean	Min.	Max.	Mean	Min.	Max.
46° N	1	51	6	116	53	11	100	59	12	136	96	13	248
	2	28	4	72	27	4	57	32	7	51	41	6	76
	3	14	3	39	17	1	36	22	3	46	27	1	56
	4	13	1	33	13	1	24	16	1	30	21	2	41
	5	10	0	23	12	1	22	14	2	25	18	2	35
44° N	1	43	7	112	51	7	99	69	9	135	102	20	241
	2	28	6	60	24	3	47	28	5	55	35	6	69
	3	14	2	28	16	2	32	19	1	41	24	5	49
	4	11	0	28	11	0	21	14	1	28	20	4	34
	5	10	3	22	10	0	22	12	1	25	15	3	34

Table 2. *Mean and extreme values of A_m (in m) for the period 1 to 15 April 1975*

		1000 mb			850 mb			700 mb			500 mb		
ϕ	m	Mean	Min.	Max.	Mean	Min.	Max.	Mean	Min.	Max.	Mean	Min.	Max.
46° N	1	39	4	116	48	7	95	70	4	130	118	27	199
	2	21	4	43	19	4	46	26	3	58	42	10	78
	3	12	1	28	12	1	32	16	2	36	25	2	52
	4	10	3	23	9	2	22	12	2	24	19	5	36
	5	8	1	18	8	1	20	9	1	24	15	1	31
44° N	1	40	9	105	52	5	102	71	6	138	118	28	188
	2	21	3	47	17	4	37	24	4	48	42	3	80
	3	12	3	24	12	1	31	16	2	38	24	6	51
	4	8	2	19	9	1	22	13	1	27	20	7	39
	5	8	1	23	6	1	14	10	0	20	15	3	33

Table 3. *Mean and extreme values of A_m (in m) for the period 1 to 15 July 1975*

		1000 mb			850 mb			700 mb			500 mb		
ϕ	m	Mean	Min.	Max.	Mean	Min.	Max.	Mean	Min.	Max.	Mean	Min.	Max.
46° N	1	29	1	104	35	7	100	40	12	102	54	12	142
	2	13	2	38	13	2	28	12	3	31	16	2	40
	3	8	3	21	8	2	17	9	2	20	9	2	21
	4	8	1	18	7	0	17	7	0	16	6	1	15
	5	5	1	13	6	1	14	6	1	11	5	1	13
44° N	1	26	3	89	34	5	92	44	4	109	65	16	163
	2	13	2	41	13	4	33	14	1	39	16	4	45
	3	7	2	18	7	1	14	8	1	16	9	3	23
	4	5	0	13	5	1	12	6	1	13	7	1	18
	5	5	0	13	5	0	13	5	1	12	5	2	18

Table 4. *Mean and extreme values of A_m (in m) for the period 1 to 15 October 1975*

		1000 mb			850 mb			700 mb			500 mb		
ϕ	m	Mean	Min.	Max.	Mean	Min.	Max.	Mean	Min.	Max.	Mean	Min.	Max.
46° N	1	29	6	62	35	3	75	51	6	100	87	22	183
	2	20	7	41	21	3	41	22	2	42	32	3	65
	3	10	3	30	11	3	24	14	1	31	20	7	48
	4	8	2	20	9	2	19	10	2	21	15	3	32
	5	8	2	18	8	1	14	9	1	17	12	1	24
44° N	1	30	5	80	34	2	84	48	4	98	80	12	160
	2	19	5	37	16	1	36	19	5	32	28	6	60
	3	10	0	26	11	1	21	12	2	23	18	2	43
	4	6	2	19	7	1	16	9	1	19	12	1	29
	5	6	0	15	6	1	12	8	2	17	12	0	26

amplitude as the planetary waves with wavelengths three to five times larger. Comparing, further, the mean values of the amplitudes for $m = 5$ in Tables 1 to 4 with the mean values of the amplitude for $m = 10$ in Eliasen's paper (1958), it can be seen that these values are very close. It means that waves in the Mediterranean area with a wavelength of about 600 km have the same intensity of amplitude as a planetary wave of the wavelength of 3000 km at the same latitude taken around the globe.

(4) In order to analyze the time variation of the amplitudes for five wavenumbers in January and July, these are presented in Figs. 2 and 3. From these figures it can be seen that in any 12-h period the amplitudes of all the wavenumbers at the 1000 mb level can be changed more than the amount of their mean values. At the 500 mb level, the 12-h variation of the amplitude cannot reach the mean value of the wave amplitude even after several days. Studying the time variations in different months it was found that, on the average, the amplitudes for all wavenumbers change rather fast up to 48 h, and after that the variations are practically insignificant.

(5) Weather situations in which the maximum intensity of the amplitude appears are characterized by strong cellular circulation in the lower part and pronounced wave circulation in the middle of the troposphere over the Mediterranean. The minimum intensity of the amplitude arises in weather situations

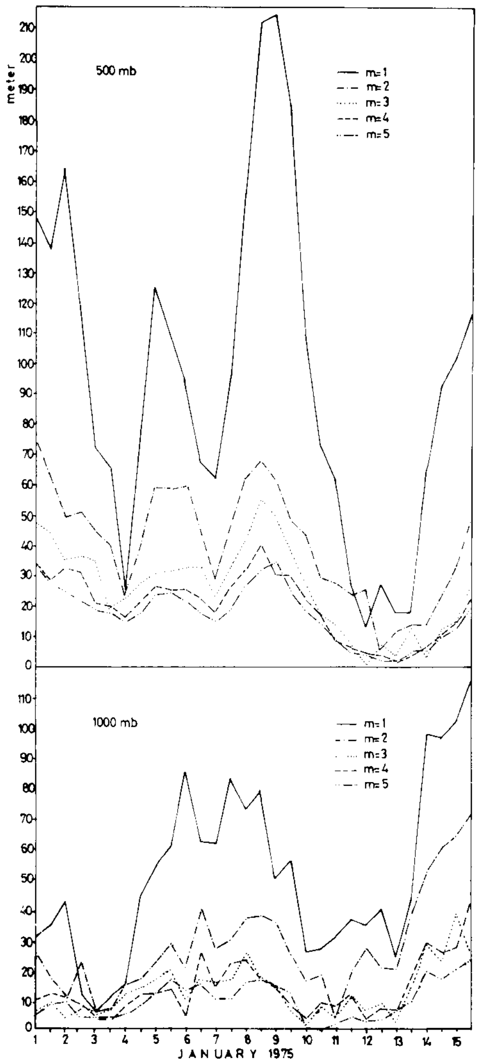

Fig. 2.

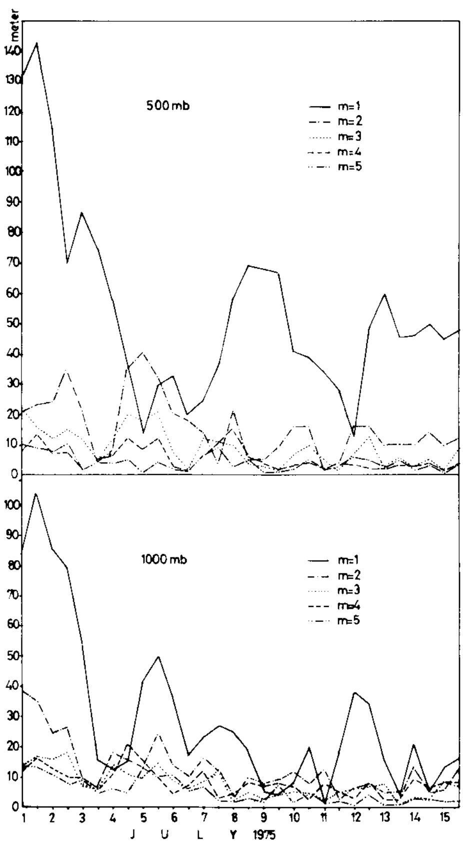

Fig. 3.

characterized by the zonal air flow or a very weak horizontal pressure gradient in the same area.

3. The spectral distribution of kinetic energy

The spectral distribution of the kinetic energy can be expressed as

$$\frac{1}{2\Pi}\int_0^{2\Pi} \tfrac{1}{2}(u^2+v^2)\,d\lambda = E_0^{(u)} + \sum_m [E_m^{(u)} + E_m^{(v)}]$$

with

$$E_0^{(u)} = \tfrac{1}{2}\bar{u}^2, \quad E_m^{(u)} = \tfrac{1}{4}[a_m^{(u)^2} + b_m^{(u)^2}],$$

$$E_m^{(v)} = \tfrac{1}{4}[a_m^{(v)^2} + b_m^{(v)^2}]$$

These quantities have been computed at the latitude 45° N, using the formulas

$$a_m(45°) = [\tfrac{1}{2}a_m(46°) + a_m(44°)]$$

$$\frac{\partial a_m}{\partial \phi} = \frac{90}{\Pi}[a_m(46°) - a_m(44°)]$$

and analogous expressions for $b_m(45°)$ and $\partial b_m/\partial\phi$. The mean and extreme values of $E_0^{(u)}$ for the first 15 days of January, April, July and October 1975 are shown in Table 5. For the same periods the 12-h variations of $E_0^{(u)}$ are shown in Table 6.

From Table 5 it can be seen that $E_0^{(u)}$ is increasing with height in all seasons. However, in some cases the mean values of $E_0^{(u)}$ at the 850 mb level are smaller than at 1000 mb. The increase of $E_0^{(u)}$ with height is especially pronounced between the 700 and 500 mb levels. Comparing the mean values of $E_0^{(u)}$ in different months it is seen that the energy of the zonal flow in the lower part of the troposphere is larger in January than in July. In the higher part of the troposphere the situation is reversed. It is also evident that the mean values of $E_0^{(u)}$ in April and October are greater than in January and July.

Regarding the extreme values of $E_0^{(u)}$, the maximum values arise, as one can expect, in the situation of strong zonal flow over the Mediterranean area, and the minimum appears in the situations of pronounced meridional circulation or small horizontal pressure gradient.

Table 5. *Mean and extreme values of $E_0^{(u)}$ in m^2 sec^{-2}*

	Months											
	I			IV			VII			X		
Mb	Mean	Min.	Max.	Mean	Min.	Max.	Mean	Min.	Max.	Mean	Min.	Max.
1000	21	1	84	16	0	53	5	0	41	20	0	87
850	18	0	72	30	0	97	7	0	32	15	0	48
700	22	0	84	54	8	134	18	0	75	34	0	142
500	39	0	226	162	16	459	45	0	237	101	0	231
Average	25	0	116	66	6	186	19	0	96	42	0	172

Table 6. *Mean and extreme values of 12-h $E_0^{(u)}$ variations in m^2 sec^{-2}*

	Months											
	I			IV			VII			X		
Mb	Mean	Min.	Max.	Mean	Min.	Max.	Mean	Min.	Max.	Mean	Min.	Max.
1000	22	1	82	15	0	47	6	0	41	20	1	77
850	12	0	69	21	0	80	7	0	32	15	0	40
700	15	0	55	25	0	104	19	0	69	34	0	104
500	28	0	106	95	9	238	33	0	174	101	2	102
Average	19	0	78	39	2	117	16	0	79	43	1	81

Table 7. *Mean values of* $E_m^{(u)}$ *and* $E_m^{(v)}$ *in* $m^2\ sec^{-2}$

m	Component of energy	January 1000 mb	January 850 mb	January 700 mb	January 500 mb	April 1000 mb	April 850 mb	April 700 mb	April 500 mb	July 1000 mb	July 850 mb	July 700 mb	July 500 mb	October 1000 mb	October 850 mb	October 700 mb	October 500 mb
1	E_u	19.9	10.3	12.6	30.4	15.8	14.5	18.8	28.7	5.6	7.2	11.3	26.7	12.9	8.9	9.6	24.4
	E_v	0.3	0.4	0.7	1.5	0.2	0.4	0.7	1.8	0.2	0.2	0.3	0.5	0.1	0.2	0.4	1.0
2	E_u	8.5	3.2	3.8	5.9	5.3	2.7	5.2	6.0	2.8	2.7	3.4	5.1	4.6	4.1	4.7	7.5
	E_v	0.5	0.4	0.5	0.8	0.2	0.2	0.4	1.0	0.1	0.1	0.1	0.2	0.2	0.2	0.2	0.5
3	E_u	5.4	2.1	2.1	3.1	6.3	2.2	2.2	3.1	2.6	2.3	2.4	3.1	3.3	2.4	1.9	5.5
	E_v	0.2	0.4	0.6	0.9	0.2	0.2	0.4	0.8	0.1	0.1	0.1	0.1	0.1	0.2	0.2	0.4
4	E_u	3.3	1.3	1.5	1.8	3.0	1.5	1.4	3.6	1.7	1.1	2.0	2.0	2.4	1.3	1.3	2.3
	E_v	0.4	0.3	0.5	1.0	0.2	0.2	0.4	0.9	0.1	0.1	0.1	0.1	0.1	0.1	0.2	0.4
5	E_u	1.2	1.0	0.8	0.9	2.3	0.9	1.1	1.4	0.7	1.2	1.2	0.9	1.8	1.0	1.0	1.5
	E_v	0.4	0.4	0.6	1.0	0.2	0.2	0.4	0.8	0.1	0.1	0.1	0.1	0.2	0.2	0.2	0.5
	Σ_1^5	40.1	19.8	23.7	47.3	33.7	23.0	31.0	48.1	14.0	15.1	21.0	38.8	25.7	18.6	19.7	44.0

Table 8. *Relative (R) and absolute (A) mean velocities of the 12-h wave displacements (in km)*

Isobaric surface	ϕ'o		January 1	2	3	4	5	April 1	2	3	4	5	July 1	2	3	4	5	October 1	2	3	4	5
1000 mb	46	*R*	154	−13	101	69	−12	134	170	197	189	73	246	106	102	2	185	172	87	−5	26	78
		A	186	211	334	232	225	286	320	318	356	506	337	451	520	354	379	272	258	271	265	229
	44	*R*	55	80	−23	52	51	112	282	129	81	118	199	158	258	234	136	172	144	218	193	150
		A	175	218	285	303	173	235	376	357	318	369	278	411	513	554	516	252	259	373	523	369
850 mb	46	*R*	65	45	140	170	43	145	94	108	−23	133	232	54	97	165	37	33	−11	88	−33	142
		A	92	97	220	250	181	211	283	330	213	317	294	448	438	431	431	221	241	257	223	284
	44	*R*	55	161	15	69	44	63	2	196	−6	60	225	34	232	141	165	133	83	24	73	2
		A	112	244	192	272	177	215	315	399	345	408	324	356	397	430	435	263	441	324	297	417
700 mb	46	*R*	51	53	119	45	101	50	44	35	48	6	236	69	175	265	129	17	−26	−7	−81	62
		A	100	99	216	139	209	168	197	316	162	267	263	298	406	423	442	211	214	251	183	179
	44	*R*	93	57	140	−94	104	58	42	76	27	85	184	61	139	83	196	156	−40	0	104	12
		A	143	115	223	220	220	199	183	272	204	273	214	328	427	548	349	238	242	209	250	179
500 mb	46	*R*	48	41	47	68	49	56	48	45	43	48	235	56	101	212	15	87	−42	−66	65	22
		A	89	126	117	105	108	133	163	197	115	151	263	329	340	552	421	232	200	198	192	219
	44	*R*	142	52	156	−23	48	54	−1	46	39	43	211	299	150	177	193	123	−41	−24	−27	−12
		A	161	139	186	147	128	146	221	182	154	163	246	368	431	483	519	218	226	241	213	237

Comparing the mean values of $E_0^{(u)}$ in Table 5 with the similar values obtained by Eliasen (1958) it can be seen that the kinetic energy contained in the zonal flow around the globe at 500 mb is about two times larger than at the same latitude in the Mediterranean area. It is likely that the very high mountain barriers deflect the zonal flow, destroying it and creating the small-scale wave disturbances.

From Table 6 it is apparent that the mean variation of $E_0^{(u)}$ in the 12-h intervals is nearly the same as the average amount of the kinetic energy of the zonal flow. At the same time, in this table it is shown that the limits in which $E_0^{(u)}$ varies in the 12-h intervals are rather large, and goes from zero to amounts which are several times larger than its mean value. Further, the mean variations of $E_0^{(u)}$ in April and October are more than two times greater than in January and July.

The spectral distributions of kinetic energy are shown in Table 7. $E_m^{(u)}$ represents the contributions from the eastward and $E_m^{(v)}$ from the northward component of the geostrophic wind. From this table, it can be seen that $E_m^{(v)}$ is one to two orders of magnitude smaller than the $E_m^{(u)}$ component. These results can be compared with Eliasen's (1958). He determined, for $m = 1$ and 2, that essentially more energy is contained in the u-component than in the v-component of the velocity, whereas for $m \geqslant 4$ most of the energy is contained in the v-component.

In the same paper (Eliasen, 1958) it is shown that $E_m = E_m^{(u)} + E_m^{(v)}$ has a maximum for $m = 5$, which at 47.5 N nearly corresponds to the wavelength 5400 km, and for the 500 mb level is about 20 m^2 sec^{-2}.

From Table 7 it is seen that E_m for $m = 1$ (wavelength about 3000 km) exceeds this value. At the same time it may be seen that the total kinetic energy of the waves with $1 \leqslant m \leqslant 4$ and the waves with $5 \leqslant m \leqslant 10$ obtained by Eliasen is of the same order of magnitude, as well as the total kinetic energy presented in Table 7 for the group of waves with wavelengths smaller than 3000 km.

Finally, it could be noted from Table 7 that the u-component is decreasing rapidly with increasing wavenumber, especially from $m = 1$ to $m = 2$, whereas the v-component seems to be independent of m. At the same time it is evident that the kinetic energy for all waves concerned has its maximum in April and minimum in July.

4. The motion of the waves

The displacement of the different waves can be described by

$$a_m(\phi, t) \cos m\lambda + b_m(\phi, t) \sin m\lambda = A_m(\phi, t) \cos m[\lambda - \delta_m(\phi, t)]$$

where $\delta_m(\phi, t)$ is phase. The relative (R) and absolute (A) mean velocities of the 12-h wave displacements are shown in Table 8. The relative velocity denotes the average value calculated from eastward (+) and westward (−) movements of the waves, whereas absolute velocity denotes the average value calculated from the whole wave displacements without regard to sign.

From Table 8 we can see:

(1) all waves at all levels and in all four months, on average, are moving eastward;

(2) the greatest absolute velocity has wave $m = 3$, and slowest wavenumber $m = 1$;

(3) the speed of all waves and in all months is decreasing with the height.

5. Conclusion

From these comparisons and analyses it seems that in regions with large mountain barriers the wave study could be of particular interest, because the waves with a wavelength smaller than 3000 km can have the amplitude and kinetic energy of the same order of magnitude as the planetary waves with wavelengths three to five times larger.

6. Acknowledgements

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АТМОСФЕРНЫЕ ВОЛНЫ НАД ОБЛАСТЬЮ СРЕДИЗЕМНОМОРЬЯ НА ОСНОВЕ АНАЛИЗА ЗОНАЛЬНЫХ ГАРМОНИК

На основе зонального гармонического анализа синоптических карт погоды области Средиземноморья оцениваются характерные особенности атмосферных волн длиною меньше 3000 км. Для уровней 1000, 850, 700 и 500 мб представлены средние и предельные величины амплитуд волн с волновыми числами от 1 до 5. Вычисляются спектральные распределения кинетической энергии для западной и южной компонент геострофического ветра на тех же уровнях, так же как и скорости волн с различными длинами волн.